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A

PRACTICAL TREATISE

ON

ARITHMETIC,

WHEREIN

EVERY PRINCIPLE TAUGHT IS EXPLAINED IN A SIMPLE AND
OBVIOUS MANNER;

CONTAINING

NUMEROUS QUESTIONS,

AND

COMBINING THE USEFUL PROPERTIES OF FORMER WORKS
WITH THE MODERN IMPROVEMENTS;

BEING

A COMPLETE SYSTEM

TO WHICH IS ADDED

TWO METHODS OF BOOK-KEEPING;

WITH EXAMPLES FOR EXERCISE.

BY GEORGE LEONARD, JR.

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Evander H. Sawyer

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Evander E. Lacy

How many ones or units make 10? 10 and how many ones or units make 12? 10 and how many units make 13? 14? 16? 19? 15? 18? 17? 11?

How many tens are there in 20? In 30? 40? 60? 80? 70? 50? 90? 100?

How many tens and units are there in 21? In 23? 28? 26? 32? 35? 37? 44? 49? 41? 53? 57? 62? 65? 68? 71? 76? 79? 85? 87? 88? 92? 94? 99?

Observe that thirteen is a contraction of three and ten; fourteen, of four and ten; fifteen, of five and ten; sixteen, of six and ten; seventeen, of seven and ten; eighteen, of eight and ten; and nineteen, of nine and ten.

Twenty is a contraction of two tens; thirty, of three tens; forty, of four tens; fifty, of five tens; sixty, of six tens; seventy, of seven tens; eighty, of eight tens; and ninety, of nine tens.

What do you call 10 and 1? 10 and 3? 10 and 7? 10 and 9? 2 tens? 2 tens and 1? 2 tens and 5? 2 tens and 7? 3 tens? 3 tens and 2? 3 tens and 8? 4 tens? 4 tens and 6? 5 tens? 5 tens and 3? 5 tens and 5? 6 tens? 6 tens and 4? 7 tens? 8 tens? 8 tens and 6? 9 tens? 9 tens and 2? 9 tens and 9? 10 tens?

How do you write one, two, three, &c., up to ten, in figures, on your slate? How do you write in figures, on your slate, fourteen, sixteen, seventeen, twenty, twenty-seven, thirty, thirty-three, thirty-six, forty, forty-one, forty-five, fifty, fifty-two, fifty-four, sixty, sixty-nine, seventy, seventy-one, seventy-eight, eighty, eighty-three, eighty-four, ninety, ninety-six, one hundred?

LESSON 2.

If we have a great many things to number, say a large quantity of silver dollars, we count out a heap containing one hundred in the preceding manner; we then count out another heap of one hundred, making with the first, two hundred, and so proceed counting out heaps of one hundred dollars each, making three hundred, four hundred,

---

What is thirteen a contraction of? Fourteen? Seventeen? Fifteen? Nineteen? Sixteen? Eighteen?

What is twenty a contraction of? Thirty? Sixty? Eighty? Forty? Seventy? Fifty? Ninety?

five hundred, six hundred, seven hundred, eight hundred, nine hundred, till we get ten hundred, which is called a thousand.

Ten heaps of a thousand dollars each make ten thousand; ten heaps of ten thousand each make one hundred thousand; and ten heaps of one hundred thousand each make ten hundred thousand, or a thousand thousand, called a million.

Ten heaps of a million each make ten millions; ten heaps of ten millions each make one hundred millions; and ten heaps of one hundred millions each make one thousand millions, called a billion.

In like manner, a thousand billions make a trillion; a thousand trillions a quatrillion, and so we go on to quintillions, sextillions, septillions, octillions, nonillions, decillions, &c., each number being a thousand times the preceding one.

How many dollars are there in 2 heaps of a thousand dollars each? 3 heaps? 4 heaps? 5 heaps? 6 heaps? 7 heaps? 8 heaps? 9 heaps? 10 heaps?

How many dollars are there in 2 heaps of ten thousand dollars each? 3 heaps? 4 heaps? 5 heaps? 6 heaps? 7 heaps? 8 heaps? 9 heaps? 10 heaps?

How many dollars are there in 2 heaps of one hundred thousand dollars each? 3 heaps? 4 heaps? 5 heaps? 6 heaps? 7 heaps? 8 heaps? 9 heaps? 10 heaps?

Make one thousand marks on your slate. Show how many make twenty-five, fifty, seventy-five, ninety-seven, one hundred and seventeen, one hundred and sixty, one hundred and eighty-four, two hundred, two hundred and seven, three hundred, four hundred, five hundred, five hundred and fifty, six hundred, seven hundred, seven hundred and seventy, eight hundred, eight hundred and fifty, nine hundred.

How should we proceed to count a thousand silver dollars?

How many do ten heaps of one thousand dollars each make? Of ten thousand dollars each? Of one hundred thousand dollars each? Of a million each? Of ten millions each? Of one hundred millions each?

How many billions make a trillion? How many trillions a quatrillion? How then do we go on? How large is each number?

One hundred and one is made in figures thus	101
One hund. and two	102
One hund. and three	103
One hund. and four	104
One hund. and five	105
One hund. and six	106
One hund. and seven	107
One hund. and eight	108
One hund. and nine	109
One hund. and ten	110
One hund. and eleven	111
One hund. and twelve	112
One hund. and thirteen	113
One hund. and fourteen	114
One hund. and twenty-three	123
One hund. and thirty-seven	137
One hund. and forty	140
One hund. and fifty-six	156
One hund. and eighty-one	181
Two hundred	200
Two hund. and one	201
Two hund. and two	202
Two hund. and thirty-one	231
Three hundred	300
Three hund. and seventy-six	376
Four hundred	400
Five hundred	500
Six hundred	600
Seven hundred	700
Eight hundred	800
Nine hundred	900
One thousand	1,000
Two thousand five hundred	2,500

Lesson 3.

The preceding numbers, and all others, are expressed by only ten figures ; figures 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. 0 always stands for nothing, and is called *nought*.

If you have some cents, and write a figure 5 to express the number, how many will you appear by this to have ?

Does a single figure then, say 5, stand for five units, five tens, five hundreds, or what ?

When you write any single figure, say 1, what do you mean, one unit, one ten, one hundred, or what ? What do you mean when you write 2 ? 3 ? 4 ? 5 ? 6 ? 7 ? 8 ? 9 ? 0 ?

Write fourteen on your slate in figures. What does the 4 at the right hand side stand for ? What does the 1 stand for, one unit, one ten, one hundred, or what ?

How do you make one hundred and one in figures, on your slate ? One hundred and two ? So proceed with questions through the other numbers, and embrace some of the intermediate numbers not here expressed.

By how many figures are the preceding numbers and all others expressed ? What are these figures ? What does 0 always stand for ? What is it called ?

Write ten on your slate in figures. Does the 0 stand for any number? What does the 1 stand for?

Write thirty-seven on your slate in figures. What does the 7 stand for? What does the 3 stand for?

Suppose that instead of writing 37 you write 73, what number do you express? What does the 3 now stand for? The 7?

Write one hundred and thirty-eight on your slate in figures. What does the 8 stand for? The 3? The 1? On which side of 138 do you find the 8 units, on the right hand side, or on the left hand side? How many figures left of the units do you find the 3 tens? The 1 hundred?

Write one hundred on your slate in figures. There being no odd ones or units, what stands in the units' place? There being no odd tens, what stands in the tens' place? What does the 1 stand for?

Write six hundred and four on your slate in figures. What does the 4 stand for? What shows there are no tens? What does the 6 stand for?

Write one thousand on your slate in figures. There being no odd units, tens, or hundreds, what stands in the units' place, the tens' place, and the hundreds' place? What does the 1 stand for? How many figures left of the units is the 1 in the thousands' place?

When one figure stands by itself, as 6, what does it mean, 6 units, 6 tens, 6 hundreds, or what?

A figure standing at the right of another, or of others, as 8 in 958, is units, tens, hundreds, or what? A figure standing one place left of units, as 5 in 958, is units, tens, hundreds, or what? A figure standing two places left of units, as 9 in 958, is units, tens, hundreds, or what? A figure standing three places left of units, as 2 in 2,500, is units, tens, hundreds, or what?

So we find the units at the right hand side of a number, the tens one place left of units, the hundreds two places left of units, and the thousands three places left of units.

LESSON 4.

How many times is 1 in 10? How many times is 10 in 100? How many times is 100 in 1,000? How many times

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In what part of a number do we find the units? The tens? The hundreds? The thousands?

is 2 in 20 ? How many times is 3 in 30 ? 4 in 40 ? 5 in 50 ? 6 in 60 ? 7 in 70 ? 8 in 80 ? 9 in 90 ?

How many times is 20 in 200 ? How many times is 30 in 300 ? 40 in 400 ? 50 in 500 ? 60 in 600 ? 70 in 700 ? 80 in 800 ? 90 in 900 ?

In 11, how much more does the 1 in the tens' place stand for than the 1 in the units' place ?

In 22, how much more does the 2 in the tens' place stand for than the 2 in the units' place ?

In 55, how much more does the 5 in the tens' place stand for than the 5 in the units' place ?

In 99, how much more does the 9 in the tens' place stand for than the 9 in the units' place ?

In 330, how much more does the 3 in the hundreds' place stand for than the 3 in the tens' place ?

In 880, how much more does the 8 in the hundreds' place stand for than the 8 in the tens' place ?

It is just so in all cases ; therefore

A figure at the left of another, stands for ten times as much as it would in the place of that other figure.

From the preceding principles we see that

7 stands for seven.
27 stands for twenty-seven.
127 stands for one hundred and twenty-seven.
5,127 stands for five thousand, one hundred and twenty-seven.
35,127 stands for thirty-five thousand, one hundred and twenty-seven.
835,127 stands for eight hundred and thirty-five thousand, one hundred and twenty-seven.
4,835,127 stands for four millions, eight hundred and thirty-five thousand, one hundred and twenty-seven.

A figure at the left of another stands for how much more than it would in the place of that other figure ?

How do you make twenty-seven, in figures, on your slate ? One hundred and twenty-seven ? Five thousand, one hundred and twenty-seven ? Thirty-five thousand, one hundred and twenty-seven ? Eight hundred and thirty-five thousand, one hundred and twenty-seven ? Four millions, eight hundred and thirty-five thousand, one hundred and twenty-seven ?

NUMERATION TABLE.

The following Numeration Table shows plainly the manner in which figures increase in value as they are placed further and further to the left.

Names of the places occupied by each figure; begin at ONES OR UNITS, and read up.		The way to read this number.
QUATRILLIONS.	6,	Six quatrillions,
Hundreds of trillions.	4	four hundred and
Tens of trillions.	8	eighty-
TRILLIONS.	5,	five trillions,
Hundreds of billions.	0	and
Tens of billions.	2	twenty-
BILLIONS.	7,	seven billions,
Hundreds of millions.	9	nine hundred and
Tens of millions.	1	thir-
MILLIONS.	3,	teen millions,
Hundreds of thousands.	5	five hundred and
Tens of Thousands.	4	forty
THOUSANDS.	0,	thousand,
Hundreds.	2	two hundred and
Tens.	7	seventy-
ONES OR UNITS.	6	six.

Numbers to be easily read, should be divided like these, by commas, into parts of three figures each, beginning at units.

The ten figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, together with the present manner of using them, were invented in India, many years ago. Some knowledge of them was given us by the Arabs, about 800 years since, they having obtained their information from the Indians. All civilized people now reckon with these figures.

Copy the Numeration Table on your slate, and name the places occupied by each figure, beginning at ones or units. Read the figures in the table you have copied, beginning at six quatrillions.

How should numbers be divided to be easily read?

Where were the ten figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 invented? Who gave us some knowledge of them? How long since? From whom did they obtain their information? What people now reckon with these figures?

Lesson 5.

Copy the following numbers on your slate in words.

1. 444
2. 708
3. 2,010
4. 10,000
5. 100,000
6. 1,000,000
7. 9,045,275
8. 6,004
9. 20,001
10. 370,004
11. 5,000,000
12. 67,000,000
13. 4,895,476
14. 77,000,691
15. 397,538,411
16. 6,348,745,877
17. 45,001
18. 1,112
19. 200,004
20. 75,000,473,000
21. 800,111,000,333
22. 3,756,235,974,212
23. 111,111,222
24. 505,505,505,555
25. 30,303
26. 10,001
27. 476,766
28. 45,000,000
29. 347
30. 901

When you write a number in figures that has no odd units, as one hundred and fifty, what do you put in the units' place? When there are no odd tens, as in one hundred and five, what do you put in the tens' place. When there are no odd hundreds, as in four thousand and forty, what do you put in the hundreds' place? When there are no odd thousands, as in forty thousand, five hundred and twelve, what do you put in the thousands' place?

If 0 stands at the left of a figure, say of figure five; thus, 05, how many tens are there in these figures? How many units? Can the 0 be omitted, then, without changing the value of the number?

If you put 0 at the right of figure five, thereby causing the 5 to stand in the tens' place; thus, 50, what do these figures now stand for? How many times 5 do they stand for? If you put 0 at the right of 50; thus, 500, what do these figures stand for? How many times 50 do they express?

Therefore, 0 *placed at the left of a number, has no effect, and may be omitted; placed at the right of a number, it increases it ten times.*

What effect has 0 placed at the left of a number? Placed at the right of a number?

LESSON 6.

Copy the following numbers on your slate in figures, and divide them into parts so as to be easily read.

1. Six hundred and five.
2. Nine hundred and eighty-three.
3. Five thousand, four hundred and thirty-two.
4. Eight thousand, and ninety-five.
5. Forty thousand, and one.
6. Sixty-five thousand.
7. Eighty-one thousand, two hundred and twelve.
8. One hundred and twenty-five thousand, two hundred.
9. Four hundred and seventy-seven thousand, eight hundred.
10. One million.
11. One million, one thousand, five hundred.
12. Fourteen millions, seven hundred and five thousand, six hundred and forty-one.
13. Eighty-seven millions, two hundred and seventy-five thousand, one hundred and twelve.
14. Three hundred and thirty-three thousand, and three.
15. Nine hundred millions.
16. Two billions, seventeen millions, one thousand, and thirteen.
17. Twenty-five billions.
18. Four hundred and forty billions, three hundred and twenty-one thousand.
19. Eight trillions, two hundred thousand.
20. Five hundred and forty-seven trillions, three hundred and eleven millions, eight hundred and thirty-four thousand, two hundred and eighty-eight.
21. Two thousand, three hundred and four.
22. Fifty-five trillions, eight hundred and seven billions.
23. Six hundred trillions, four hundred and fourteen billions, and three.
24. Forty-five billions, three hundred and twenty-seven millions.
25. Ten thousand, and ten.
26. One hundred thousand, two hundred.
27. Forty-four thousand, four hundred and forty-four.
28. Five billions, and two.
29. Twenty-seven millions.
30. Four thousand, two hundred and twelve.

LESSON 7.

There is another kind of numbers which it will be well to understand. The following table explains these numbers, and their use.

If you are making snow-balls, after you finish
One, that will be the first you make ; after you finish

Two, the last will be	the second
Three	the third
Four	the fourth
Five	the fifth
Six	the sixth
Seven	the seventh
Eight	the eighth
Nine	the ninth
Ten	the tenth
Eleven	the eleventh
Twelve	the twelfth
Thirteen	the thirteenth
Fourteen	the fourteenth
Fifteen	the fifteenth
Sixteen	the sixteenth
Seventeen	the seventeenth
Eighteen	the eighteenth
Nineteen	the nineteenth
Twenty	the twentieth
Twenty-one	the twenty-first
Twenty-two	the twenty-second
Twenty-three	the twenty-third
Thirty	the thirtieth
Forty	the fortieth
Fifty	the fiftieth
Sixty	the sixtieth
Seventy	the seventieth
Eighty	the eightieth
Ninety	the ninetieth
One hundred	the one hundredth
One hundred and one	the one hundred and first

If you are making snow-balls, after you finish one, will that be the first, second, or third, that you make? After you finish two, what will the last be? Three? Four? So proceed with questions through the other numbers.

One hundred and two	the one hundred and second
One hundred and three	the one hundred and third
Two hundred	the two hundredth
Three hundred	the three hundredth
One thousand	the one thousandth
Two thousand	the two thousandth
Ten thousand	the ten thousandth
Ten thousand five hundred and twenty-six	the ten thousand five hundred and twenty-sixth
One hundred thousand	the one hundred thousandth
One million	the one millionth

These numbers are called *ordinal numbers*; they are often employed in books and conversation, though not directly in any calculation.

If you are in a class, standing on the floor, and the master counts the scholars in the class, beginning at the head, and you are counted nine, what will be your place in the class; the first, second, or what ? What will your place in the class be if you are counted 5 ? 6 ? 10 ? 13 ?

If the page in the book where you are reading, is marked 31, will that page be the first, second, or third, in the book ? What page will it be ? If the page is marked 52, what page of the book will it be ? What if it is marked 45 ? 28 ? 14 ? 12 ? 8 ? 2 ? 60 ? 78 ? 91 ? 101 ? 300 ? 500 ? 756 ? 1,000 ?

A man counted out 10,000 dollars ; what do you call the last dollar ; the one hundredth, the one thousandth, or what ? What do you call the last dollar after he has counted out 12,000 dollars ? 13,571 ? 100,212 ? 625,471 ? 8,000,000 ?

What are these numbers called, and how employed?

ADDITION.

LESSON 8.

To be performed in the mind.

Note. The learner may be taught to reckon with his fingers, or what is better, with twenty marks, arranged by five and tens on his slate; thus.

||||| ||||| ||||| |||||

1. If you have 2 quills in one hand, and 3 in the other, how many have you in both hands? How many then are 2 and 3? How many are 3 and 2?

2. James has 4 cents in a box; if he puts 4 cents more into the box, what number will it then contain?

3. A laborer worked for me 5 hours one day, and 3 hours the next; how long did he work for me in both days? 5 and 3 are how many then? 3 and 5?

4. 4 men engaged on a piece of work, were joined by 6 more; how many were engaged on it then?

5. I have 5 cents, and my brother has 8 cents; how many have we both? What number of cents have we both, if I have 8 cents, and my brother 5 cents?

6. There are 7 sheep in one part of a pasture, and 7 in another part; how many would there be if they were all together? What number then do 7 and 7 make?

7. A little girl bought a picture book for 8 cents, some paper for 3 cents, and a quill for 2 cents; how much must she pay for the whole?

8. William had 9 apples, when he found 3 more; how many did he then have?

9. If you buy a barrel of flour for 6 dollars, and a plough for 9 dollars, how many dollars must you pay for both?

10. A farmer, after selling 10 pounds of butter to a man, had 5 pounds left; what quantity had he at first? What is the sum of 10 and 5 then? Of 5 and 10?

LESSONS 9 AND 10.

ADDITION TABLE.

Note. Questions in this table should not be asked in rotation, because when they are so asked the learner can answer by merely counting, without the least exertion of memory.

In one part of the table we find 2 and 4 are 6, and in another part, 4 and 2 are 6. Each operation in the table is repeated, in this way, except when 1 is added to a number, or a number is added to itself. Make the learner observe this.

2 and 1 are 3
2 and 2 are 4
2 and 3 are 5
2 and 4 are 6
2 and 5 are 7
2 and 6 are 8
2 and 7 are 9
2 and 8 are 10
2 and 9 are 11
2 and 10 are 12

3 and 1 are 4
3 and 2 are 5
3 and 3 are 6
3 and 4 are 7
3 and 5 are 8
3 and 6 are 9
3 and 7 are 10
3 and 8 are 11
3 and 9 are 12
3 and 10 are 13

4 and 1 are 5
4 and 2 are 6
4 and 3 are 7
4 and 4 are 8
4 and 5 are 9
4 and 6 are 10
4 and 7 are 11
4 and 8 are 12
4 and 9 are 13
4 and 10 are 14

5 and 1 are 6
5 and 2 are 7
5 and 3 are 8
5 and 4 are 9
5 and 5 are 10
5 and 6 are 11
5 and 7 are 12
5 and 8 are 13
5 and 9 are 14
5 and 10 are 15

6 and 1 are 7
6 and 2 are 8
6 and 3 are 9
6 and 4 are 10
6 and 5 are 11
6 and 6 are 12
6 and 7 are 13
6 and 8 are 14
6 and 9 are 15
6 and 10 are 16

7 and 1 are 8
7 and 2 are 9
7 and 3 are 10
7 and 4 are 11
7 and 5 are 12
7 and 6 are 13
7 and 7 are 14
7 and 8 are 15
7 and 9 are 16
7 and 10 are 17

8 and 1 are 9
8 and 2 are 10
8 and 3 are 11
8 and 4 are 12
8 and 5 are 13
8 and 6 are 14
8 and 7 are 15
8 and 8 are 16
8 and 9 are 17
8 and 10 are 18

9 and 1 are 10
9 and 2 are 11
9 and 3 are 12
9 and 4 are 13
9 and 5 are 14
9 and 6 are 15
9 and 7 are 16
9 and 8 are 17
9 and 9 are 18
9 and 10 are 19

10 and 1 are 11
10 and 2 are 12
10 and 3 are 13
10 and 4 are 14
10 and 5 are 15
10 and 6 are 16
10 and 7 are 17
10 and 8 are 18
10 and 9 are 19
10 and 10 are 20

LESSON 11.

To be performed in the mind.

1. John bought an apple for 3 cents, and an orange for 4 cents; how many cents did he pay for both?

2. If you have 4 walnuts in your pocket, and 5 in your hat, how many have you in both places?

3. Henry's father gave him 6 cents, and his sister gave him 6 more? how many had he then?

4. A man walked 7 miles in the forenoon, and 4 in the afternoon; how far did he walk during the day? How many then are 7 and 4? 4 and 7?

5. What is the sum of 2 and 5? 4 and 3? 4 and 8? 5 and 6? 6 and 7? 8 and 7? 8 and 9? 9 and 3? 9 and 6? 9 and 9? 10 and 7? 10 and 9?

6. Mark caught 4 speckled trout in a little pond, 3 in a brook, and a boy gave him 5; how many had he then?

7. If you have 6 marbles in your hat, 2 in one pocket, 2 in another, and 5 in your hand, how many will you have if they are all put into a heap?

8. If you buy a picture book for 11 cents, and a top for 4 cents, how many cents must you pay for them?

9. What sum do 12 and 7 make? 15 and 3 and 2? 19 and 2? 24 and 5 and 6 and 2? 37 and 6? 52 and 8? 66 and 2? 70 and 9? 88 and 6? 93 and 3 and 4?

10. A little boy gathered 54 cherries from one tree, 9 from another, and he picked up 6 from the ground; how many had he then?

LESSON 12.

To be performed in the mind.

1. A man had 10 dollars in his pocket, when one of his neighbors paid him 13 dollars; what number of dollars had he then? *Explanation.* How many are 10 and 10 and 3?

2. How many are 10 and 11? 10 and 15? 10 and 19? 10 and 27? 10 and 33? 10 and 46? 10 and 59? 10 and 62? 10 and 78? 10 and 81? 10 and 95? 10 and 110? 10 and 120? 10 and 359? 10 and 476?

3. Mary bought a quire of paper for 20 cents, and a book for 30 cents; how many cents must she pay for the paper and book? *Explanation.* How many tens are 2 tens and 3 tens? What are 5 tens called?

4. If you ride 50 miles in one day, 30 miles the next, and 10 miles the next, how far do you ride in the three days ?

5. A company of 50 men were joined by 60 more ; how many were there then ? *Explanation.* What are 10 tens called ? What then are 10 tens and 1 ten, making 11 tens, called ?

6. What is the sum of 40 and 70 ? 80 and 90 ? 90 and 90 ? 60 and 120 ; that is, 6 tens and 12 tens ?

7. Alexander found 17 apples under one tree, and 20 under another ; if he puts these with 5 and 2, how many will there be ?

8. If a man has 30 acres of land, and buys 45 more, how many will he then have ?

9. 24 bales of cotton are piled up with 38 ; how many bales do both quantities make ? *Explanation.* 4 and 8 are how many ? 2 tens and 3 tens are how many ? How many then are 5 tens and 12 ?

10. What is the sum of 11 and 17 ? 21 and 43 ? 52 and 38 ? 82 and 91 ? 77 and 85 ? 40 and 180 ? 300 and 400 ? 9,000 and 500 ?

Lesson 13.

For the Slate.

1. A man paid 213 dollars to various creditors on Monday, 402 dollars on Tuesday, and 21 dollars on Wednesday ; what was the whole sum paid ?

OPERATION.

Hundreds.	Tens.	Units.
2	1	3
4	0	2
	2	1
6	3	6 dollars. Answer.

Explanation. We first write these numbers under one another, with units under units, tens under tens, &c., and draw a line beneath. We now add the units, proceeding from the bottom up ; thus, 1 and 2 are 3 and 3 are 6 ; this 6 is placed under the column of units. The tens are added in the same way, and their sum placed under the column of tens, and so on with the hundreds.

In example 1, lesson 13, how do we write the numbers to be added ? What is the next thing done, before beginning to add ? What do we add first ? How do we proceed in adding them ? Where do we place the sum of the units ? What is said of the tens and hundreds ?

2. If I have 100 pounds of butter, buy 201 pounds more of one farmer, 322 of another, and receive 1,265 in payment of a debt, what number of pounds do I then have ? Ans. 1,888.

3. A farmer sold 3 barrels of cider to one trader, 2 barrels to another, 3 barrels to a neighboring farmer, and 1 to his blacksmith ; how many barrels did he sell to all of them ? Ans. 9.

4. A young farmer began business on 23 acres of land, he soon after bought 12 acres, and in a year or two more obtained 113 by inheritance; how many acres had he then ? Ans. 148.

5. Four partners furnished money to purchase merchandise, as follows. The first 30,112 dollars, the second 23,010, the third 14,234, and the fourth 2,322 ; how much did they all furnish ? Ans. 69,678 dollars.

LESSON 14.

1. A rich farmer had four pieces of land ; on the first he had 646 sheep, on the second 300, on the third 29, and on the fourth 127; how many sheep had he on all the pieces?

OPERATION.

Hundreds. Tens. Units.
6 4 6
3 0 0
2 9
1 2 7
———
1,1 0 2 sheep. Ans.

Explanation. The column of units, when added, makes 22 ; that is, 2 tens and 2 units ; we put the 2 odd units in the units' place, and carry the 2 tens to the next column, and add them with the other tens. These 2 tens and the others make 10 tens, that is, 1 hundred ; there being no odd tens, we put 0 in the tens' place, and carry the 1 hundred to the next column, and add it with the other hundreds. This 1 hundred and the other hundreds, make 11 hundred ; that is, 1 thousand and 1 hundred ; the 1 hundred being written in the hundreds' place, and the 1 thousand in the thousands' place finishes.

In example 1, lesson 14, the column of units, when added, making 22, that is, 2 tens and 2 units, what do we put in the units' place? What is done with the 2 tens? The sum of the tens being 10; that is, 10 tens, or 1 hundred, what do we put in the tens' place? Why? What is done with the 1 hundred? The sum of the hundreds being 11, that is, 1 thousand and 1 hundred, what do we put in the hundreds' place? What is done with the 1 thousand?

2. William had 8 walnuts in one pocket, 9 in another, and besides these he had 15 in his trunk, and 7 in the table drawer; what was the whole number he had? Ans. 39.

3. A dealer in lumber has 37,276 feet of boards in one pile, 9,536 in another, 45,092 lying on his wharf, and 8,870 in a vessel; how many feet of boards has he in all these places? Ans. 100,774.

4. A merchant owes one man in Boston 975 dollars, one in New York 483, another in New York 237, one in New Orleans 87, and various other persons 689; what is the amount of all his debts? Ans. 2,471 dollars.

5. If you buy a yoke of oxen for 75 dollars, a cart for 57, three cows for 88, and a plough for 10, how much must you pay for the whole? Ans. 230 dollars.

LESSON 15.

From what precedes we get the following

RULE FOR ADDITION.

Write the numbers to be added under one another, with units under units, tens under tens, &c., and draw a line beneath. Add the right hand column from the bottom upwards; place the units in the sum of the column beneath it, and carry all the tens one place to the left. So proceed to add up and carry in all the columns.

If you add the work twice to prove its correctness, you will not be apt to detect a mistake, if you proceed the second time the same as the first; thus, if you are adding the units in example 1, lesson 14, and say, by mistake, 7 and 9 are 15, and 6 are 21, you will be very liable to say 7 and 9 are 15 the second time, instead of saying 7 and 9 are 16. We can prove the work, therefore,

By adding each column from the top downwards, proceeding in other respects as before. If the second sum be equal to the first, the work will generally be right.

Note. Each example should now be proved.

How do you write the numbers to be added? What do you add first? What do you do with the units in the sum of the column? With the tens? How then do you proceed?

If you add the work twice to prove its correctness, when will you not be apt to detect a mistake? Explain this by example 1, lesson 14. How can we prove the work in Addition?

Numbers to add.	Numbers to add.	Numbers to add.
(1.)	(2.)	(3.)
456321783	576377	88745
283683001	6892203	3431
477054212	7203578	48032
336864326	4349	23324
	643481	73782

LESSON 16.

Numbers to add.	Numbers to add.	Numbers to add.
(1.)	(2.)	(3.)
858	230007321	5
375	562077899	37
9	125766232	463
4507	440488552	5398
23	234711143	93424
1		643779
456		29944
678		

4. A merchant owns a sloop worth 1,000 dollars, a schooner worth 5,675 dollars, a brig worth 8,340 dollars, and a ship worth 12,345 dollars; what is the value of all these vessels? Ans. 27,360 dollars.

5. A man owns one farm containing 45 acres, a second containing 18 acres, a third containing 156 acres, a fourth containing 225 acres, and a fifth containing 9 acres; what number of acres are contained in the five farms? Ans. 453.

6. Benjamin Franklin was born in the year 1706, and was 84 years old when he died; in what year did he die? Ans. in 1790.

7. A teamster hauled a quantity of wheat in different loads containing the following numbers of bushels; 34, 28, 27, 32, 42, 29, 33, 35, and 30; what was the whole quantily hauled? Ans. 290 bushels.

8. Add the following numbers, 3,575,412, 900, 8, 27, 8,208, 450,275, 633, 44, 65,000, and 1,225. Ans. 4,101,732.

9. A merchant paid 15,255 dollars for a store, 9,237 dollars for a vessel, 12,676 dollars for goods, and 275 dollars for a horse and chaise; how much did he pay for the whole? Ans. 37,443 dollars.

10. The owner of a coal mine agreed to furnish some iron manufacturers with one hundred and sixty-five thou-

sand, two hundred bushels of coal; a merchant, with forty-seven thousand, six hundred and ninety-five bushels; another person with eight thousand, two hundred and seventy-nine bushels, and he wants himself three thousand, five hundred and forty-five bushels; what quantity will supply himself and the others? Ans. 224,719 bushels.

Lesson 17.

1. A farmer bought an ox wagon for 135 dollars, a yoke of oxen for 75 dollars, three ploughs for 27 dollars, six cows for 96 dollars, and thirteen sheep for 65 dollars; what sum of money must he pay for the whole? Ans. 398 dollars.

2. If a man be 27 years old when his first son is born, of what age will he be when his son is 21 years old? Ans. 48 years.

3. According to the census of 1830, Maine contained 399,462 inhabitants, New Hampshire 269,533, Vermont 280,679, Massachusetts 610,014, Rhode Island 97,210, and Connecticut 297,211; how many inhabitants were there in all of these states, which are called the New England States? Ans. 1,954,109.

4. If I pay 85 dollars for a gold watch, 37 for a coat, 6 for a hat, 5 for a pair of boots, and 275 for a horse and chaise, what does the whole cost me? Ans. 408 dollars.

5. A merchant sold a lot of coffee for 2,327 dollars, and lost 637 dollars on it; how much did the coffee cost him? Ans. 2,964 dollars.

6. Charles paid twenty-five cents for some paper, fifty-eight cents for a penknife, twelve cents for some quills, and seventy-five cents for a pair of gloves; how much did he pay for the whole? Ans. 170 cents.
As 100 cents make a dollar, how many dollars and cents did he pay for the whole?

7. What is the sum of 7, 87,455,383, 67,914,533, 29, 456, and 500,000. Ans. 155,870,408.

8. A merchant bought a brig for 6,236 dollars; he paid 614 dollars for repairs on the hull, and 869 dollars for repairs on the rigging; for what price must he sell the brig to gain 325 dollars? Ans. 8,044 dollars.

9. If your debts to different persons are as follows, 2,756 dollars, 1,000 dollars, 75 dollars, 467 dollars, 395 dollars, and 5,832 dollars, how much is the whole that you owe? Ans. 10,525 dollars.

10. If you travel 115 miles from Portland to Boston, 42 miles from Boston to Providence, 186 miles from Providence to New York, 145 miles from New York to Albany, 16 miles from Albany to Schenectady, and 24 miles from Schenectady to Saratoga; how far do you travel in going from Portland to Saratoga? Ans. 528 miles.

SUBTRACTION.

Lesson 18.

To be performed in the mind.

1. If you have 5 cents, and give away 2, how many will you have left? 2 from 5 leaves how many then? 3 from 5 leaves how many?

2. Edwin took 3 walnuts from a heap that contained 6; how many did he leave?

3. A man walked 7 miles from home, and afterwards returned 4 miles; how far was he then from home? How many then does 4 from 7 leave? Why does 4 from 7 leave 3? *Answer.* Because 4 and 3 are 7.

4. Maria counted 8 robins on a tree, but 3 shortly flew away; how many remained? 3 from 8 leaves what number then? Why?

5. I had 9 cents, but soon after spent 4 of them for some apples; how many did I keep?

6. A boy bought a pear for 3 cents, and handed the seller a 10 cent piece; how many cents must he receive back?

7. A fisherman returning home with 12 shad, sold 5 of them by the way; how many were left? How many does 5 from 12 leave then? 7 from 12?

8. If you have 16 walnuts, and your brother has 6, which has the most, and how many?

9. A farmer who had 14 cows, sold 5? how many did he keep?

10. A ship had a crew of 20 men, but 10 deserted before she sailed; how many remained? 10 from 20 leaves how many then? Why?

LESSONS 19 AND 20.

SUBTRACTION TABLE.

Note. Questions in this table should not be asked in rotation, because when they are so asked the learner can answer by merely counting, without the least exertion of memory.

In one part of the table we find 3 from 8 leaves 5, and in another part, 5 from 8 leaves 3. Each operation in the table is repeated in this way, except when the number left is 1, or is the same as the number subtracted. Make the learner observe this.

2 from 3 leaves 1
2 from 4 leaves 2
2 from 5 leaves 3
2 from 6 leaves 4
2 from 7 leaves 5
2 from 8 leaves 6
2 from 9 leaves 7
2 from 10 leaves 8
2 from 11 leaves 9
2 from 12 leaves 10

3 from 4 leaves 1
3 from 5 leaves 2
3 from 6 leaves 3
3 from 7 leaves 4
3 from 8 leaves 5
3 from 9 leaves 6
3 from 10 leaves 7
3 from 11 leaves 8
3 from 12 leaves 9
3 from 13 leaves 10

4 from 5 leaves 1
4 from 6 leaves 2
4 from 7 leaves 3
4 from 8 leaves 4
4 from 9 leaves 5
4 from 10 leaves 6
4 from 11 leaves 7
4 from 12 leaves 8
4 from 13 leaves 9
4 from 14 leaves 10

5 from 6 leaves 1
5 from 7 leaves 2
5 from 8 leaves 3
5 from 9 leaves 4
5 from 10 leaves 5
5 from 11 leaves 6
5 from 12 leaves 7
5 from 13 leaves 8
5 from 14 leaves 9
5 from 15 leaves 10

6 from 7 leaves 1
6 from 8 leaves 2
6 from 9 leaves 3
6 from 10 leaves 4
6 from 11 leaves 5
6 from 12 leaves 6
6 from 13 leaves 7
6 from 14 leaves 8
6 from 15 leaves 9
6 from 16 leaves 10

7 from 8 leaves 1
7 from 9 leaves 2
7 from 10 leaves 3
7 from 11 leaves 4
7 from 12 leaves 5
7 from 13 leaves 6
7 from 14 leaves 7
7 from 15 leaves 8
7 from 16 leaves 9
7 from 17 leaves 10

8 from 9 leaves 1
8 from 10 leaves 2
8 from 11 leaves 3
8 from 12 leaves 4
8 from 13 leaves 5
8 from 14 leaves 6
8 from 15 leaves 7
8 from 16 leaves 8
8 from 17 leaves 9
8 from 18 leaves 10

9 from 10 leaves 1
9 from 11 leaves 2
9 from 12 leaves 3
9 from 13 leaves 4
9 from 14 leaves 5
9 from 15 leaves 6
9 from 16 leaves 7
9 from 17 leaves 8
9 from 18 leaves 9
9 from 19 leaves 10

10 from 11 leaves 1
10 from 12 leaves 2
10 from 13 leaves 3
10 from 14 leaves 4
10 from 15 leaves 5
10 from 16 leaves 6
10 from 17 leaves 7
10 from 18 leaves 8
10 from 19 leaves 9
10 from 20 leaves 10

LESSON 21.

To be performed in the mind.

1. Augustus had 5 apples, but he soon after dropped 3 and lost them ; how many had he left ?

2. Benjamin has 6 cents and Samuel 4 ; which has the most ? How many the most ?

3. If you have 7 pears, and give your sister 3 of them, how many will you have left ? 3 from 7 leaves how many then ? 4 from 7 ?

4. I have 5 dollars ; how many more must I get to have 8 ? 5 from 8 leaves how many then ? Why ?

5. What is the difference between 4 and 8 ? 4 and 6 ? 5 and 9 ? 4 and 7 ? 8 and 3 ? 8 and 6 ? 9 and 5 ? 10 and 7 ? 6 and 12 ? 13 and 7 ? 15 and 8 ? 9 and 17 ?

6. My father gave me 5 cents and my brother 4, but I soon after lost 3 of them ; how many did I then have ?

7. A boy having 10 cents, spent 2 cents for some apples, and 6 cents for a little book; how many cents did he keep ?

8. Julia has 16 plums; how many will be left if she gives away 4 ?

9. If you take 7 dollars from 15, how many will be left ? How many will be left if you take 3 from 11 ? 9 from 19 ? 3 from 24 ? 6 from 33 ? 8 from 41 ? 5 from 55 ? 9 from 62 ? 2 from 70 ? 4 from 85 ? 7 from 105 ? 3 from 120 ?

10. George's mother gave him 75 cents ; after he had spent 4 cents and 5 cents, how many had he left ?

LESSON 22.

To be performed in the mind.

1. If you have 23 marbles, and lose 10, how many will you have left ? *Explanation.* 1 ten from 2 tens and 3 leaves how many ?

2. What remains after taking 10 apples from 15 ? 10 from 19 ? 10 from 27 ? 10 from 32 ? 10 from 46 ? 10 from 54 ? 10 from 63 ? 10 from 75 ? 10 from 81 ? 10 from 99 ? 10 from 100 ? 10 from 108 ? 10 from 215 ? 10 from 455 ? 10 from 1,000 ?

3. A trader sold 20 pounds of sugar out of a box that contained 30 pounds ; how much was left ? *Explanation.* 2 tens taken from 3 tens leave how many ?

4. A farmer set out 40 trees in an orchard, and his neighbor 70; which set out the most, the farmer or his neighbor? How many the most?

5. What is the difference between 30 and 60? 20 and 50? 60 and 80? 30 and 45? 40 and 78? 50 and 120; that is, between 5 tens and 12 tens? 70 and 150?

6. A grocer who had 20 cheeses sold 15; how many had he left? *Explanation.* 5 from 20 leaves how many? 10 from 15 leaves how many?

7. If you have 40 cents, and pay away 25 of them for a book, how many will you have left?

8. A man who lives 16 miles from Boston, travelled 12 miles towards that city; how far was he from it then?

9. If you take 19 eggs from a basket that contains 32, how many will you leave?

10. If you take 13 chestnuts from 18, how many will you leave? How many will you leave if you take 21 from 37? 32 from 40? 26 from 35? 57 from 72? 62 from 70? 200 from 300? 150 from 230?

LESSON 23.

For the Slate.

1. A young man having 665 dollars, paid 464 dollars for a piece of woodland; how much money had he left?

OPERATION.

$$\begin{array}{r} 665 \\ 464 \\ \hline 201 \end{array}$$
dollars. Ans.

Explanation. We first write 464, the smaller number, under 665 the larger, with units under units, tens under tens, &c., and draw a line beneath. We now say 4 from 5 leaves 1, we place this 1 under the units, and so proceed to take 6 tens from 6 tens, and 4 hundreds from 6 hundreds; thus, 6 from 6 leaves 0, 4 from 6 leaves 2.

The smaller number, which we subtract, is called the *subtrahend*, the larger number the *minuend*, and the number obtained, the *difference* or *remainder;* thus, in the preceding example, 464 is the subtrahend, 665 the minuend, and 201 the difference or remainder.

How do we proceed in example 1, lesson 23, to subtract 464 from 665? What is called the subtrahend? Minuend? Difference or remainder? In example 1, lesson 23, what is the subtrahend? Minuend? Difference or remainder?

2. A planter, who had 96 bales of cotton, sent 45 of them to Charleston ; how many were left ? Ans. 51.

3. If you have 688 mill-logs, and get 133 of them sawed, what number will remain ? Ans. 555.

4. A man who had 45,647 feet of boards, sent away 21,546 feet; what quantity did he keep ? Ans. 24,101 feet.

5. A merchant, worth 8,946 dollars, lost 805 dollars in a speculation ; how much was he then worth ?
Ans. 8,141 dollars.

6. A farmer in Michigan raised 1,507 bushels of wheat, and sold 1,103 bushels ; what quantity did he retain ?
Ans. 404 bushels.

Lesson 24.

1. A man who had 43 acres of land, sold 26 acres ; how many acres did he keep ?

OPERATION.
4 3
2 6
——
1 7 acres. Ans.

Explanation. Being unable to take 6 units from 3, we borrow 1 of the 4 tens, add it to 3, and take 6 from 13 ; having now only 3 tens left in the upper number, we take 2 tens from 3 tens.

Another better way. Being unable to take 6 units from 3, we add 10 to 3, and take 6 from 13 ; as we have added 1 ten to the upper number, to balance it, we now add 1 ten to the lower number, and take 3 tens from 4 tens. We proceed in subtracting thus ; 6 from 13 leaves 7, 1 carried to 2 makes 3, 3 from 4 leaves 1.

2. A farmer raised 636 bushels of potatoes, and after saving 457 bushels for himself, sold the remainder ; how many bushels did he sell ? Ans. 179.

Explanation. Add 1 hundred, or 10 tens, to 3 tens, in order to subtract, and then carry 1 hundred to the 4 hundred.

3. A ship had 35 persons on board, including the passengers and crew ; how many passengers were there if the crew consisted of 9 men ? Ans. 26.

4. If you have 8,005 dollars, and buy a farm for 5,126 dollars, how much money will you have left ?
Ans. 2,879 dollars.

Explain how we subtract 26 from 43, in example 1, lesson 24. How do we proceed in subtracting 26 from 43?

5. A company of merchants commenced trade with one hundred and twelve thousand three hundred and five dollars; at the end of a year they had eighty-one thousand three hundred and seven dollars remaining; how much had they lost? Ans. 30,998 dollars.

LESSON 25.

From what precedes we get the following

RULE FOR SUBTRACTION.

Write the smaller number under the larger, with units under units, tens under tens, &c., and draw a line beneath. Subtract the units in the lower number from the figure above, and put the remainder directly below; proceed in the same way with the tens, hundreds, &c. If a figure in the lower number be larger than the figure above, add 10 *to the one above, subtract, and then carry* 1 *to the next figure or place below.*

If you take 3 walnuts from a heap that contains 8, there will be 5 left. If you now add the 3 walnuts to the remainder 5, there will be 8 in the heap again.

Therefore, as we sometimes make a mistake in subtracting, we can prove the work,

By adding the smaller number to the remainder; if the sum be equal to the larger number, the work will generally be right.

1. Subtract 673 from 1,782.

OPERATION.	PROOF.	
1 7 8 2	To 1 1 0 9	remainder,
6 7 3	Add 6 7 3	smaller number,
1,1 0 9 remainder.	1,7 8 2	same as the greater number.

Note. Each example should now be proved.

(2.)		(3.)	
From	6 0 0 2 5	From	3 2 4 5 7 3 8 1 9 7 4 6
take	4 6	take	1 4 7 2 3 3 5 1 8 8 6 9

How do you write the numbers in Subtraction? What do you subtract first? Where do you put the remainder? How then do you proceed? What if a figure in the lower number be larger than the figure above?

If you take 3 walnuts from a heap that contains 8, how many will be left? If you now add the 3 walnuts to the remainder 5, how many will there be in the heap again?

How, then, can we prove the work in Subtraction?

(4.)	(5.)
From 5 1 0 0 0 2	From 1 7 4 8 3 3
take 1 2 0 0 0 5	take 5 2 0 0 3

Lesson 26.

1. From 64,321,535,488,327 take 10,035,780,321,199.

2. From 9,000,052 take 1,643,215.

3. From 750,081 take 3,011.

4. 81 men were engaged in quarrying stone; after a number were discharged, there were 33 left; how many were discharged? Ans. 48.

5. George Washington died in the year 1799, aged 67 years; in what year was he born? Ans. in 1732.

6. A man who had 11,375 dollars in his possession, paid all his debts, amounting to 3,287 dollars; how much money had he left? Ans. 8,088 dollars.

7. By how much do 48,323 dollars exceed 19,295 dollars? Ans. 29,028 dollars.

8. A brickmaker had a kiln containing 43,000 bricks; after he had sold 25,376, how many remained? Ans. 17,624.

9. If you buy 275 bushels of wheat for 400 dollars, and sell 125 bushels for 198 dollars, how many bushels have you left, and what have they cost you?

Ans., you have 150 bushels left, and they have cost you 202 dollars.

10. If you have 23 apples, and a boy gives you 16 more, how many will you have after giving your sister 15, and your brother 8? Ans. 16.

Lesson 27.

1. A merchant who had 5,635 pounds of coffee, sold 1,210 pounds to one man, and 3,145 to another; how much had he then left? Ans. 1,280 pounds.

2. A tree seventy-five feet high, was broken off by the wind, and the part that fell was thirty-nine feet long; how high was the stump left standing? Ans. 36 feet.

3. A grocer sold a quantity of butter for 103 dollars, and made 27 dollars on it; how much did it cost him?

Ans. 76 dollars.

4. How many years is it since the Pilgrims landed at Plymouth, in the year 1620?

5. If a clerk collects 2,750 dollars from one person, and 1,385 from another, and pays out of it 275 dollars to one man, and 323 to another, how much will be left?

Ans. 3,537 dollars.

6. In 1830 France contained 32,052,465 inhabitants, and the British Empire 22,297,621; how many more people had France than the British Empire? Ans. 9,754,844.

7. If I travel 2,332 miles from home, and then return 1,421 miles, how far am I from home then? Ans. 911 miles.

8. A man gave 15,235 dollars for a ship, and sold her for 12,250 dollars, how much did he lose?
Ans. 2,985 dollars.

9. If you buy a horse and chaise for 235 dollars, and sell the chaise for 147 dollars, what will be the cost of the horse to you? Ans. 88 dollars.

10. There is a bin that holds 2,000 bushels; if there are 9 bushels of corn in it, how many more will it hold?
Ans. 1,991 bushels.

MULTIPLICATION.

LESSON 28.

To be performed in the mind.

1. If one orange costs 3 cents, what will 2 oranges cost? 2 times 3 are how many then? 3 times 2 are how many?

2. Mary recites 3 lessons a day; how many will she recite in 5 days?

3. A merchant bought 4 barrels of flour at 6 dollars a barrel; what sum must he pay for the whole? What number then are 4 times 6? 6 times 4?

4. What shall I pay for 6 yards of cloth, at 2 dollars a yard?

5. What sum must you pay for 7 weeks' board, at 3 dollars a week?

6. A man walked 4 miles an hour for 8 hours; how far did he go in that time? 8 times 4 are how many then? 4 times 8.?

7. George bought 5 camels' hair pencils at 5 cents apiece; what did he pay for them?

8. A gardener gave 9 children 2 apples apiece; what number did they all receive?

9. How many days are there in 10 weeks, there being 7 days in a week?

10. What sum must I give for 6 pounds of sugar at 8 cents a pound? How many are 6 times 8 then? 8 times 6?

LESSONS 29 AND 30.

MULTIPLICATION TABLE.

In one part of the table we find 4 times 7 are 28, and in another part, 7 times 4 are 28. Each operation in the table is repeated in this way, except when a number is multiplied by 1, or itself. Make the learner observe this.

2 times 1 are 2
2 times 2 are 4
2 times 3 are 6
2 times 4 are 8
2 times 5 are 10
2 times 6 are 12
2 times 7 are 14
2 times 8 are 16
2 times 9 are 18
2 times 10 are 20

3 times 1 are 3
3 times 2 are 6
3 times 3 are 9
3 times 4 are 12
3 times 5 are 15
3 times 6 are 18
3 times 7 are 21
3 times 8 are 24
3 times 9 are 27
3 times 10 are 30

4 times 1 are 4
4 times 2 are 8
4 times 3 are 12
4 times 4 are 16
4 times 5 are 20
4 times 6 are 24
4 times 7 are 28
4 times 8 are 32
4 times 9 are 36
4 times 10 are 40

5 times 1 are 5
5 times 2 are 10
5 times 3 are 15
5 times 4 are 20
5 times 5 are 25
5 times 6 are 30
5 times 7 are 35
5 times 8 are 40
5 times 9 are 45
5 times 10 are 50

6 times 1 are 6
6 times 2 are 12
6 times 3 are 18
6 times 4 are 24
6 times 5 are 30
6 times 6 are 36
6 times 7 are 42
6 times 8 are 48
6 times 9 are 54
6 times 10 are 60

7 times 1 are 7
7 times 2 are 14
7 times 3 are 21
7 times 4 are 28
7 times 5 are 35
7 times 6 are 42
7 times 7 are 49
7 times 8 are 56
7 times 9 are 63
7 times 10 are 70

8 times 1 are 8
8 times 2 are 16
8 times 3 are 24
8 times 4 are 32
8 times 5 are 40
8 times 6 are 48
8 times 7 are 56
8 times 8 are 64
8 times 9 are 72
8 times 10 are 80

9 times 1 are 9
9 times 2 are 18
9 times 3 are 27
9 times 4 are 36
9 times 5 are 45
9 times 6 are 54
9 times 7 are 63
9 times 8 are 72
9 times 9 are 81
9 times 10 are 90

10 times 1 are 10
10 times 2 are 20
10 times 3 are 30
10 times 4 are 40
10 times 5 are 50
10 times 6 are 60
10 times 7 are 70
10 times 8 are 80
10 times 9 are 90
10 times 10 are 100

LESSON 31.

To be performed in the mind.

1. Alfred has 3 little boxes, in each of which he has 4 cents; how many cents has he?

2. Nathan has 6 plums, and Robert 5 times as many; how many has Robert?

3. A man bought 7 sheep, at 4 dollars apiece; how many dollars did he give for all of them?

4. If a picture book costs 9 cents, what will 6 such books cost? How many then are 6 times 9? 9 times 6?

5. How many are 3 times 2? 2 times 4? 2 times 9? 3 times 7? 5 times 4? 8 times 3? 5 times 6? 6 times 6? 7 times 4? 8 times 9? 9 times 6? 9 times 9?

6. A farmer bought 3 pigs of one man, and 4 of another, paying 3 dollars apiece for them; what did they all cost him?

7. A boy had 2 melons, which he sold at 10 cents apiece; what sum did he get for them? How many are 3 times 10? 5 times 10? 4 times 10? 8 times 10? 7 times 10? 6 times 10? 9 times 10?

8. There are 2 rows of plum trees in a garden, with 20 trees in a row; how many trees are there in both rows?

9. If you buy 6 books at 40 cents apiece, how many cents must you pay for them? *Explanation.* 6 times 4 tens are how many tens? What sum do 10 tens make? What sum do 20 tens, that is, 2 times 10 tens make? What sum then do 2 hundred and 4 tens make?

10. How many are 2 times 30? 2 times 50? 3 times 20? 4 times 30? 6 times 50? 5 times 80? 7 times 20? 9 times 30? 2 times 200? 4 times 400?

LESSON 32.

To be performed in the mind.

1. A laborer earned 2 dollars a day for 10 days; how much did his wages amount to in that time?

2. How many are 10 times 3? 10 times 6? 10 times 4? 10 times 5? 10 times 9? 10 times 7? 10 times 8? 10 times 10?

3. Joseph has 20 filberts, and Daniel 10 times as many; what number has Daniel?

4. How many are 10 times 30? 10 times 40? 10 times 70? 10 times 80? 10 times 60? 10 times 50? 10 times 90? 10 times 100? 10 times 25? 10 times 44?

5. 2 men were paid 16 dollars each; what sum did they both receive? *Explanation.* 2 times 6 are how many? 2 times 1 ten are how many tens? How many, then, are 2 tens and 12, or 2 tens, 1 ten, and 2?

6. If a team hauls 32 bushels of corn in one load, how many bushels can it haul in 2 loads?

7. How many lines are there in 5 pages of your book? if there are 15 lines in a page?

8. A boy has 4 25-cent pieces; how many cents are they worth?

9. There are 63 gallons in a hogshead; how many gallons are there in 2 hogsheads?

10. How many are 3 times 11? 2 times 12? 4 times 21? 3 times 38? 5 times 22? 6 times 13? 2 times 91? 8 times 42?

LESSON 33.

For the Slate.

1. 4 sons inherited 352 dollars apiece; how much did they all inherit?

OPERATION.

$$\begin{array}{r} 352 \\ 4 \\ \hline 1{,}408 \end{array}$$

dollars. Ans.

Explanation. We first write the smaller number under the larger, with units under units, and draw a line beneath. We now say 4 times 2 are 8; and place the 8 under the units; then 4 times 5 tens are 20 tens, or 2 hundred. There being no odd tens, we put 0 in the tens' place, carry the 2 hundred one place to the left, and add it to the 12 hundreds which are obtained by multiplying the 3 hundreds by 4, thereby making 14 hundred, or 1 thousand 4 hundred. The 4 hundred we put in the hundreds' place, and the 1 thousand we carry to the thousands' place.

The lower number, which we multiply by, is called the *multiplier*, the upper number, which we multiply, the *multiplicand*, and the number produced the *product*; thus, in

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In example 1, lesson 33, how do we proceed to multiply 352 by 4? What is called the multiplier? Multiplicand? Product?

the preceding example, 4 is the multiplier, 352 the multiplicand, and 1,408 the product.

2. How many pounds are there in 15 2-pound weights ? Ans. 30.

3. How much flour is there in 9 barrels, each of which contains 196 pounds ? Ans. 1,764 pounds.

4. There are 5,280 feet in one mile, and 3 miles in one league ; now how many feet are there in 6 leagues. Ans. 95,040.

5. Suppose that there are one hundred and eighty-five millions of people in Europe, and that the Earth contains four times as many ? how many are there on the Earth ? Ans. 740,000,000.

Lesson 34.

1. How much will it cost to build a road 127 miles long, at 705 dollars a mile ?

OPERATION.

$$\begin{array}{r} 705 \\ 127 \\ \hline 4935 \\ 1410 \\ 705 \\ \hline 89{,}535 \end{array}$$

dollars. Ans.

Explanation. We multiply first by the 7 units, then by the 2 tens, and as the product is ten times as much as it would have been had we multiplied by 2 units, we put it beneath the first product, one place to the left ; afterwards we multiply by the 1 hundred, and as the product is a hundred times as much as it would have been had we multiplied by 1 unit, we put it beneath the other products two places to the left ; finally, adding up the three products, we get the whole product.

This is plainly the answer, since we have taken 705, 7, 20, and 100 times, and have added the products together.

Observe that the right hand figure of each product is placed directly under the figure we multiply by.

2. How much are 134 tons of hay worth, at 25 dollars a ton ? Ans. 3,350 dollars.

3. What sum must I pay for 2,327 chaldrons of orrel coal, at 14 dollars a chaldron ? Ans. 32,578 dollars.

4. How many gallons of molasses have I in three hun-

In example 1, lesson 33, what is the multiplier ? Multiplicand ? Product ?

How do we proceed to multiply 705 by 127 in example 1, lesson 34 ? Where is the right hand figure of each product placed ?

dred and forty-five hogsheads, each hogshead containing ninety-four gallons ? Ans. 32,430.

5. If 112 emigrants to the West are provided with 1,235 dollars apiece, how much money have they all ?

Ans. 138,320 dollars.

LESSON 35.

From what precedes we get the following

RULE FOR MULTIPLICATION.

Write one number under the other, with units under units, tens under tens, &c., and draw a line beneath. Then beginning at the right of the multiplicand, multiply each figure in it by the units in the multiplier, carry as in Addition, and write the product below. If the multiplier contain but one figure, the operation is now done. If it contain more than one figure, multiply in the same manner by the tens, hundreds, &c., in the multiplier, taking care to put the first figure of each product directly beneath the figure by which you multiply, and finish by adding up the several products.

Note. It is generally best to make the smaller number the multiplier.

5 times 9 are 45, and 9 times 5 are 45.

Therefore, as we sometimes make a mistake in multiplying, we can prove the work,

By multiplying the former multiplier by the multiplicand; if the product be equal to the first, the work will generally be right.

1. Multiply 412 by 18.

OPERATION.	PROOF.
412	18
18	412
3296	36
412	18
	72
7,416 product.	7,416 product, same as before.

How do you write the numbers in Multiplication ? How then do you multiply, carry, and write the product ? What if the multiplier contain but one figure ? What if the multiplier contain more than one figure ? What number should generally be made the multiplier ?

How many are 5 times 9 ? 9 times 5 ?

How then can we prove the work in Multiplication ?

Note. **Each example should now be proved.**

(2.)	(3.)	(4.)
Multiply 6 3 5 7	Multiply 4 8 2 1	Multiply 4 3 0 0 2
by 2 3 3 5	by 1 1 3 1	by 2 5

Lesson 36.

1. Multiply 254,420,335 by 3,347,889.

2. Multiply 815,555 by 5,542.

3. Multiply 5,001 by 357.

4. How much must I give for 234 hogsheads of molasses, at 19 dollars a hogshead? Ans. 4,446 dollars.

5. 8 men trading in company, furnished 5,237 dollars apiece; how much did they all furnish? Ans. 41,896 dollars.

6. I have a book which contains 421 pages, and each page 42 lines; how many lines are there in the book? Ans. 17,682.

7. What number will you get if you take 646,325, 5,335 times? Ans. 3,448,143,875.

8. What must I give for 35 yards of broadcloth, at 5 dollars a yard? Ans. 175 dollars.

9. How much must a man give for twenty-five barrels of flour, at twelve dollars a barrel? Ans. 300 dollars.

10. A certain army contains 13 regiments, with 832 men in a regiment; how many men are there in the whole army? Ans. 10,816.

Lesson 37.

1. What is the product of twelve thousand two hundred and twelve by three hundred and seventy-five? Ans. 4,579,500.

2. 225 men did a piece of work in 313 days; how long would it have taken one man to do it? Ans. 70,425 days.

3. How much will it cost to build a rail-road 26 miles long, at 8,231 dollars a mile? Ans. 214,006 dollars.

4. If you buy 473 tons of iron, at 116 dollars a ton, how much must you pay for it? Ans. 54,868 dollars.

5. A man hired 12 persons to labor for him at 1 dollar a day, apiece; how much did their wages amount to in 303 days? Ans. 3,636 dollars.

6. If a chaise wheel turns round 351 times in a mile,

how many times will it turn round in going from Boston to Providence, 41 miles? Ans. 14,391 times.

7. How much must I pay for 9 cows, at 18 dollars apiece? Ans. 162 dollars.

8. Suppose a steamboat between Providence and New York moves 15 miles an hour; how far will she go at this rate in 12 hours? Ans. 180 miles.

9. What sum must a man pay for 459 barrels of cider, at 3 dollars a barrel? Ans. 1,377 dollars.

10. 13 men received 189 dollars apiece; how much did they all receive? Ans. 2,457 dollars.

LESSON 38.

1. There is a book that has 21 pages, each page contains 16 lines, and each line 8 words; how many words are there in the book?

OPERATION.	OR
2 1 pages.	1 6 lines in a page.
1 6 lines in a page.	8 words in a line.
1 2 6	1 2 8 words in a page.
2 1	2 1 pages.
3 3 6 lines in the book.	1 2 8
8 words in a line.	2 5 6
2,6 8 8 words. Ans.	2,6 8 8 words. Ans.

Therefore, when several numbers are to be multiplied together to obtain an answer,

We can multiply the numbers together in any order we please, and the answer will always be the same.

2. If I hire 35 men at 4 dollars a week, apiece, what sum will their wages amount to in 3 years, there being 52 weeks in a year? Ans. 21,840 dollars.

3. Multiply 7, 19, 27, 236, and 11 together. Ans. 9,322,236.

4. Multiply 33, 5,321, and 424 together. Ans. 74,451,432.

5. How many bushels of potatoes are there in 12 loads, each load containing 10 barrels, and each barrel 3 bushels? Ans. 360 bushels.

When several numbers are to be multiplied together to obtain an answer, how can we multiply them?

6. If a vessel sail 9 miles an hour, how far will she sail in a week; there being 24 hours in a day, and 7 days in a week? Ans. 1,512 miles.

7. What is the product of 5, 6, 7, 8, 9, and 25 multiplied together? Ans. 378,000.

CONTRACTIONS IN MULTIPLICATION.

LESSON 39.

1. If I buy 103 pieces of cloth, at 256 dollars apiece, how much do I pay for the whole?

OPERATION.

$$\begin{array}{r} 256 \\ 103 \\ \hline 768 \\ 256 \\ \hline 26{,}368 \end{array}$$

dollars. Ans.

0s therefore, between the figures of the multiplier, are not used.

2. Multiply 245,181 by 6,005? Ans. 1,472,311,905.

3. How many bricks did a team haul away from a brick-yard in 206 loads, each load containing 1,296 bricks? Ans. 266,976.

4. Multiply 3,837 by 2,108. Ans. 8,088,396.

5. If a cow is worth 25 dollars, what are 10 such cows worth? Ans. 250 dollars.

Explanation. Put 0 at the right of 25, and it increases its value 10 times, or multiplies it by 10. See Numeration, latter part of lesson 5.

6. What must I give for 4 horses, at 100 dollars apiece? Ans. 400 dollars.

7. If 1,000 men are paid 165 dollars apiece, how much are the whole paid? Ans. 165,000 dollars.

So when two numbers are to be multiplied together, if one is 10, 100, &c.

We put the 0s in it at the right of the other to get the product.

What is done with the 0s between the figures of the multiplier?

Explain how example 5, lesson 39, is performed.

What is done when two numbers are to be multiplied together, if one is 10, 100, *&c.*?

8. 1,000 men before going into battle, were furnished with 45 cartridges apiece; how many cartridges did they receive?

9. There being 100 cents in a dollar, how many cents are there in 3 dollars ?

10. What is the product of 1,000 by 255.

Lesson 40.

1. If a vessel sail 120 miles a day for 6 days, how far will she go in that time ?

OPERATION.
1 2 0
6
7 2 0 miles. Ans.

Explanation. We here multiply 12 tens by 6, and put the 0 in 120 at the right of the product to show that it is 72 tens.

2. A drover has 300 cows worth 25 dollars apiece ; what is the value of all of them ?

OPERATION.
2 5
3 0 0
7,5 0 0 dollars. Ans.

Explanation. 25 taken 3 hundred times, or which is the same thing, 3 hundred taken 25 times, makes 75 hundred ; we therefore put the 0s in 300 at the right of 75.

3. If 200 men have 150 dollars apiece, how much have they all ?

OPERATION.
1 5 0
2 0 0
3 0,0 0 0 dollars. Ans.

Explanation. 15 tens taken 2 times make 30 tens; we therefore put the 0 in 150 at the right of 30, and as 2 hundred times 15 tens are a hundred times as much as 2 times 15 tens, we put the two 0s in 200 at the right of 300.

Therefore, when there are 0s at the right of the multiplier, or multiplicand, or both,

Omit them entirely in multiplying, but place them at the right of the product.

4. What quantity of shad are there in 65 barrels, each of which contains 200 pounds ? Ans. 13,000 pounds.

5. Multiply 432,000 by 2,100. Ans. 907,200,000.

Explain how example 1, lesson 40, is performed.

Explain how example 2, lesson 40, is performed.

Explain how example 3, lesson 40, is performed.

What course do we take when there are 0s at the right of the multiplier, or multiplicand, or both?

6. What sum are 160 hogsheads of molasses worth, at 20 dollars a hogshead? Ans. 3,200 dollars.

7. A company of men bought 6 acres of land at 2,000 dollars an acre; how much did they give for it? Ans. 12,000 dollars.

8. How much must a merchant pay for 250 bales of cotton, at 80 dollars a bale? Ans. 20,000 dollars.

9. What is the product of eighty-three thousand five hundred, and nine hundred and seventy-seven? Ans. 81,579,500.

10. If one ton of hay is worth 20 dollars, what are 9 tons worth? Ans. 180 dollars.

DIVISION.

Lesson 41.

To be performed in the mind.

1. How many apples can you get for 6 cents, at 2 cents apiece? 2 is in 6 how many times then? 3 is in 6 how many times?

2. Emma had 9 roses, which she divided equally among 3 children; how many did she give to each?

3. I rode 15 miles in 3 hours; how far did I ride in one hour? How many times is 3 in 15 then? Why is 3 in 15 5 times? *Answer.* Because 3 times 5 are 15.

4. A trader paid 16 dollars for 4 hats? what was the price of one hat? 4 is contained in 16 how many times then? Why?

5. If you have 18 sugar plums, 6 times as many as Charles, how many has Charles?

6. 7 boys have 21 pears; if they divide them equally, what will be the share of each?

7. How many hours will a traveller be in going 40 miles, if he rides 8 miles an hour? What number then does 40 divided by 8 give? 40 divided by 5?

8. A man earned 54 dollars in 9 weeks; how much did he earn in one week?

9. If you divide 60 cents into 10 equal heaps, how many will there be in each heap?

10. 6 men did a piece of work for 42 dollars; what was each one's share of the pay? How many times then is 6 in 42? Why?

Lessons 42 and 43.

DIVISION TABLE.

Note. Questions in this table should not be asked in rotation, because when they are so asked the learner can answer by merely counting, without the least exertion of memory.

In one part of the table we find 5 in 35, 7 times, and in another part, 7 in 35, 5 times. Each operation in the table is repeated, in this way, except when a number is divided by itself, or is contained in another as many times as 1 is in itself. Make the learner observe this.

2 in 2, 1 time
2 in 4, 2 times
2 in 6, 3 times
2 in 8, 4 times
2 in 10, 5 times
2 in 12, 6 times
2 in 14, 7 times
2 in 16, 8 times
2 in 18, 9 times
2 in 20, 10 times

3 in 3, 1 time
3 in 6, 2 times
3 in 9, 3 times
3 in 12, 4 times
3 in 15, 5 times
3 in 18, 6 times
3 in 21, 7 times
3 in 24, 8 times
3 in 27, 9 times
3 in 30, 10 times

4 in 4, 1 time
4 in 8, 2 times
4 in 12, 3 times
4 in 16, 4 times
4 in 20, 5 times
4 in 24, 6 times
4 in 28, 7 times
4 in 32, 8 times
4 in 36, 9 times
4 in 40, 10 times

5 in 5, 1 time
5 in 10, 2 times
5 in 15, 3 times
5 in 20, 4 times
5 in 25, 5 times
5 in 30, 6 times
5 in 35, 7 times
5 in 40, 8 times
5 in 45, 9 times
5 in 50, 10 times

6 in 6, 1 time
6 in 12, 2 times
6 in 18, 3 times
6 in 24, 4 times
6 in 30, 5 times
6 in 36, 6 times
6 in 42, 7 times
6 in 48, 8 times
6 in 54, 9 times
6 in 60, 10 times

7 in 7, 1 time
7 in 14, 2 times
7 in 21, 3 times
7 in 28, 4 times
7 in 35, 5 times
7 in 42, 6 times
7 in 49, 7 times
7 in 56, 8 times
7 in 63, 9 times
7 in 70, 10 times

8 in 8, 1 time
8 in 16, 2 times
8 in 24, 3 times
8 in 32, 4 times
8 in 40, 5 times
8 in 48, 6 times
8 in 56, 7 times
8 in 64, 8 times
8 in 72, 9 times
8 in 80, 10 times

9 in 9, 1 time
9 in 18, 2 times
9 in 27, 3 times
9 in 36, 4 times
9 in 45, 5 times
9 in 54, 6 times
9 in 63, 7 times
9 in 72, 8 times
9 in 81, 9 times
9 in 90, 10 times

10 in 10, 1 time
10 in 20, 2 times
10 in 30, 3 times
10 in 40, 4 times
10 in 50, 5 times
10 in 60, 6 times
10 in 70, 7 times
10 in 80, 8 times
10 in 90, 9 times
10 in 100, 10 times

LESSON 44.

To be performed in the mind.

1. William had 8 cents, which he laid on the table in 2 equal heaps; how many were there in each heap? If he had had 7 cents, how many would there have been in each heap, and how many over?

2. A man received 12 dollars for 4 days' labor; what were his wages a day?

3. How many heaps of 4 cents in a heap can you make out of 14 cents, and how many will there be over?

4. If you divide 32 apples equally among 8 boys, how many will each receive? How many times then is 8 in 32? 4 in 32?

5. How many times is 2 in 12? 3 in 27? 4 in 28? 4 in 36? 5 in 40? 7 in 28? 6 in 54? 6 in 36? 8 in 32? 8 in 64? 9 in 36? 10 in 80?

6. If you have 43 pounds of butter, and sell it in parcels of 5 pounds each, how many of such parcels will there be, and how many pounds over?

7. 2 men had equal shares in 20 dollars; what was each one's portion?

8. 30 cents were equally divided among 2 boys; how many did each one receive? *Explanation.* 30 is 3 tens; how many tens did each of the 2 boys receive, and how many cents were there over? How many times is 2 in 10? How many then are 1 ten and 5?

9. How many times is 2 contained in 24? In 26? In 32? In 46? In 60? In 48? In 82? In 100?

10. A man bought 3 pounds of butter for 42 cents; what was the price a pound? What would the price have been a pound if he had given 39 cents?

LESSON 45.

To be performed in the mind.

1. How many times is 3 contained in 30? In 36? In 48? In 54? In 60? In 75? In 90?

2. How many yards of broadcloth do I get for 48 dollars, if I pay 4 dollars a yard?

3. How many times is 4 contained in 40? In 52? In 64? In 80? In 96? In 100?

4. How many times is 5 contained in 55? 5 in 70? 6 in 78? 6 in 96? 7 in 70? 7 in 91? 8 in 96? 9 in 99?

5. 2 shipwrecked sailors divided 110 biscuits equally between them; what was each one's share? *Explanation.* 110 is 11 tens; now how many tens did each receive, and how many were there over? How many times is 2 contained in 10? How many then are 5 tens and 5?

6. How many barrels of apples can you buy for 120 dollars, at 3 dollars a barrel?

7. How many times is 2 contained in 200? 3 in 300? 4 in 400? 4 in 440? 5 in 600? 7 in 700? 9 in 540?

8. If you give 30 cents for 10 oranges, what is the price of one orange?

9. How many times is 10 contained in 40? In 60? In 90? In 120? In 160? In 170? In 200? In 300? In 370? In 450? In 540?

10. How many times is 2 contained in 25, and what is left? How many times is 3 contained in 38, and what is left? How many times is 5 contained in 67, and what is left? How many times is 8 contained in 100 and what is left? How many times is 10 contained in 96, and what is left?

LESSON 46.

For the Slate.

Way of dividing when the number we divide by consists of only one figure.

1. A merchant having 527 dollars in silver, wished to get the amount in 2-dollar bills; how many of such bills ought he to receive in exchange?

OPERATION.
2)5 2 7
———
2 6 3   1 remainder.
Ans. 263 2-dollar bills, and 1 dollar over.

Explanation. We place 2 at the left of 527, separate them by a curved line, and draw a line beneath. Now 2 is in the 5 hundred 2 even hundred times, and 1 hundred over; 2 is in the 1 hundred over, and the 2 tens, or in 12 tens, 60 times, or 6 ten times exactly; 2 is in the 7, 3 times, and 1 over. The 2 hundreds, 6 tens, and 3 put under 527, make 263, evidently the answer, there being 1 remainder, which we place at the right. We proceed in dividing thus; 2 is in 5, 2 times and 1 over, 2 is in 12, 6 times, 2 is in 7, 3 times and 1 over.

Explain how example 1, lesson 46, is performed. How do we proceed in dividing 527 by 2?

The number we divide by is called the *divisor*, that which we divide, the *dividend*, that which we obtain by dividing, the *quotient*, and the number left, the *remainder* ; thus, in the preceding example, 2 is the divisor, 527 the dividend, 263 the quotient, and 1 the remainder.

2. If I buy 126 dollars' worth of broadcloth, at 3 dollars a yard, how many yards do I obtain for my money ? Ans. 42.

3. A gentleman gave his 2 sons 264 dollars in equal shares ; how much did each receive ? Ans. 132 dollars.

4. A vessel owned by 5 persons in equal shares, earned 1,230 dollars in one year ; what was each one's part of the gain ? Ans. 246 dollars.

5. If I sell 8 horses for 688 dollars, how much do I get apiece for them ? Ans. 86 dollars.

6. How many barrels of flour can I buy for 64,459 dollars, at 9 dollars a barrel, and how many dollars will be left ? Ans. 7,162 barrels, and there will be 1 dollar left.

7. A teamster hauled two hundred and sixteen kegs of lard in four equal sized loads ; how many kegs were there in each load ? Ans. 54.

8. A drover wished to put 1,305 sheep in 6 equal flocks; how many will there be in a flock, and how many will there be over ? Ans. there will be 217 in a flock, and 3 over.

Lesson 47.

Way of dividing when the divisor consists of more than one figure.

1. 13 men gained 1,963 dollars, and divided it equally ; what was each one's share ?

```
OPERATION.
13)1963(151 dollars. Ans.
   13
   ——
    66          TRIAL
    65           66
    ——           52
     13          ——
     13          14
     ——
```

Explanation. We divide as before, but for the sake of ease, write down all of the work thus. We make a curved line each side of 1,963, and place 13 at the left. Now 19 hundreds divided by 13 give 1 hundred, which we place at the right of

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What is called the divisor ? Dividend ? Quotient ? Remainder ?

In example 1, lesson 46, what is the divisor ? Dividend ? Quotient ? Remainder ?

1,963, and multiplying 13 by the 1 hundred, subtract the product, 13 hundreds, from 19 hundreds ; we have 6 hundreds left, and bringing down the 6 tens we have 66 tens to divide by 13. Let us see if the quotient be 4 tens ; multiplying 13 by 4 tens we get 52 tens, which subtracted from 66 tens leave 14 tens. The quotient is evidently larger. Let us try 5 tens. Multiplying 13 by 5 tens we get 65 tens, which subtracted from 66 tens, leave 1 ten ; we now place the 5 tens at the right of the quotient, and bringing down the 3, we have 13, which divided by 13 gives 1. This we put at the right of the other quotients, and multiplying 13 by it, subtract the product, which leaves no remainder.

151 is plainly the whole quotient, since 13 is contained in 1,963, 1 hundred times, 5 ten times, and 1 time.

2. A fisherman sold 12 shad for 108 cents ; how much did he get apiece for them ? Ans. 9 cents.

3. How many times is 25 contained in 1,043, and how large is the remainder ?
Ans. 41 times, and the remainder is 18.

4. A man earns 26 dollars a month ; how long will it take him to earn 1,638 dollars ? Ans. 63 months.

5. If 112 muskets are worth 1,344 dollars, what is the value of each ? Ans. 12 dollars.

6. A man gave 73 dollars for 6,205 alewives ; how many could he have bought for 1 dollar ? Ans. 85.

7. 87 cattle slaughtered at Brighton, were found to weigh 35,757 pounds ; if they were all of a size, what was the weight of each ? Ans. 411 pounds.

8. A teacher wished to divide 412 nuts equally among 34 boys ; how many can he give each, and how many will be left, that is, what will be the remainder ?
Ans. he can give 12 to each, and there will be 4 left.

LESSON 48.

From what precedes we get the following

RULE FOR DIVISION.

Write the divisor at the left of the dividend. Then if the divisor consists of only one figure, draw a line beneath the

---

Explain how you divide 1,963 by 13.
Why is 151 plainly the whole quotient?
How do you write the numbers in Division? How, then, do you proceed if the divisor consists of only one figure?

dividend, and divide the left hand figure or two left hand figures, placing the quotient beneath; the next figure of the dividend joined to the right of the remainder, if any, must now be divided, and the quotient put at the right of the other, and so on. If the divisor consists of two or more figures, find how many times it is contained in the smallest possible number of figures at the left of the dividend, and place the quotient at the right of the dividend; then multiply the divisor by the quotient, and subtract the product from the figures divided. Bring down the next figure of the dividend, join it to the right of the remainder, if any, and divide as before, putting the quotient at the right of the other, and so on.

3 is in 7, 2 times and 1 over. 3 times 2 are 6 and 1 are 7. Therefore, as we sometimes make a mistake in dividing, we can prove the work,

By multiplying the divisor and quotient together, and adding the remainder, if any, to the product; if the result be equal to the dividend, the work will generally be right.

1. Divide 636 by 15.

OPERATION.

```
15)636(42 quotient.
   60
   ——
    36
    30
    ——
     6 remainder.
```

PROOF.

```
 42 quotient.
 15 divisor.
 ———
210
42
———
630
  6 remainder.
———
636 dividend.
```

Note. **Each example should now be proved.**

2. Divide 457,408 by 1,021.
3. Divide 10,985 by 21.
4. Divide 3,584 by 32.
5. Divide 895 by 8.

Lesson 49.

1. Divide 3,647,819 by 7.
2. Divide 46,200,981 by 3,975.

How do you proceed if the divisor consists of two or more figures.

How many times is 3 in 7, and how many over? How many are 3 times 2 added to 1?

How, then, can we prove the work in Division?

3. Divide 115,692,192 by 1,894.

4. An owner in a privateer was to receive one dollar out of every 6 that she made; after property to the amount of 22,302 dollars had been taken by her, what was his share? Ans. 3,717 dollars.

5. Suppose the salary of the bishop of Durham, in England, to be 96,666 dollars a year, how much does he receive a day, and what is the remainder, there being 365 days in a year?

Ans. he receives 264 dollars a day, and the remainder is 306 dollars.

6. If there are 136 furlongs in 17 miles, how many furlongs are there in 1 mile? Ans. 8.

7. A man has 6 chaises built in the same way; all of them are worth 750 dollars; what is the value of each? Ans. 125 dollars.

8. Divide 975 by 75. Ans. 13.

9. Divide 5,310 by 45. Ans. 118.

10. A trader bought 3,472 pounds of butter for 434 dollars; how many pounds did he get for one dollar? Ans. 8.

LESSON 50.

1. A bank having 28,400 dollars in silver, the officers put them in 25 bags, with the same quantity in each; how much was that? Ans. 1,136 dollars.

2. If 4 yards of cloth make one suit of clothes, how many suits will 108 yards make? Ans. 27.

3. A man bought 44 pounds of cheese for 528 cents; how much did he give a pound? Ans. 12 cents.

4. A man who had 625 pounds of butter, put it in 25 firkins, with the same number of pounds in each; what number was that? Ans. 25 pounds.

5. What is the quotient of 225 divided by 15? Ans. 15.

6. How long will 57,202 pounds of flour last, if 37 pounds are used daily? Ans. 1,546 days.

7. Divide 6,665 by 1,333. Ans. 5.

8. Divide 28,791 by 7. Ans. 4,113.

9. If 40 bushels of potatoes are worth 1 ton of hay, how many tons are 364,920 bushels worth? Ans. 9,123.

10. Suppose I want to go 873 miles in 9 days, how far must I travel each day, if I ride at a uniform speed? Ans. 97 miles.

LESSON 51.

1. How many are 3 times 4? 3 is in 12 how many times? 4 is in 12 how many times?

Therefore, by dividing the product of two numbers by one of them, we obtain the other.

2. If you get 252 dollars for 6 loads of cider, with 14 barrels to a load, what price a barrel are you paid?

OPERATION.

$$\begin{array}{r} 14 \\ 6 \\ \hline 84 \text{ barrels.} \end{array} \qquad \begin{array}{r} 84)252(3 \text{ dollars. Ans.} \\ 252 \\ \hline \end{array}$$

Therefore, by dividing the product of several numbers by the product of all of them but one, we obtain this one.

3. 7 men were paid 105 dollars for labor done at 1 dollar apiece a day; how many days did they work? Ans. 15.

4. A teamster hauled 65 bushels of corn at a load until he had carried away 1,625 bushels; how many loads had he taken? Ans. 25.

5. 2 horses eat 4,872 pounds of hay in 3 months; how much did each of them consume in a day, there being 7 days in a week, and 4 weeks in a month? Ans. 29 pounds.

6. 17,160 is the product of 13, 8, 11, 3, and another number multiplied together; what is that other number? Ans. 5.

7. I paid 85,120 cents for a load of butter in 95 firkins, giving 16 cents a pound for it; how many pounds were there in a firkin? Ans. 56 pounds.

CONTRACTIONS IN DIVISION.

LESSON 52.

1. A coal merchant bought 1,133 dollars' worth of anthracite coal, at 11 dollars a ton; how many tons did he buy?

What do we obtain by dividing the product of two numbers by one of them?

What do we obtain by dividing the product of several numbers by the product of all of them but one?

OPERATION.

$$\begin{array}{r l} 11)\,1133\,(\,103 & \text{tons. Ans.} \\ 11 & \\ \hline 33 & \\ 33 & \\ \hline \end{array}$$

Explanation. After bringing down the 3 tens we find that 11 is contained in them 0 ten times, which we put in the quotient; and then bring down the 3, and divide 33 by 11.

So after bringing down a figure, if the number made be less than the divisor,

Put 0 *in the quotient, and bring down the next figure, and divide.*

2. 23 horses, each of the same value, are worth 2,300 dollars; how much is one of them worth ? Ans. 100 dollars.

3. 42,230 bricks were hauled in 41 equal loads ; how many were taken in each load ? Ans. 1,030.

4. Divide 89,760 by 17. Ans. 5,280.

5. 27,054 acres of land were divided into 27 equal sized lots for sale ; how many acres were put in each lot ? Ans. 1,002.

6. James had 230 chestnuts to divide among 10 children; how many must he give to each ?

OPERATION.

2 3 | 0

2 3 chestnuts. Ans.

Explanation. Since by putting one 0 at the right of a number we multiply it by 10, by cutting one off we divide by 10. See Numeration, latter part of lesson 5. If James had had 234 apples, we should have cut off one figure from the right ; thus 2 3 | 4, leaving the answer 23, with a remainder 4.

7. There being 100 cents in a dollar, how many dollars are there in 53,000 cents ? Ans. 530.

8. If 1,000 soldiers have 2,311 flints, how many can each one have, and how many will there be over ?
Ans. each one can have 2, and there will be 311 over.

So when the divisor is 10, 100, &c.

Cut off as many figures from the right of the dividend as there are 0s *in the divisor. The figures cut off are the remainder, the others the quotient.*

Explain how example 1, lesson 52, is performed.

If after bringing down a figure the number made be less than the *divisor,* what is done ?

Explain how example 6, lesson 52, is performed.

What if the divisor be 10, 100, &c. ?

9. Divide 599,843 by 100.

10. 10 children inherited 15,220 dollars; what was the share of each ?

11. If you divide 372,000 into 1,000 parts, how large will each part be ?

12. There are 21,300 pounds of beef to be distributed among 10,000 men ; how many pounds can each receive, and how much will remain ?

Lesson 53.

1. A man had 330 pounds of lard which he wished to put into 20 equal sized pots , how many pounds must be put in each ?

OPERATION.

2|0) 3 3|0

1 6 1 0

Ans. 16 pounds, and there are 10 pounds over.

Explanation. To multiply by 20 we multiply by 2, and put the 0 at the right of the product ; therefore to divide 330 by 20 we cut off the 0 from 330, and divide the rest by 2. The remainder 1 being 1 ten, we put the 0 cut off, at the right of it. If he had had 337 pounds, we should have cut off the 7, and put it at the right of the 1 ten for a remainder.

2. How many piles of 600 dollars can you take out of a heap containing 12,200, and how many will there be over ?
Ans. 20 piles, and 200 over.

So when there are 0s at the right of the divisor,

Cut them off ; also cut off the same number of figures from the right of the dividend, divide what remains of the dividend by what remains of the divisor, and place the figures cut off from the dividend at the right of the remainder.

3. How many times is 4,000 contained in 2,010,000 ?
Ans. 502 times, and 2,000 over.

4. 450 men were paid 31,500 dollars ; what was the share of each ? Ans. 70 dollars.

5. Divide 3,541 by 30. Ans. 118, and 1 over.

6. How many divisions of 90 feet each can I make in one mile, or 5,280 feet ? Ans. 58, and there are 60 feet over.

7. A party of 20 men having obtained 720 dollars, divided them equally ; what was each one's share ?
Ans. 36 dollars.

Explain how example 1, lesson 53, is performed.

When there are 0s at the right of the divisor, how do you get the *answer?*

8. What is the quotient of 75,000 divided by 500? Ans. 150.

9. How many acres of land, at 310 dollars an acre, can I buy for 1,240 dollars? Ans. 4 acres.

10. How long will it take a man to travel 2,800 miles, if he goes 70 miles a day? Ans. 40 days.

PROMISCUOUS QUESTIONS
IN
ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

LESSON 54.

To be performed in the mind.

1. Rufus having 27 cents, wishes to buy as many oranges at 6 cents apiece as possible, and to spend the rest in apples at 1 cent apiece; how many oranges and apples can he buy?

2. If he has 25 cents, how many oranges and apples can he buy?

3. James picked up 15 apples, and William 21, but William soon after gave James 7; how many did each have then?

4. How much money do 35 10-dollar bills make?

5. A man having 97 sheep, purchased 35 more; how many had he then?

6. A farmer sold 2 loads of cider; each load contained 4 barrels, and each barrel 30 gallons; how many gallons did he sell?

7. A boy having 75 cents, spent 28 for a knife; how many did he then have?

8. If a man has an income of 2 dollars a day, and spends 5 dollars a week, how much will he save in 4 weeks?

9. Suppose a man on foot to be 25 miles ahead of a man on horseback, and to walk 5 miles an hour; how long before the man on horseback will overtake the other, if he rides 10 miles an hour?

10. A farmer having 81 dollars, bought 10 sheep at 3 dollars apiece, and 9 sheep at 4 dollars apiece; how much money did he have after paying for them?

11. If I buy a yoke of oxen, a cart, and three cows for 240 dollars, how many 5-dollar bills must I give in payment?

LESSON 55.

1. If you pay 4 cents for 2 oranges, what do you give apiece for them? If you pay 2 cents for 4 oranges, how many do you get for one cent?

So we divide the price by the quantity to get the value of one thing, and the quantity by the price in cents to get what one cent will buy, and by the price in dollars to get what one dollar will buy.

For the Slate.

2. 147 tons of hay sold for 3,087 dollars; what did it bring a ton? Ans. 21 dollars.

3. A man bought 3,684 pounds of sugar for 307 dollars; how many pounds did he get for one dollar? Ans. 12.

4. If you buy 160 acres of land at 25 dollars an acre, and expend 100 dollars upon it, at how much an acre must you sell it to gain 220 dollars? Ans 27 dollars.

Explanation. The whole cost added to 220 dollars, is what all the land must be sold for to gain 220 dollars.

5. A merchant having 45 barrels of pork, sold 25 of them at 25 dollars a barrel, but the price falling he sold the rest at 21 dollars a barrel; what did he get for the whole? Ans. 1,045 dollars.

6. Multiply 4,829,001 by 300,128.
Ans. 1,449,318,412,128.

7. A merchant bought 4,960 yards of coarse cotton cloth for 310 dollars; how many yards did he get for one dollar?
Ans. 16.

8. What number multiplied by 20 produces 2,740?
Ans. 137

Explanation. Remember that the divisor and quotient multiplied together produce the dividend.

9. What number multiplied by 12 produces 144?
Ans. 12.

10. What number added to 192 makes 271? Ans. 79.

How do we get the value of one thing from the price and quantity? How do we get *what one* cent or one dollar will buy, from the price *and quantity?*

LESSON 56.

1. A merchant bought 37,520 dollars worth of flour, and after selling it all, found he had gained 4,281 dollars; what did he sell it for? Ans. 41,801 dollars.

2. A man bought a farm for 3,230 dollars, paying with a farm worth 1,240 dollars, a yoke of oxen worth 75 dollars, a wagon worth 83 dollars, three cows worth 54 dollars, and the rest in money; how much money did he pay? Ans. 1,778 dollars.

3. A barrel of pork contains 200 pounds; now if you buy 3 barrels, each of which lacks 4 pounds of being full, at 16 cents a pound, what must you give for the whole? Ans. 9,408 cents.

How many dollars must you give, 100 cents making a dollar? Ans. 94 dollars and 8 cents.

4. A merchant bought a ship for 11,200 dollars, and after expending 1,247 dollars in repairs, sold her for 14,000 dollars; what did he gain? Ans. 1,553 dollars.

5. 10 children inherited 4,535 dollars apiece; what did they all inherit? Ans. 45,350 dollars.

6. If I give 141 dollars for a piece of cloth containing 47 yards, how much must I sell it for to gain 1 dollar a yard? Ans. 188 dollars.

7. A man who owned the following quantities of land, 500 acres, 17 acres, 121 acres, and 98 acres, sold 325 acres; how much did he have left? Ans. 411 acres.

8. What number subtracted from 2,521 leaves 178? Ans. 2,343.

9. How long will it take a steamboat to go 345 miles, if she moves 15 miles an hour? Ans. 23 hours.

10. A man sold a cargo of salt for 2,300 dollars, and made on it 625 dollars; what did it cost him? Ans. 1,675 dollars.

LESSON 57.

1. If I spend 35 dollars a month, how long will 700 dollars last me? Ans. 20 months.

2. If I spend 35 dollars a month, and earn 25, how long before I shall spend 700 dollars more than I earn? Ans. 70 months.

3. A man has a farm worth 2,300 dollars, stock worth

450, and 575 in cash ; he owes one man 1,125 dollars, and various others 323 dollars ; what property will he have when all his debts are paid ? Ans. 1,877 dollars.

4. Divide 4,725,000 by 5,000. Ans. 945.

5. 4 children inherited 2,250 dollars apiece, but one having died, the remaining 3 inherited the whole ; what was each one's part ? Ans. 3,000 dollars.

6. If a man rides 60 miles a day, how far will he go in 150 days ? Ans. 9,000 miles.

7. How long can 5 men live on 475 crackers, if they consume 5 apiece each day ? Ans. 19 days.

8. Add 2,581, 37, 583, 10,000 and 12. Ans. 13,213.

9. 12 men are hired at 1 dollar a day ; how much will their wages amount to in 300 days ? Ans. 3,600 dollars.

10. Subtract 1,287 from 1,287,000. Ans. 1,285,713.

The preceding rules, Addition, Subtraction, Multiplication, and Division, comprise all the operations ever performed in arithmetic ; other rules are only different ways of applying these.

FRACTIONS,

OR BROKEN NUMBERS.

COMMON FRACTIONS,

OFTEN CALLED VULGAR FRACTIONS.

LESSON 58.

If you cut an apple into 2 equal parts, each part will be a half of the apple ; if you cut it into 3 equal parts, each part will be a third of the apple ; if you cut it into 4 equal parts, each part will be a fourth of the apple ; and if you

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What is said of the preceding rules?

If you cut an apple into 2 equal parts, what portion of the apple will each part be? What portion of the apple will each part be if *you cut it into 3 equal parts?* 4 equal parts?

cut it into any other number of equal parts, the *ordinal number*, corresponding to it, is used to express the size of a part; thus, if you cut it into 23 equal pieces, each one will be a twenty-third part; if you divide it into 78 equal pieces, each one will be a seventy-eighth part.

If you have a number of apples, say 6, and divide them into 2 equal heaps, each heap will be a half of the whole; if you divide them into 3 equal heaps, each heap will be a third of the whole. If you have 20 apples, and divide them into 4 equal heaps, each heap will be a fourth of the whole; if you divide them into 5 equal heaps, each heap will be a fifth of the whole, and so on.

Into how many equal parts must you divide a pear so that each division may be a half? how many halves are there in any thing then?

Into how many equal parts must Joseph divide a piece of gingerbread so that each portion may be a third of the whole? How many thirds are there in any thing then?

Into how many equal parts must you divide any thing so that each part may be a fourth? How many fourths are there in any thing then?

Into how many equal parts must you divide any thing so that each portion may be a fifth? A sixth? A seventh? An eighth? A thirteenth? A twenty-first part? A fortieth? A hundred and fifty-sixth? A five hundredth?

How many fifths are there in any thing? Sixths? Sevenths? Ninths? Twenty-fourths? Forty-sevenths? Seventy-sixths? Two hundred and fifteenths? Thousandths?

To be performed in the mind.

1. Lucius had 2 peaches, and he gave Samuel one half of them; what number was that? How many halves of 2 are there then?

2. What number is one half of 4? Of 8? Of 10? Of 12? Of 14? Of 20? Of 30? Of 46? Of 52? Of 78? Of 192? How many halves of 4 are there then? Of 8? Of 10? Of 12? Of 14? Of 30? Of 78? Of any number?

If you cut it into any other number of equal parts, what is used to express the size of a part?

What if you cut it into 23 equal pieces? 78 equal pieces?

If you divide 6 apples into 2 equal heaps, what part of the whole will each heap be? What part will each heap be if you divide them into 3 equal heaps? If you divide 20 apples into 4 equal heaps? 5 equal heaps?

3. Sarah had 6 sugar plums; after giving away all but one third of them, how many had she left? How many thirds of 6 are there?

4. A man having 24 dollars, spent one fourth of it; what sum was that? How many fourths of 24 are there?

5. What is one fifth of 45? One sixth of 12? One seventh of 56? One tenth of 100?

6. How many halves are there in any number? Thirds? Fourths? Fifths? Sixths? Sevenths? Eighths? Ninths? Twelfths? Twentieths? Fifty-fourths? Ninety-thirds?

7. Elias had 12 cents, and he gave Robert one third of them, and Caleb one fourth; how many did Robert get? Caleb? Which then is the greatest, a third or a fourth of 12?

8. Which is the most, a half of 18 nuts or a third of them? A fourth of 40 nuts or a fifth of them? A fifth of 60 nuts or a sixth of them?

9. William had an eighth of 72 cherries, and Daniel a ninth of them; which had the most, and how many?

10. Which is the greatest, a half of an apple or a third? A third or a fourth? A fourth or a fifth? A fifteenth or a sixteenth? A half or a fourth? A third or a sixth? A fourth or a sixth?

Lesson 59.

To be performed in the mind.

1. Mary had one half of a dollar, and her father gave her another half; how many halves had she then? What part of a dollar had she then?

2. A man had one fifth of an acre of land, when he bought two fifths of an acre more; what part of an acre had he then?

3. Levi bought three eighths of a sheet of gingerbread for himself, and two eighths for his playmate; what part of a sheet did he buy for both?

4. If you take one third of an apple from two thirds what part of an apple will be left?

5. Augustus had four fourths, or quarters, of an apple in his hand when he dropped one fourth; what part of an apple remained in his hand?

6. A merchant owned six tenths of a ship, when he sold three tenths; what part did he keep?

7. A farmer having 10 dollars, bought a number of arti-

cles, and then counted his money ; he found two fifths of it left ; what sum was that ? *Explanation.* What is one fifth of 10 dollars ? Then what are two fifths ?

8. How many fifths of his money did he spend ?

9. Ephraim paid three fourths of 40 cents for a knife ; how many cents did his knife cost ? How many fourths of his money did he save ?

10. Richard had 63 walnuts, but he lost two sevenths of them ; what number remained ?

11. If a watermelon be worth 24 cents, how much is five eighths of it worth ?

12. Oliver had two pears, each of which he cut into halves ; how many halves were there ?

13. If you cut each of 3 apple-pies into fourths, how many fourths will there be ?

14. How many sixths of 1 are there in 2 ? In 5 ? In 4 ? In 7 ? In 9 ? In 6 ?

15. If you divide one half of an apple into 2 parts, how large will each part be ? How large will each part be if you divide one third of an apple into 2 parts ? One third of an apple into 3 parts ?

LESSON 60.

It is usual to write fractions in figures ; thus, one half is written $\frac{1}{2}$, one third $\frac{1}{3}$, one fourth $\frac{1}{4}$, one fifth $\frac{1}{5}$, one twentieth $\frac{1}{20}$, two thirds $\frac{2}{3}$, three fourths $\frac{3}{4}$, seven tenths $\frac{7}{10}$, three eighteenths $\frac{3}{18}$.

That is, *the number showing the size of a part is placed below the line, and the number of parts is placed above the line.*

The number below the line is called the *denominator*, and the number above the line the *numerator.*

How do you write in figures one ninth, one twelfth, one thirty-third, two sevenths, two forty-fifths, three eighteenths, four fifths, five elevenths, seven fourteenths, twelve one hundredths, twenty-five forty-firsts, ninety-two one hundredths, seventy-seven ninetieths, one hundred and twenty-four one hundred and fiftieths, three halves, five

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How is it usual to write fractions ? How is one half written ? One third ? One fourth ? One fifth ? One twentieth ? Two thirds ? Three fourths ? Seven tenths ? Three eighteenths ?

What number of a fraction do we put below the line ? Above the line ? *Which is the denominator ?* Numerator ?

thirds, eight fifths, seventeen twelfths, twenty-five sixteenths ?

How do you read $\frac{3}{7}$, $\frac{4}{11}$, $\frac{1}{6}$, $\frac{5}{8}$, $\frac{7}{11}$, $\frac{13}{25}$, $\frac{19}{20}$, $\frac{33}{40}$, $\frac{17}{55}$, $\frac{90}{100}$, $\frac{150}{280}$, $\frac{12}{45}$, $\frac{88}{98}$, $\frac{73}{91}$, $\frac{113}{220}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{6}{4}$, $\frac{9}{10}$, $\frac{15}{16}$, $\frac{45}{9}$, $\frac{125}{25}$?

A fraction whose numerator is smaller than its demominator, is called a *proper fraction.* A fraction whose numerator is larger than its denominator, is called an *improper fraction.*

To be performed in the mind.

1. If you cut 2 apples into thirds, and take $\frac{1}{3}$ of each, how many thirds of 1 apple will you have ? Are $\frac{2}{3}$ of 1 the same as $\frac{1}{3}$ of 2 then ?

2. Moses cut 3 oranges into fourths, and took $\frac{1}{4}$ of each; how many fourths of 1 orange did he get ? What part of 1 then is $\frac{1}{4}$ of 3 ? What part of 3 are $\frac{3}{4}$ of 1 ?

3. If you have 5 melons to divide among 6 boys, what part of a melon is each boy's share ? *Explanation.* If you had but 1 melon to divide, what part would each boy have? How many sixth parts can each have in all the 5 melons ? What part of 1 is $\frac{1}{6}$ of 5 ? What part of 5 are $\frac{5}{6}$ of 1 ?

4. 10 men have equal shares in 4 large pumpkins ; how large a part must each have ? What part of 1 is $\frac{1}{10}$ of 4 ? What part of 4 is $\frac{4}{10}$ of 1 ?

5. 3 boys have 7 pears ; what is each one's share ? What part of 1 is $\frac{1}{3}$ of 7 ? What part of 7 are $\frac{7}{3}$ of 1 ?

The numerator is always supposed to be divided by the denominator, and the value of the fraction is the quotient. A quotient may always be expressed by writing the divisor as a denominator under the dividend ; thus $\frac{1}{3}$ is 1 divided by 3, $\frac{2}{5}$ is 2 divided by 5. This last fraction may be considered as two fifths of 1, one fifth of 2, or 2 divided by 5.

6. In what different ways may you consider and read $\frac{3}{4}$? $\frac{4}{5}$? $\frac{3}{7}$? $\frac{15}{6}$? $\frac{105}{12}$? $\frac{16}{123}$? $\frac{212}{309}$? $\frac{3}{8}$? $\frac{9}{16}$? $\frac{11}{25}$?

7. How can you express the quotient of 1 divided by 2 ? 7 divided by 3 ? 14 divided by 25 ? 10 divided by 307 ? 120 divided by 9 ? 19 divided by 20 ? 3 divided by 47 ? 101 divided by 1,250 ? 33 divided by 55 ? 6 divided by 50 ?

What is called a proper fraction ? An improper fraction ?

What is the numerator always supposed to be divided by ? What is the quotient ? How may a quotient always be expressed ? What then is $\frac{1}{3}$? $\frac{2}{5}$? How may $\frac{2}{5}$ be considered ?

LESSON 61.

To find what part of one number another number is.

To be performed in the mind.

1. If there are 2 apples on the table, and 1 of them belongs to you, what part of the 2 apples belongs to you, $\frac{1}{2}$, $\frac{1}{3}$, or what part? If you have 1 of 3 apples, what part of the whole are yours? 1 of 4? 1 of 5? 1 of 6? 1 of 7? 1 of 8?

2. If you have 2 of 3 apples, what part of the whole are yours? *Explanation.* If you have 1 of 3 apples, what part are yours? Then what part are yours if you have 2 of 3?

3. What part of 4 apples are 3 apples? What part of 5 is 3? What part of 4 is 5? What part of 6 is 2?

Therefore, to find what part of one number another number is,

Make the first the denominator of a fraction of which the last is the numerator.

4. A man owns 7 acres in a piece of land that contains 11 acres; what part of the piece does he possess?
5. William has 3 oranges, and Samuel 4; what part of Samuel's number has William?
6. What part of 4 is 5? What part of 7 is 9?
7. John has 8 cents, and Jacob 5; what part of Jacob's number has John? What part of John's number has Jacob?
8. What part of 28 is 13? What part of 13 is 28?
9. A captain of a ship received 95 dollars, and the mate 45; what part of the captain's share is the mate's?
10. What part of 19 is 4? What part of 7 is 70?

LESSON 62.

To find the exact quotient when there is a remainder.

To be performed in the mind.

1. 2 boys have 3 apples between them; how many apples, and what part of an apple is the share of each?
2. If you divide 4 pears into 3 equal portions, how many pears, and what part of a pear will there be in each portion?

How do we find what part of one number another number is?

3. If you divide 5 bushels of potatoes among 3 persons, how many bushels, and what part of a bushel will each receive? How many bushels, and what part of a bushel will each receive if you divide 5 bushels among 4 persons? 6 bushels? 7 bushels? 9 bushels? 10 bushels?

For the Slate.

4. 9 dollars are to be paid to 4 men in equal shares; what will each one have?

OPERATION.

4)9

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$2\frac{1}{4}$ dollars. Ans.

Explanation. We divide 9 into 4 parts, and get 2 dollars for each part, and there is 1 dollar over. This divided into 4 parts gives $\frac{1}{4}$ of a dollar for each; the $\frac{1}{4}$ being put at the right of the 2, makes $2\frac{1}{4}$ dollars for the answer.

Therefore, when there is a remainder after dividing,

Write the divisor beneath it; the fraction thus made, is a part of the quotient, and must be put at the right of it.

Any number composed of a whole number and a fraction, like $2\frac{1}{4}$, is called a ***mixed number.***

5. A man gave 6 dollars for 5 bushels of corn; what was the price a bushel? Ans. $1\frac{1}{5}$ dollar.

6. 3 boys divided 5 oranges between them; what was the portion of each? Ans. $1\frac{2}{3}$.

7. Divide 35 by 8. Ans. $4\frac{3}{8}$.

8. Divide 911 by 6. Ans. $151\frac{5}{6}$.

9. Divide 73 by 21. Ans. $3\frac{10}{21}$.

10. A laborer gets one bushel for every 10 that he threshes; what quantity will he have after threshing 27 bushels? Ans. $2\frac{7}{10}$ bushels.

11. Divide 33 by 12. Ans. $2\frac{9}{12}$.

12. What is the quotient of 2,700 by 185? Ans. $14\frac{110}{185}$.

LESSON 63.

To change a whole or mixed number to an improper fraction.

To be performed in the mind.

1. How many halves of a pear are there in 2 pears? In $2\frac{1}{2}$ pears? In 3? In 4? In 7? In $7\frac{1}{2}$? In 15? In 20?

Explain how example 4, lesson 62, is performed.
What is done when there is a remainder after dividing?
What is called a mixed number?

In 100? How many thirds are there in 3 pears? In $4\frac{2}{3}$ pears? In 6? In $7\frac{1}{3}$? In 9?

2. How many eighths of an inch are there in $3\frac{3}{8}$ inches? In $4\frac{5}{8}$ inches? In 8 inches? In $9\frac{1}{8}$ inches? In 12 inches?

3. 2 loaves were each broken into 10 equal parts to distribute among some starving sailors; how many tenth parts were there in both loaves? How many tenth parts would there have been in 3 loaves? In 4? In 6? In $5\frac{3}{10}$? In 9?

For the Slate.

4. A merchant having 13 pounds of cloves, put them in parcels of $\frac{1}{4}$ of a pound each for sale; how many parcels, or fourths of a pound did he make?

OPERATION.

$$\begin{array}{r} 13 \\ 4 \\ \hline 52 \end{array}$$

$\frac{52}{4}$ of a pound. Ans.

Explanation. There are 4 fourths in 1 pound, and 13 times as many in 13 pounds.

5. If he had had $13\frac{3}{4}$ pounds, how many parcels or fourths of a pound would he have made?

OPERATION.

$$\begin{array}{rl} & 13\tfrac{3}{4} \\ & 4 \\ \cline{2-2} & 52 \text{ fourths of a pound in 13 pounds.} \\ \text{add} & 3 \text{ fourths of a pound.} \\ \cline{2-2} & 55 \end{array}$$

$\frac{55}{4}$ of a pound. Ans.

Explanation. There are 4 fourths in 1 pound, and 13 times as many in 13 pounds. Adding the 3 fourths to the 52 fourths we get 55 fourths.

Therefore, to change a whole or mixed number to an improper fraction,

Multiply the whole number by the denominator, add the numerator to the product, if it be a mixed number, and place the denominator beneath the result.

6. Change $4\frac{1}{20}$ to an improper fraction. Ans. $\frac{81}{20}$.
7. Change $873\frac{4}{7}$ to an improper fraction. Ans. $\frac{6115}{7}$.
8. Change 35 to an improper fraction, of which the denominator shall be 3. Ans. $\frac{105}{3}$.

Explain how example 4, lesson 63, is performed.
Explain how example 5, lesson 63, is performed.
How do we change a whole or mixed number to an improper fraction?

9. Joshua has $12\frac{5}{8}$ dollars in ninepences or eighths; how many eighths of a dollar has he? Ans. $\frac{101}{8}$.

10. A number of men paid $\frac{1}{16}$ of a dollar apiece for admittance to see a juggler; he obtained $15\frac{6}{16}$ dollars; how many sixteenths of a dollar were there in that sum?

Ans. $\frac{246}{16}$.

11. In 43 dollars how many sixths of a dollar are there?

Ans. $\frac{258}{6}$.

12. A farmer divided 4 bushels, 5 bushels, and $\frac{3}{8}$ of a bushel of corn among some poor persons, giving each $\frac{1}{8}$ of a bushel; how many eighths of a bushel did he give away?

Ans. $\frac{75}{8}$.

LESSON 64.

To change an improper fraction to a whole or mixed number.

To be performed in the mind.

1. How many apples are there in $\frac{2}{2}$ of an apple? In $\frac{3}{2}$ of an apple? In $\frac{4}{2}$? In $\frac{5}{2}$? In $\frac{3}{3}$? In $\frac{6}{3}$? In $\frac{9}{3}$? In $\frac{5}{4}$? In $\frac{12}{4}$? In $\frac{24}{4}$? In $\frac{30}{6}$? In $\frac{35}{6}$? In $\frac{41}{7}$? In $\frac{80}{8}$?

2. A man cut up some pies into fourths to sell; after selling a number of pieces, or fourths, he found he had $\frac{12}{4}$ left; how many whole pies would these pieces make?

3. A New England ninepence is $\frac{1}{8}$ of a dollar; now how many dollars are there in 8 ninepences, or in $\frac{8}{8}$ of a dollar? In $\frac{9}{8}$ of a dollar? In $\frac{16}{8}$? In $\frac{19}{8}$? In $\frac{30}{8}$?

For the Slate.

4. A merchant had $\frac{52}{4}$ of a pound of cloves; how many pounds had he?

OPERATION.

$$\begin{array}{r} 4\,)\,5\,2 \\ \hline 1\,3 \end{array}$$ pounds. Ans.

Explanation. $\frac{4}{4}$ of a pound make 1 pound; $\frac{52}{4}$, then, evidently, make as many pounds as 4 goes times in 52.

5. If the merchant had had $\frac{55}{4}$ of a pound, how many pounds would that quantity have been?

OPERATION.

$$\begin{array}{r} 4\,)\,5\,5 \\ \hline 1\,3\,\frac{3}{4} \end{array}$$ pounds. Ans.

Explanation. 4 is in 55, $13\frac{3}{4}$ times.

Explain how examples 4 and 5, lesson 64, are performed.

Therefore, to change an improper fraction to a whole or mixed number,

Divide the numerator by the denominator, and the quotient will be the whole or mixed number.

6. William had $\frac{480}{10}$ of a dollar; how many dollars did he have? Ans. 48.

7. Some sailors, cast away on an island, lived $\frac{39}{24}$ of a day without food; how many days was that time? Ans. $1\frac{15}{24}$ day.

8. A grocer has 27 half-barrels of flour, that is, $\frac{27}{2}$ of a barrel; how many barrels are there in this quantity? Ans. $13\frac{1}{2}$.

9. What whole number is equal to $\frac{18883}{23}$? Ans. 821.

10. What whole number is equal to $\frac{900}{1}$? Ans. 900.

11. Change $\frac{86457}{192}$ to a mixed number. Ans. $450\frac{57}{192}$.

12. Change $\frac{137}{12}$ to a mixed number. Ans. $11\frac{5}{12}$.

LESSON 65.

To change a fraction to a simpler form.

If we multiply the denominator and numerator of a fraction, say of $\frac{1}{3}$, by 2, for instance, it becomes $\frac{2}{6}$, and is plainly of the same value as $\frac{1}{3}$; since 3 times 1 third make $\frac{3}{3}$, or 1, and 3 times 2 sixths make $\frac{6}{6}$, or 1. Now if we divide both denominator and numerator of $\frac{2}{6}$ by 2, we get $\frac{1}{3}$ again, of the same value as $\frac{2}{6}$, but in a simpler form.

So, if the denominator and numerator of a fraction be both multiplied, or both divided by the same number, the value of the fraction will not be altered; but if both be divided by the same number, without a remainder, the new fraction is simpler.

Therefore, to change a fraction to a simpler form,

Divide the denominator and numerator by any number that is contained in both without a remainder.

How do we change an improper fraction to a whole or mixed number?

If we multiply the denominator and numerator of $\frac{1}{3}$ by 2, what fraction does it become? What is its value? Why? Now if we divide both denominator and numerator of $\frac{2}{6}$ by 2, what fraction do we get? What is said of its value and form?

What if the denominator and numerator of a fraction be both multiplied, or both divided by the same number? What if both be divided by the same number, without a remainder?

How do we change a fraction to a simpler form?

To be performed in the mind.

1. If you have $\frac{2}{4}$ of an apple, in what simpler form can you express that part ?

2. In what simpler form can you express $\frac{4}{6}$? $\frac{10}{20}$? $\frac{18}{36}$?

3. Henry has $\frac{12}{16}$ of a dollar ; in what simpler form can you express this amount of money ?

4. A boy has $\frac{4}{12}$ of a melon ; how much is that, expressed in a simpler form ?

5. In what simpler form can you express $\frac{2}{10}$?

6. In what simpler form can you express $\frac{100}{200}$? $\frac{200}{300}$?

7. In what simpler form can you express $\frac{400}{600}$?

8. In what simpler form can you express $\frac{7}{21}$?

9. In what simpler form can you express $\frac{10}{15}$?

10. In what simpler form can you express $\frac{12}{15}$? $\frac{4}{2}$? $\frac{6}{2}$? $\frac{6}{3}$? $\frac{12}{4}$? $\frac{12}{8}$? $\frac{9}{24}$?

We frequently meet with fractions composed of a large number of figures, rendering calculations tedious, but which can be changed to very simple forms ; thus $\frac{124}{248}$ is equal to $\frac{1}{2}$, because if we divide both denominator and numerator by 124 we get $\frac{1}{2}$.

A fraction is in its simplest form when no number but 1 will divide both denominator and numerator without a remainder.

The greatest number that will divide both denominator and numerator, without a remainder, is called the *greatest common divisor ;* and after dividing by such a number, the fraction is evidently changed to its simplest form.

LESSON 66.

To find the greatest common divisor, and change a fraction to its simplest form.

For the Slate.

1. A ship was divided into 42 shares, and sold to various persons ; one bought 12 shares, and of course then owned $\frac{12}{42}$ of the ship ; what is the simplest form to express what part of the ship he owned ?

In what simpler form can we write $\frac{124}{248}$? Why ?

When is a fraction in its simplest form ?

What is called the greatest common divisor of the denominator and numerator of a *fraction*, and when is the fraction changed to its simplest *form ?*

OPERATION.

```
12)42(3
   36
   --
    6)12(2
      12
      --
```

6 greatest common divisor.

12 divided by 6 gives 2
42 divided by 6 gives 7 Ans. $\frac{2}{7}$

Explanation. 12 goes in 42, 3 times, and 6 remains. 6 goes in 12, 2 times, with no remainder. 6 will therefore divide 3 times 12, or 36, without a remainder, and also 36 and 6, or 42; moreover it is the *greatest* common divisor of 12 and 42; for the greatest common divisor of 12 and 42 will divide 3 times 12, or 36, and 42, without a remainder, and it must be contained at least once more in 42 than in 36, and no number greater than their difference, 6, will go in 42 one time more than in 36.

Therefore, to find the greatest common divisor of any two numbers, say of the denominator and numerator of a fraction,

Divide the larger number by the smaller, and then the divisor by the remainder, if there be any, and so on, always dividing the last divisor by the last remainder until nothing is left, when the last divisor will be the greatest common divisor.

Note. The denominator and numerator of many fractions have no common divisor except 1.

2. If a man buys 12 shares in a ship in which there are 44 shares, and therefore owns $\frac{12}{44}$ of her, what is the simplest form to write his part in? Ans. $\frac{3}{11}$.

3. Change $\frac{116}{224}$ to its simplest form. Ans. $\frac{29}{56}$.

4. Change $\frac{36}{24}$ to its simplest form as a fraction? Ans. $\frac{3}{2}$.

5. If I am building 35 rods of stone wall, after I have finished 14 rods, what part of the whole, expressed in its simplest form, have I built? Ans. $\frac{2}{5}$.

6. What is the simplest form in which to express $1,000\frac{128}{156}$? Ans. $1,000\frac{32}{39}$.

7. Change $\frac{1250}{35}$ to its simplest form as a fraction? Ans. $\frac{250}{7}$.

8. A man owns 10 out of 100 shares in a factory; what is his part of the property expressed in the simplest way? Ans. $\frac{1}{10}$.

Explain how example 1, lesson 66, is performed.

How do we find the greatest common divisor of any two numbers?

What is said of the denominator and numerator of many fractions?

9. Change $\frac{55}{165}$ to its simplest form. Ans. $\frac{1}{3}$.
10. Change $\frac{343}{7280}$ to its simplest form. Ans. $\frac{49}{1040}$.
11. Change $\frac{128}{12}$ to its simplest form as a fraction. Ans. $\frac{32}{3}$.
12. Change $\frac{162}{567}$ to its simplest form. Ans. $\frac{2}{7}$.

LESSON 67.

To change fractions to a common denominator.

If we have $\frac{1}{3}$ to add to $\frac{2}{3}$, we obviously add the numerators, and get $\frac{3}{3}$ or 1. In this case the fractions have a common denominator 3; but we cannot so easily add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$, where the denominators are different. In cases like this, before adding, we must contrive to make the denominators alike without altering the value of the fractions.

For the Slate.

1. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$ to a common denominator.

OPERATION.	first numerator.	second numerator.	third numerator.
2 first denominator.	1	2	2
3 second denominator.	3	2	2
—	—	—	—
6	3	4	4
5 third denominator.	5	5	3
—	—	—	—
30 common denominator.	15	20	12

Ans. $\frac{15}{30}$, $\frac{20}{30}$ and $\frac{12}{30}$

Explanation. By examining the preceding operation, we find each denominator is multiplied with the others for a common denominator, and each numerator with the same numbers as its denominator for a new numerator; so that the values of the fractions are not altered. See first part of lesson 65.

Therefore, to change fractions to a common denominator,

Multiply all the denominators together for a common denominator, and each numerator by all the denominators, except its own, for a new numerator.

How do we add $\frac{1}{3}$ and $\frac{2}{3}$?

Where the denominators are different, as in $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$, what must we contrive to do before adding?

Explain how you change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$ to a common denominator.

How then do we change fractions to a common denominator?

2. Change $\frac{5}{6}$ and $\frac{3}{7}$ to a common denominator.
Ans. $\frac{35}{42}$, and $\frac{18}{42}$.

3. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to a common denominator.
Ans. $\frac{60}{120}$, $\frac{80}{120}$, $\frac{90}{120}$, and $\frac{96}{120}$.

4. Change $2\frac{1}{4}$ and $\frac{5}{8}$ to a common denominator.
Ans. $\frac{72}{32}$, and $\frac{20}{32}$.

Explanation. Change $2\frac{1}{4}$ to an improper fraction first.

5. Change $3\frac{1}{3}$ and $6\frac{1}{5}$ to a common denominator.
Ans. $\frac{50}{15}$, and $\frac{93}{15}$.

6. Change $\frac{4}{3}$, $\frac{5}{7}$, and $\frac{1}{2}$ to a common denominator.
Ans. $\frac{56}{42}$, $\frac{30}{42}$, and $\frac{21}{42}$.

7. Change 8 and $\frac{2}{3}$ to a common denominator.
Ans. $\frac{24}{3}$, and $\frac{2}{3}$.

8. Change $7\frac{1}{2}$, $5\frac{1}{4}$, and $1\frac{1}{3}$ to a common denominator.
Ans. $\frac{180}{24}$, $\frac{126}{24}$, and $\frac{32}{24}$.

9. Change $\frac{36}{256}$ and $\frac{81}{125}$ to a common denominator.
Ans. $\frac{4500}{32000}$, and $\frac{20736}{32000}$.

10. Change $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ to a common denominator.
Ans. $\frac{4}{8}$, $\frac{2}{8}$, and $\frac{1}{8}$.

Explanation. We change to a common denominator by merely changing $\frac{1}{2}$ and $\frac{1}{4}$ to eighths. We can often abridge the rule in this way, by multiplying the denominators and numerators by such numbers as will make the denominators alike.

11. Change $\frac{16}{2500}$, $\frac{180}{250}$, and $\frac{9}{25}$ to a common denominator.

12. Change $\frac{3}{10}$ and $\frac{2}{15}$ to a common denominator.

13. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{2}{10}$, and $\frac{3}{20}$ to a common denominator.

ADDITION OF FRACTIONS.

LESSON 68.

To be performed in the mind.

1. A little boy had $\frac{1}{10}$ of a dollar, when his father gave him $\frac{3}{10}$ more ; what part of a dollar had he then ? What is the simplest form to express the answer in ?

2. A man having $\frac{3}{5}$ of a barrel of mackerel at home, bought $\frac{2}{5}$ of a barrel ; what quantity had he then ? If he had purchased $2\frac{1}{5}$ barrels, what quantity would he have had with that at home ?

3. What is the sum of $\frac{1}{9}$, $\frac{2}{9}$, and $\frac{5}{9}$? What is the sum of $\frac{1}{3}$, $\frac{2}{3}$, and $3\frac{1}{3}$?

4. Jane has $\frac{1}{2}$ of a sheet of paper, $\frac{3}{4}$ of a sheet, and 2 sheets ; how much paper has she ?

For the Slate.

5. A farmer sold $\frac{3}{4}$ of a ton of hay at one time, and $\frac{2}{3}$ of a ton at another time; what was the whole quantity he sold?

OPERATION.

4		3	2
3		3	4
—		—	—
12	common denominator.	9	8
		$\frac{9}{12}$	$\frac{8}{12}$

$\frac{9}{12}$ and $\frac{8}{12}$ make $\frac{17}{12}$ of a ton; or, changing to a mixed number, $1\frac{5}{12}$ ton. Ans.

Explanation. We first change $\frac{3}{4}$ and $\frac{2}{3}$ to a common denominator, and then $\frac{3}{4}$ becomes $\frac{9}{12}$, and $\frac{2}{3}$, $\frac{8}{12}$; adding 9 twelfths and 8 twelfths, we get $\frac{17}{12}$ of a ton; or changing to a mixed number, $1\frac{5}{12}$ ton.

Therefore, to add fractions,

Add the numerators, taking care to change the fractions to a common denominator first, if the denominators be not alike.

6. If I sell from a piece of cloth, at different times, $\frac{5}{8}$ of a yard, $\frac{2}{7}$ of a yard, and $\frac{4}{5}$ of a yard, what is the whole quantity I dispose of? Ans. $1\frac{199}{280}$ yard.

7. A farmer sold 2 bushels of potatoes to one person, $2\frac{3}{4}$ to another, and $\frac{1}{3}$ of a bushel to a third; what was the whole quantity he sold? Ans. $5\frac{1}{12}$ bushels.

Explanation. What number do $\frac{3}{4}$ and $\frac{1}{3}$ make? What number then do 2, 2, and $1\frac{1}{12}$ make?

8. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Ans. $2\frac{43}{60}$, the fraction being expressed in its simplest form.

9. John has $5\frac{1}{4}$ dollars, $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{1}{10}$ of a dollar; what is the whole amount? Ans. $5\frac{39}{40}$.

10. A man bought of a grocer $12\frac{1}{2}$ pounds of sugar, at one time, and $9\frac{3}{4}$ at another; what was the whole number of pounds he bought? Ans. $22\frac{1}{4}$.

11. Add $\frac{4}{10}$, $\frac{17}{10}$, $\frac{3}{10}$, and $\frac{9}{10}$ together? Ans. $3\frac{3}{10}$.

12. A man owns $\frac{9}{80}$ of the property belonging to a bank, one of his sons owns $\frac{3}{20}$ of it, and another $\frac{1}{5}$ of it; what part of it do they all own? Ans. $\frac{37}{80}$.

Explain how example 5, lesson 68, is performed.
How do we add fractions?

SUBTRACTION OF FRACTIONS.

LESSON 69.

To be performed in the mind.

1. A grocer had $\frac{3}{5}$ of a hogshead of cider ; after he had sold $\frac{2}{5}$ of a hogshead how much remained ?

2. If you take $\frac{5}{8}$ of a dollar from $\frac{7}{8}$, what part of a dollar will be left ? What part of a dollar will be left if you take $\frac{5}{8}$ of a dollar from 1 dollar ? 2 dollars ? $2\frac{1}{8}$ dollars ? $2\frac{5}{8}$ dollars ?

3. Subtract $\frac{7}{11}$ from $\frac{10}{11}$? $\frac{2}{5}$ from $\frac{8}{5}$? $\frac{12}{7}$ from $\frac{15}{7}$?

4. A boy having $\frac{5}{2}$ of an orange, gave away $\frac{3}{4}$ of one ; what part of an orange had he left ?

For the Slate.

5. Alfred had $\frac{3}{4}$ of a dollar, when he gave his brother $\frac{3}{5}$ of a dollar ? what sum had he left ?

OPERATION.

4	3	3
5	5	4
——	——	——
20 common denominator.	15 $\frac{15}{20}$	12 $\frac{12}{20}$

$\frac{12}{20}$ from $\frac{15}{20}$ leaves $\frac{3}{20}$. Ans.

Explanation. We first change $\frac{3}{4}$ and $\frac{3}{5}$ to a common denominator, and then $\frac{3}{4}$ becomes $\frac{15}{20}$, and $\frac{3}{5}$, $\frac{12}{20}$; now subtracting 12 twentieths from 15 twentieths we get $\frac{3}{20}$.

Therefore, to subtract one fraction from another,

Subtract one numerator from the other, taking care to change the fractions to a common denominator first, if the denominators be not alike.

6. A man who had $\frac{18}{20}$ of an acre of land, sold $\frac{3}{4}$ of an acre ; how much did he keep ? Ans. $\frac{1}{5}$ of an acre.

7. A merchant having $16\frac{1}{3}$ barrels of pork, sold $12\frac{1}{2}$ barrels ; how much had he left ? Ans. $3\frac{5}{6}$ barrels.

Explanation. After changing $\frac{1}{3}$ and $\frac{1}{2}$ to a common denominator, we have $12\frac{3}{6}$ to take from $16\frac{2}{6}$; now as we cannot take $\frac{3}{6}$ from $\frac{2}{6}$, we suppose 1, or $\frac{6}{6}$, to be added to $\frac{2}{6}$, then take $\frac{3}{6}$ from $\frac{8}{6}$, and 12 from the remaining 15.

8. A man having 5 dollars, lost $1\frac{1}{4}$ dollar ; how much had he left ? Ans. $3\frac{3}{4}$ dollars.

Explain how example 5, lesson 69, is performed.
How do we subtract one fraction from another ?

9. A man owning $\frac{45}{1000}$ of the stock of a bank, sold $\frac{6}{1000}$; how much had he left? Ans. $\frac{39}{1000}$.

10. If I have $10\frac{9}{10}$ dollars, and spend $3\frac{3}{4}$ dollars, how much money have I left? Ans. $7\frac{3}{20}$ dollars.

11. What is the difference between $\frac{18}{95}$ and $\frac{48}{56}$?
Ans. $\frac{3552}{5320}$.

12. If I own $\frac{1}{12}$ of a factory, and another man $\frac{1}{16}$, which owns the most of it, and how much?
Ans. I own the most by $\frac{1}{48}$.

MULTIPLICATION OF FRACTIONS.

LESSON 70.

To multiply a whole number and a fraction.

To be performed in the mind.

1. What is $\frac{1}{2}$ of 2 dollars? Of 4 dollars? Of 10 dollars? Of 18 dollars? $\frac{1}{3}$ of 3 dollars? Of 6 dollars? Of 12 dollars? $\frac{1}{4}$ of 16 dollars? Of 24 dollars?

2. What do 2 times $\frac{1}{2}$ of a dollar make? 3 times $\frac{1}{4}$ of a dollar? 2 times $\frac{2}{5}$ of a dollar? 4 times $\frac{2}{5}$ of a dollar? 3 times $\frac{5}{4}$ of a dollar?

3. Samuel has $\frac{2}{5}$ of 5 walnuts; what number is that? *Explanation.* What is $\frac{1}{5}$ of them? Then what is $\frac{2}{5}$ of them?

4. A man bought 4 gallons of molasses at $\frac{3}{8}$ of a dollar a gallon, what sum did he give for it?

For the Slate.

5. David got $\frac{3}{4}$ of an apple at 2 different times; how much did he obtain at both times?

OPERATION.

3
2
–
6
$\frac{6}{4}$ or $\frac{3}{2}$ or $1\frac{1}{2}$ apple. Ans.

ALSO.

2)4
–
2
$\frac{3}{2}$ or $1\frac{1}{2}$ apple. Ans.

Explanation. We take the 3 fourths 2 times, which plainly gives $\frac{6}{4}$ or $\frac{3}{2}$ or $1\frac{1}{2}$ for the answer. We can also get the answer by dividing the denominator of $\frac{3}{4}$ by 2; for if we take the answer just obtained, $\frac{6}{4}$, and divide both 6 and 4 by 2, which will not alter the value of the fraction, we shall evidently get the numerator of $\frac{3}{4}$, and the denominator divided by 2.

Explain how example 5, lesson 70, is performed.

Therefore, to multiply a whole number and a fraction,

Divide the denominator by the whole number when it can be done without a remainder, and when not, multiply the numerator by the whole number.

6. A farmer agreed to give a laborer $\frac{3}{16}$ of all the potatoes he dug; after digging 48 bushels what quantity must he receive? Ans. 9 bushels.

7. 5 men owned $\frac{4}{25}$ of a tract of land apiece; what part of it do they all own? Ans. $\frac{4}{5}$.

8. What are 25 times $2\frac{38}{50}$? Ans. 69.

Explanation. What are 25 times 2 and 25 times $\frac{38}{50}$?

9. 8 men possess 2,400 dollars in equal shares; what is each one's part, or $\frac{1}{8}$ of the whole? Ans. 300 dollars.

10. Multiply $\frac{4}{5}$ by 30. Ans. 24.

11. Multiply 45 by $\frac{17}{900}$. Ans. $\frac{17}{20}$.

12. 16,000 acres of land were divided into lots, each of which contained $\frac{3}{150}$ of the whole; how much land was there in each lot? Ans. 320 acres.

LESSON 71.

To multiply a fraction by a fraction.

To be performed in the mind.

1. If you cut an apple into 3 equal portions, what part of an apple will each portion be? If you now cut each of these portions into halves, what part of an apple will each of the new portions be? What is $\frac{1}{2}$ of $\frac{1}{3}$ then? What is $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{4}$?

2. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{3}{4}$?

3. What part of an orange is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{2}{3}$ of $\frac{1}{5}$? $\frac{2}{3}$ of $\frac{2}{5}$?

4. A trader sold $\frac{2}{3}$ of 3 half-barrels of flour, that is, $\frac{2}{3}$ of $\frac{3}{2}$ of a barrel; what quantity was that?

For the Slate.

5. A man having $\frac{2}{3}$ of an acre of land, sold $\frac{3}{4}$ of it; how much land did he sell?

How do we multiply a whole number and a fraction?

OPERATION.

3 denominator of $\frac{2}{3}$ 2 numerator of $\frac{2}{3}$
4 denominator of $\frac{3}{4}$ 3 numerator of $\frac{3}{4}$
— —
12 6
$\frac{6}{12}$ or $\frac{1}{2}$ of an acre. Ans.

Explanation. We first multiply the denominator 3, of $\frac{2}{3}$, by the denominator 4, of $\frac{3}{4}$, which is the same as taking $\frac{1}{4}$ of the 2 thirds; because the 2 thirds are then divided by a number 4 times as large as before, and of course the quotient is $\frac{1}{4}$ of what it was. Having now $\frac{1}{4}$ of $\frac{2}{3}$, we multiply the numerator 2, by 3, which gives 3 times $\frac{1}{4}$ of $\frac{2}{3}$, or $\frac{3}{4}$ of $\frac{2}{3}$.

Therefore, to multiply one fraction by another,

Multiply the denominators together for a new denominator, and the numerators together for a new numerator.

6. A man who owned $\frac{5}{12}$ of a ship, bequeathed to his son $\frac{4}{5}$ of his portion; what part of the ship was that ? Ans. $\frac{1}{3}$.
7. If you have $3\frac{1}{2}$ dollars, and spend $\frac{1}{2}$ of it, what sum will that be ? Ans. $1\frac{3}{4}$ dollar.

Explanation. Change $3\frac{1}{2}$ to an improper fraction first.

8. Multiply $2\frac{1}{3}$ by $1\frac{2}{5}$. Ans. $3\frac{4}{15}$.
9. Multiply $\frac{2}{5}$ of $\frac{3}{4}$ by $\frac{5}{8}$. Ans. $\frac{3}{16}$.
10. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{9}$? Ans. $\frac{7}{27}$.
11. A boy having $\frac{9}{10}$ of a dollar, spent $\frac{3}{5}$ of it ; what part of a dollar was that ? Ans. $\frac{27}{50}$.
12. A manufacturing company divided $25\frac{1}{4}$ dollars on each share owned in it ; what was paid on $7\frac{1}{2}$ shares ? Ans. $189\frac{3}{8}$ dollars.

DIVISION OF FRACTIONS.

Lesson 72.

To divide a fraction by a whole number.

To be performed in the mind.

1. If 3 pecks of corn are worth $\frac{3}{4}$ of a dollar, what is one peck worth ?
2. If 2 apples are worth $\frac{1}{2}$ of a cent, what is one apple worth ?

Explain how example 5, lesson 71, is performed.
How do we multiply one fraction by another ?

3. 2 boys have $\frac{6}{7}$ of a piece of gingerbread ; what is the share of each ? What would be the share of each if they had $\frac{1}{8}$ of a piece ? $\frac{2}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{8}{8}$?

4. $\frac{3}{4}$ of a bushel of oats was given to 4 horses ; what part of a bushel did one horse have ?

For the Slate.

5. 2 men owned equal shares of $\frac{8}{9}$ of a ship ; what part of the ship belonged to each ?

OPERATION.
2)8
–
4
$\frac{4}{9}$. Ans.
ALSO.
9
2
——
18
$\frac{8}{18}$ or $\frac{4}{9}$. Ans.

Explanation. We divide 8 ninths into 2 equal parts, and get $\frac{4}{9}$ for the answer. We can also get the answer by multiplying the denominator 9, by 2 ; for if we take the answer just obtained, $\frac{4}{9}$, and multiply both 4 and 9 by 2, which will not alter the value of the fraction, we shall evidently get the numerator of $\frac{8}{9}$, and the denominator multiplied by 2.

Therefore, to divide a fraction by a whole number,

Divide the numerator when it can be done without a remainder, and when not, multiply the denominator.

6. 10 boys share $\frac{1}{2}$ of a dollar equally ; what is the part of each ? Ans. $\frac{1}{20}$ of a dollar.

7. Divide $5\frac{3}{8}$ by 4. Ans. $\frac{43}{32}$.

Explanation. First change $5\frac{3}{8}$ to an improper fraction.

8. Divide $\frac{49}{19}$ by 7. Ans. $\frac{7}{19}$.

9. $\frac{145}{12}$ is 17 times what number ? Ans. $\frac{145}{204}$.

10. A man owning $\frac{19}{20}$ of a factory, divided his part into 5 portions for sale ; what part of the factory did he put in each portion ? Ans. $\frac{19}{100}$.

11. If 25 men are to receive $100\frac{5}{8}$ dollars in equal shares, what will be the share of each ? Ans. $4\frac{1}{40}$ dollars

12. Divide $\frac{87}{12}$ by 105. Ans. $\frac{87}{1260}$.

13 $\frac{175}{400}$ of an acre produced 25 bushels of corn ; what part of an acre produced one bushel ? Ans. $\frac{7}{400}$.

Explain how example 5, lesson 72, is performed.
How do we divide a fraction by a whole number ?

LESSON 73.

To divide a whole number by a fraction.

To be performed in the mind.

1. How many times is $\frac{1}{2}$ of an apple in 5 apples ?

2. How many times must a man carry away $\frac{2}{3}$ of a bushel of potatoes to get 12 bushels ? *Explanation.* How many times must he carry away 1 third of a bushel to get 12 bushels ? Then how many times must he carry away 2 thirds of a bushel to get 12 bushels ?

3. How many yards of cloth can I buy for 27 dollars at $\frac{3}{4}$ of a dollar a yard ?

4. How many times is $\frac{2}{5}$ contained in 20 ? $\frac{5}{6}$ in 20 ? $\frac{3}{4}$ in 12 ?

For the Slate.

5. How many perches of stone can I buy for 16 dollars, at $\frac{5}{8}$ of a dollar a perch ?

OPERATION.

```
 1 6
   8
———
1 2 8
```

$\frac{128}{5}$ or $25\frac{3}{5}$ perches. Ans.

Explanation. If the price had been $\frac{1}{8}$ of a dollar, I could evidently have bought 8 times 16, or 128 perches, but the price being 5 times as much, that is $\frac{5}{8}$ of a dollar, I must divide 128 by 5 to get the answer.

Therefore, to divide a whole number by a fraction,

Multiply by the denominator, and divide by the numerator.

6. A number of men are to be paid $\frac{7}{8}$ of a dollar apiece; how many will 56 dollars pay ? Ans. 64.

7. How many yards of broadcloth can I buy for 110 dollars at $5\frac{1}{2}$ dollars a yard ? Ans. 20.

Explanation. Change $5\frac{1}{2}$ to an improper fraction first.

8. Divide 87 by $\frac{12}{5}$. Ans. $36\frac{1}{4}$.

9. 100 is $\frac{1}{12}$ of what number ? Ans. 1,200.

10. Divide 112 by $\frac{3}{8}$. Ans. $298\frac{2}{3}$.

11. Divide 42 by $\frac{84}{160}$. Ans. 80.

12. How many barrels of flour can I buy for 62 dollars at $7\frac{3}{4}$ dollars a barrel ? Ans. 8.

13. How far can I travel for 25 dollars at $\frac{1}{16}$ of a dollar a mile ? Ans. 400 miles.

Explain how example 5, lesson 73, is performed.
How do we divide a whole number by a fraction ?

LESSON 74.

To divide a fraction by a fraction.

To be performed in the mind.

1. If you can get a pound of butter for $\frac{1}{4}$ of a dollar, how many pounds can you get for $\frac{3}{4}$ of a dollar ?

2. How many gallons of molasses can I obtain for $\frac{4}{5}$ of a dollar, the price of one gallon being $\frac{2}{5}$ of a dollar ?

3. If you give $\frac{1}{8}$ of a dollar for a sheet of drawing-paper, how many sheets of such paper can you get for $\frac{1}{2}$ of a dollar ? For $\frac{3}{4}$ of a dollar ? For $\frac{5}{8}$ of a dollar ?

4. How many times is $\frac{2}{6}$ contained in $\frac{2}{3}$?

For the Slate.

5. How many penknives at $\frac{3}{8}$ of a dollar apiece can I buy for $\frac{3}{4}$ of a dollar ?

OPERATION.

$$\begin{array}{cc} 4 & 3 \\ 3 & 8 \\ \hline 12 & 24 \end{array}$$

$\frac{24}{12}$ or 2 penknives. Ans.

Explanation. We divide $\frac{3}{4}$ by 3, the numerator of $\frac{3}{8}$, by multiplying the denominator 4 by 3 ; as we now have divided $\frac{3}{4}$ by a number 8 times too large, we multiply the numerator by 8.

Therefore, to divide a fraction by a fraction,

Divide by the numerator of the divisor, and multiply by its denominator.

Note. Divide and multiply as directed in lessons 72 and 70.

6. A man wishes to give some boys $\frac{3}{20}$ of a dollar apiece; how many can he give that sum to, if he has $\frac{9}{10}$ of a dollar ? Ans. 6.

7. Divide $6\frac{4}{5}$ by $\frac{3}{10}$. Ans. $22\frac{2}{3}$.

Explanation. Change $6\frac{4}{5}$ to an improper fraction first.

8. Divide $10\frac{5}{10}$ by $1\frac{1}{4}$. Ans. $8\frac{2}{5}$.

9. Divide $2\frac{2}{5}$ by $11\frac{7}{8}$. Ans. $\frac{96}{475}$.

10. How many yards of cloth at $\frac{1}{8}$ of a dollar a yard, can I get for $\frac{5}{6}$ of a dollar ? Ans. $6\frac{2}{3}$.

11. A man who owned $\frac{9}{15}$ of a ship, said that his part was ten times as large as that of another man, who owned $\frac{3}{40}$; was he correct ? Ans. No. How many times as large was his part ? Ans. 8 times.

12. Divide $\frac{8}{25}$ by $\frac{4}{5}$. Ans. $\frac{2}{5}$.

13. Divide $\frac{35}{230}$ by $\frac{3}{100}$. Ans. $5\frac{5}{69}$.

Explain how example 5, lesson 74, is performed.
How do we divide a fraction by a fraction ?

DECIMAL FRACTIONS.

LESSON 75.

Common fractions are sufficient for all arithmetical operations; but there is still another sort, extremely convenient and useful. We have seen in Numeration, that in whole numbers, a figure put at the right of another is ten times as small, or only one tenth as large as though it was in the place of that other figure. This regular diminution in value, is continued beyond units, so that the first figure at the right of units is one tenth part as much as though it was in the units' place, the second figure one hundredth part as much as though it was in the units' place, the third figure one thousandth part as much as though it was in the units' place, and so on. Such figures at the right of units are called *decimal fractions*.

A point is always placed at the left of decimal fractions to distinguish them from whole numbers; as in the following table.

Tens of millions.	MILLIONS.	Hundreds of thousands.	Tens of thousands.	THOUSANDS.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	THOUSANDTHS.	Ten thousandths.	Hundred thousandths.	MILLIONTHS.	Ten millionths.
4	8,	5	2	6,	4	5	9.	3	6	0	2	8	7	1

On the preceding principle we find,

25.3 to be $25\frac{3}{10}$.
25.37 to be $25\frac{3}{10}$ and $\frac{7}{100}$ or $25\frac{37}{100}$.
25.371 to be $25\frac{3}{10}$ and $\frac{7}{100}$ and $\frac{1}{1000}$ or $25\frac{371}{1000}$.
25.3714 to be $25\frac{3}{10}$ and $\frac{7}{100}$ and $\frac{1}{1000}$ and $\frac{4}{10000}$ or $25\frac{3714}{10000}$.

For what are common fractions sufficient? What have we seen in Numeration? How is this regular diminution in value continued? What are called *decimal fractions*?

What is always placed* at the left of decimal fractions? For what *purpose?

It appears, then, that we get the value of a decimal fraction in a common fraction, by writing a denominator beneath, consisting of 1, with as many 0s at the right of it as there are figures in the decimal fraction. A decimal fraction is always read as though it was written in such a common fraction.

Read the following numbers from your slate.

1. ...... .05
2. .... 3.5
3. ...... .205
4. ... 47.537
5. ...... .008
6. .. 458.00037
7. ...... .4847213
8. .. 375.802
9. .... 6.2801
10. ...... .0202
11. ...... .12222
12. .... 3.333
13. ...... .000001
14. ... 19.30003
15. ... 50.008

When there are no tenths, no hundredths, or no thousandths, &c. in a decimal fraction, 0 is put in place of tenths, hundredths, or thousandths, &c.; thus $\frac{2}{100}$ is written .02 in decimals, $\frac{2}{1000}$ is written .002, and $6\frac{102}{10000}$ is written 6.0102.

0 prefixed to a decimal fraction, that is, placed before it, makes it ten times as small as before; thus .5 is $\frac{5}{10}$, .05 is $\frac{5}{100}$, and .005 is $\frac{5}{1000}$. 0 annexed to a decimal fraction, that is, placed after it, does not alter its value; thus .5 is $\frac{5}{10}$, .50 is $\frac{50}{100}$ equal to $\frac{5}{10}$, and .500 is $\frac{500}{1000}$ equal to $\frac{5}{10}$.

Write the following numbers in figures on your slate.

1. Five, and six tenths.
2. Twelve one hundredths.
3. Four one thousandths.
4. 700, and 375 one thousandths.
5. Six ten thousandths.
6. Two hundred, and one one hundred thousandth.
7. Twenty-five, and forty-seven thousand and twelve ten millionths.
8. Seventeen hundredths.
9. Fifty-six thousandths.
10. One hundred and one thousandths.

How do we get the value of a decimal fraction in a common fraction? How is a decimal fraction always read?

When there are no tenths, no hundredths, or no thousandths, &c., what is put in the place of tenths, hundredths, or thousandths? How then is $\frac{2}{100}$ written in decimals? $\frac{2}{1000}$? $6\frac{102}{10000}$?

What is the effect if 0 be prefixed to a decimal fraction, that is, placed before it? What then is .5? .05? .005? What is the effect if 0 be annexed to a decimal fraction, that is, placed after it? What then is .5? .50? .500?

11. 12.000, and 12 ten thousandths.

12. 1.000.000, and 505 ten thousandths.

13. Two hundred and five, and six one hundred thousandths.

14. One thousand one hundred and eighteen ten thousandths.

15. 16, and 12 hundredths.

16. Eleven billionths.

LESSON 76.

To change a common fraction to a decimal fraction.

For the Slate.

1. 2 boys are to have equal shares in 1 apple; what will be each one's part expressed in decimals? *Observation.* It is plain that the share of each in common fractions will be $\frac{1}{2}$; we obtain the value of $\frac{1}{2}$ in decimals as follows,

OPERATION.

```
2)1.0(.5. Ans.
  1 0
  ——
```

Explanation. We cannot divide the numerator 1, by 2, the denominator, so we annex 0 to 1, and consider it as 10 tenths; 10 tenths divided into 2 parts give 5 tenths, or .5 for each part.

2. Change $\frac{2}{27}$ to a decimal fraction.

OPERATION.

```
27)2.00(.07407. Ans.
   189
   ———
    110
    108
    ————
      200
      189
      ———
       11
```

Explanation. We cannot divide 2 by 27, so we annex 0 to 2, and consider it as 20 tenths; but we cannot divide 20 tenths by 27, so we annex another 0 to 2, and consider the dividend as 200 hundredths, which divided into 27 parts gives 7 hundredths for each part with 11 hundredths remainder. The 11 hundredths remainder we change to thousandths by annexing 0; we then divide by 27, and get 4 thousandths with 2 thousandths remainder, and so on. There are no tenths in the quotient, the first

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Explain how example 1, lesson 76, is performed.
Explain how example 2, lesson 76, is performed.

figure being 7 hundredths, we therefore put 0 in the tenth's place. We evidently get as many decimals in the quotient as we annex 0s to the dividend and remainders.

Note. The operation can be carried further than has been done here, but the next figure obtained, being $\frac{1}{1000000}$ of 1, is too small to be noticed; indeed it is not generally necessary to carry the operation so far as we have done, for practical purposes.

Therefore, to change a common fraction to a decimal fraction,

Divide the numerator by the denominator, annexing as many 0s to the numerator, and to each remainder, as may be necessary in order to perform the division, and to carry the operation as far as desired. Place your point in the quotient so as to cut off as many figures for decimals as there have been 0s annexed to the numerator and remainders.

If there be not figures enough in the quotient for decimals, supply the deficiency by prefixing 0s.

3. 4 men had equal shares in 3 dollars; what was the portion of each in decimals? Ans. .75 of a dollar.

4. Change $\frac{3}{2}$ to decimals. Ans. 1.5.

5. Change $\frac{1}{64}$ to decimals. Ans. .015625.

6. 12 men owned 45 bushels of corn in equal shares; how many bushels and parts, in decimals, did each own? Ans. 3.75 bushels.

7. Change $\frac{3}{2400}$ to decimals. Ans. .00125.

8. 7 acres of land were divided into 5 equal parts; what portion of an acre was each part in decimals? Ans. 1.4 acre.

9. Change $\frac{5}{8}$ to decimals. Ans. .625.

10. Change $2\frac{14}{25}$ to decimals. Ans. 2.56.

LESSON 77.

To write decimals when some are omitted.

1. If 30 dollars are to be paid to 7 men in equal shares, how many dollars and cents, or hundredths of a dollar, should each receive?

How many decimals do we get?
How do we change a common fraction to a decimal fraction?
What if there be not figures enough in the quotient for decimals?

OPERATION.
7)30

4.28571 &c.
4.29 dollars, or 4 dollars and 29 cents. Ans.

Explanation. As we are to express nothing less than hundredths, we write 4.29, and not 4.28, for 4.29 is nearer 4.28571, &c. than 4.28, since the value of the first decimal omitted is .005, or half of .01, and of course the value of all the decimals omitted, or .00571, &c., is more than half of .01.

If we express but two decimals in the numbers 4.286, 4.287, 4.288, or 4.289, it is also plain that we should write 4.29 and not 4.28.

If we express but two decimals in the numbers 4.2847, 4.2837, 4.2827, or 4.2817, we must write 4.28 and not 4.29, because the value of the decimals omitted is less than .005, or half of .01.

Therefore, when we omit any decimals,

Add 1 *to the last decimal expressed, if the first decimal omitted be more than* 5, *or if there be more than one decimal omitted, and the first be* 5.

If the first decimal omitted be less than 5, the last decimal expressed should *not* be increased by 1.

2. Change $\frac{2}{3}$ to decimals. Ans. .6667, nearly.
3. Change $\frac{1}{12}$ to decimals. Ans. .0833, about.
4. 13 acres of land are to be divided into 7 equal parts; how much in acres and decimals will there be in each part? Ans. 1.857 acres, about.
5. Change $\frac{121}{1707}$ to decimals. Ans. .070885, nearly.
6. A man gave 74 dollars for 311 bushels of potatoes; how many bushels did he get for 1 dollar? Ans. 4.2, about.

To change a decimal fraction to a common fraction.

To change a decimal fraction to a common fraction, proceed as directed in lesson 75, that is,

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Explain how example 1, lesson 77, is performed.
Mention some other instances in which the last decimal expressed is to be increased by 1.
Mention some instances in which the last decimal expressed is not to be increased. Why not?
When should we add 1 to the last decimal expressed? When not?

Write a denominator beneath consisting of 1 *with as many* 0*s annexed as there are decimals.*

The common fraction thus obtained can be changed to its simplest form if desired.

7. What is the simplest form in which .75 can be expressed in a common fraction? Ans. $\frac{3}{4}$.
8. Change .00125 to a common fraction, and this fraction to its simplest form. Ans. $\frac{1}{800}$.
9. What is the simplest form in which six hundred and twenty-five one thousandths can be expressed as a common fraction? Ans. $\frac{5}{8}$.
10. Express 35.36 in a whole number and a common fraction in its simplest form. Ans. $35\frac{9}{25}$.
11. What common fraction in its simplest form is equal to .688? Ans. $\frac{86}{125}$.

ADDITION OF DECIMALS.

LESSON 78.

1. A farmer sold at different times the following quantities of hay, 14.125 tons, .75 of a ton, .5 of a ton, 1.0625 ton, and 4 tons; what was the whole quantity he sold?

OPERATION.

```
14.125
  .75
  .5
 1.0625
 4.
----------
20.4375 tons. Ans.
```

Explanation. We write the quantities with tenths under tenths, hundredths under hundredths, &c., and as ten one thousandths make one hundredth, ten one hundredths make one tenth, and ten tenths make one, it is plain that we must add and carry just as in whole numbers.

Therefore, to add when there are decimals,

Write the numbers with tenths under tenths, hundredths under hundredths, &c., and then add as in whole numbers; being careful to put a point at the left of the tenths in all cases.

2. Add .00004, .27, 451, and 13.003.
3. Rufus had $\frac{3}{4}$ of a dollar, Josiah $\frac{2}{3}$ of a dollar, Elias $\frac{4}{5}$

How do we change a decimal fraction to a common fraction?
What can be done to the common fraction thus obtained?
Explain how example 1, lesson 78, is performed.
How do we add when there are decimals?

of a dollar, and Philip $\frac{1}{2}$ of a dollar; change each of these fractions to decimals, add them, and tell what they all had.
Ans. 2.717 dollars, nearly.

4. Change the following fractions to decimals, and add them, and the whole numbers ; $7\frac{9}{10}$, $\frac{12}{4}$, $\frac{4}{9}$, $\frac{17}{20}$.
Ans. 12.1944, about.

5. A man sold .875 of a cord of wood to one person, 2.0625 cords to another, and .25 of a cord to a third ; how much did he sell to all of them ? Ans. 3.1875 cords.

6. Add 27.01, 251, .3801, and 3.8. Ans. 282.1901.

7. What is the sum of .75, .25, and .50 expressed in whole numbers and common fractions ? Ans. $1\frac{1}{2}$.

8. What is the sum of five, and seventy-five hundredths, four, and eight thousandths, and two hundred and three ten thousandths ? Ans. 9.7783.

9. Add $\frac{4}{10}$, $17\frac{20}{100}$, $6\frac{2}{1000}$, and $\frac{11}{10000}$, the fractions being first changed to decimals. Ans. 23.6031.

10. James had $\frac{1}{2}$ of a dollar, $\frac{1}{8}$ of a dollar, $\frac{3}{4}$ of a dollar, $\frac{1}{10}$ of a dollar, and $\frac{1}{20}$ of a dollar ; change each of these fractions to decimals, and then find what sum he had.
Ans. 1.525 dollar.

SUBTRACTION OF DECIMALS.

LESSON 79.

1. A man who owned 4.5 acres of land, sold .875 of an acre ; what quantity of land had he left ?

OPERATION.

```
4.5
 .8 7 5
———————
3.6 2 5 acres. Ans.
```

Explanation. We write the smaller number under the larger, with tenths under tenths, &c., and subtract as in whole numbers, for the same reason that we add decimals as whole numbers. The upper number is of just the same value as though two 0s were placed at the right of it. We may imagine them there, or place them there before subtracting if we please.

Therefore, to subtract where there are decimals,

Write the smaller number under the larger, with tenths under tenths, hundredths under hundredths, &c., and then

---

Explain how example 1, lesson 79, is performed.
***How do we subtract* when there are decimals?**

subtract as in whole numbers ; being careful to put a point at the left of the tenths in all cases.

2. Subtract 3.0175 from 4.

3. Ephraim having $\frac{5}{8}$ of a dollar, spent $\frac{3}{20}$ of a dollar ; change these fractions to decimals, and tell me how much he had left. Ans. .475 of a dollar.

4. What is the difference between .875 of a dollar, and .75 of a dollar ? Ans. .125 of a dollar.

5. A merchant who owned $\frac{4}{5}$ of a ship, sold $\frac{2}{3}$ of her ; what part of her, expressed in decimals, did he retain ? Ans. .1333, about.

6. From 5 take .5. Ans. 4.5.

7. A man who had three hundred and seventy-five one thousandths of a dollar, gave a boy six hundred and twenty-five ten thousandths of a dollar ; how much had he left ? Ans. .3125 of a dollar.

8. What is the difference between 17.375 and 11.00005 ? Ans. 6.37495.

9. Subtract .5625 from .625, and give the answer in a common fraction in its simplest form. Ans. $\frac{1}{16}$.

10. Andrew promised to be gone on an errand only $\frac{1}{12}$ of an hour, but stayed $\frac{55}{60}$ of an hour ; what part of an hour, in decimals, was he gone longer than he promised to be ? Ans. .833 of an hour, about

MULTIPLICATION OF DECIMALS.

LESSON 80.

1. A man having 1.3 acre of land, sold .02 of it ; how much was that ?

OPERATION.
1.3
.0 2
———
0 2 6 of an acre. Ans.

Explanation. 2 times 1.3 are 2.6, but as we multiply by .02, only $\frac{1}{100}$ of 2, the product is only $\frac{1}{100}$ of 2.6, or .026. We can show this result to be correct by another method ; thus, changing 1.3 and .02 to common fractions, we have $\frac{13}{10}$ and $\frac{2}{100}$, now multiplying these fractions together, we get $\frac{26}{1000}$, equal in decimals to .026.

The product .026, must contain as many decimals as

Explain how example 1, lesson 80, is performed.
How can we show this result to be correct by another method ?

there are decimals in 1.3 and .02; for .026 must contain as many decimals as there are 0s in the denominator of $\frac{26}{1000}$; see lesson 75, and the denominator of $\frac{26}{1000}$ must contain as many 0s as there are 0s in the denominators of $\frac{13}{10}$ and $\frac{2}{100}$; and the denominators of $\frac{13}{10}$ and $\frac{2}{100}$ must contain as many 0s as there are decimals in 1.3 and .02.

So there will always be as many decimals in a product as in both multiplier and multiplicand.

Therefore, to multiply numbers in which there are decimals,

Multiply as in whole numbers. In the product point off as many decimals as there are in both multiplier and multiplicand.

If there be more decimals in the multiplier and multiplicand than figures in the product, prefix 0s to the product to supply the deficiency.

2. What is the product of .043 by 12?

3. A farmer sold at different times, 8 parcels of butter, each of which contained 6.25 pounds; what was the whole number of pounds he disposed of? Ans. 50.

4. What must I give for five tenths of a bushel of corn, if the price be seventy-five hundredths of a dollar a bushel? Ans. .375 of a dollar.

5. How much is 1.2 bushel of wheat worth, at 1.75 dollar a bushel? Ans. 2.1 dollars.

6. 10 boys have .6 of a dollar apiece; how much have they all? Ans. 6 dollars.

7. Multiply .003 by .0009. Ans. .0000027.

8. A man had 21, .125 of a dollar; what sum had he? Ans. 2.625 dollars.

9. A farmer sold $6\frac{2}{3}$ tons of hay for $18\frac{1}{3}$ dollars; how much did he get for the whole in dollars and decimals? Ans. 122.22 dollars, about

10. What is the amount of .25 of .0125? Ans. .003125

11. 100 men were paid .75 of a dollar apiece; what sum did they all get? Ans. 75 dollars.

How many decimal figures must the product .026 contain? Explain why.

How many decimals will there always be in a product?

How do we multiply numbers in which there are decimals?

What if there be more decimals in the multiplier and multiplicand than *figures in the product?*

DIVISION OF DECIMALS.

LESSON 81.

The product of the divisor and quotient is the dividend, so, according to lesson 80, there must be the same number of decimals in the divisor and quotient as in the dividend, and of course as many decimals in any quotient as those in the dividend exceed those in the divisor.

1. 3.5 dollars are to be divided equally among 12 persons; what is each one's portion?

OPERATION.

12)3.5(.29 of a dollar, about. Ans.
24
——
110
108
——
2

Explanation. We cannot divide the 3 dollars by 12, so we consider the sum as 35 tenths of a dollar, which divided into 12 parts, give 2 tenths for each part with 11 tenths remainder. We annex 0 to the remainder 11, and considering it 110 hundredths, divide by 12, and get 9 hundredths, with a remainder 2, which being small, is neglected. The 0 annexed to 11 must be considered as one of the decimals of the dividend, as it is used just as though it had been originally placed in it.

2. How many times is .03 contained in 6?

OPERATION.

.03)6.00(200 times. Ans.
6
——
00

Explanation. We first change the 6 to 600 hundredths, which contain 3 hundredths 200 times.

3. .01 of an acre of land is .7 of a certain quantity; what is that quantity?

How many decimals are there in any quotient? Why?
Explain how example 1, lesson 81, is performed.
Explain how example 2, lesson 81, is performed.

OPERATION.

.7) .0 1 0 (.0 1 4 3 of an acre, nearly.
7 Ans.
———
3 0
2 8
———
2 0
2 1
—

Explanation. 7 tenths not being contained in the tenths, we get no whole number; not being contained in the hundredths, we get no tenths, but we get 1 hundredth, 4 thousandths, and nearly 3 ten thousandths.

Therefore, to divide one number by another, when there are decimals in both or either,

Proceed as in whole numbers, annexing as many 0s to the dividend and to each remainder, as may be necessary to perform the division, and to carry the operation as far as desired. In the quotient, point off as many decimals as the number of decimals in the dividend, including the 0s annexed to it, and to the remainders, exceeds the number of decimals in the divisor.

If the divisor have more decimals than the dividend, supply the deficiency by annexing 0s to the dividend before dividing.

If there be not figures enough in the quotient for decimals, supply the deficiency by prefixing 0s.

4. Divide .05 by .003.

5. 10 men are to receive equal shares in 27.25 dollars; what is each one's portion? Ans. 2.725 dollars.

6. A man bought 2.5 barrels of flour for 16 dollars; what price did he give a barrel? Ans. 6.4 dollars.

7. If you give .75 of a dollar for .125 of a yard of cloth, what price do you pay a yard? Ans. 6 dollars.

8. .43 of a mile was divided into 6 equal parts; what portion of a mile was there in each part?
Ans. .07167 of a mile, nearly.

Explain how example 3, lesson 81, is performed.

How do we divide one number by another when there are decimals in both or either?

What if the divisor have more decimals than the dividend?

What if there be not figures enough in the quotient for decimals?

9. Divide .0004 by .2. Ans. .002.

10. Divide 37 by .0403. Ans. 918.114, about.

11. A man gave 9 cents for .75 of a pound of cheese; how much must he have given for a pound? Ans. 12 cents.

12. If 375 bushels of potatoes be obtained for 7.5 tons of hay, how many bushels should you receive for 1 ton of hay? Ans. 50.

CONTRACTIONS IN THE USE OF DECIMALS.

Lesson 82.

When we have either of the decimal fractions in the following table to multiply or divide by, it will be much shorter and easier to employ the corresponding common fraction.

Recite the following Table.

.0625	is	$\frac{1}{16}$.
.125	is	$\frac{1}{8}$.
.25	is	$\frac{1}{4}$.
5	is	$\frac{1}{2}$.
.1666 &c.	is	$\frac{1}{6}$.
.333 &c.	is	$\frac{1}{3}$.
.666 &c.	is	$\frac{2}{3}$.
.8333 &c.	is	$\frac{5}{6}$.

1. What is .333 &c. of 27.15 dollars? Ans. 9.05 dollars.

2. Multiply 45 by .1666 &c. Ans. 7.5

3. Multiply 3.68 by .0625. Ans. .23.

4. A man owned .5 of 432.54 acres of land; what quantity was that? Ans. 216.27 acres.

5. What is .8333 &c. of .048? Ans. .04

6. Divide 7.61 by .0625. Ans. 121.76.

7. Divide 36 by .666 &c. Ans. 54.

8. Divide .037 by .25. Ans. .148.

9. .125 of a hogshead of molasses cost 4 dollars; what did the whole hogshead cost? Ans. 32 dollars.

10. Multiply 5640 by .125. Ans. 705.

11. What sum is .25 of 11.6 dollars? Ans. 2.9 dollars.

12. Divide .43 by .333 &c. Ans. 1.29.

PROMISCUOUS QUESTIONS IN FRACTIONS.

LESSON 83.

1. The head of a fish is $\frac{1}{4}$ of his whole length, his body is $\frac{1}{2}$ of his whole length, and his tail is 2 feet long; what is the length of the fish? Ans. 8 feet.

2. 4 boys found 7 large apples, which they agreed to share equally; what was each one's part in a whole number and a common fraction, and in a whole number and decimals? Ans. $1\frac{3}{4}$ and 1.75.

3. A man saved out of the wreck of his fortune 450 dollars, which was only $\frac{1}{20}$ of what he had possessed; what was his fortune before his loss? Ans. 9,000 dollars.

4. Subtract .875 from .9. Ans. .025.

5. Express $32\frac{2}{3}$ in a whole number and decimals. Ans. 32.667, nearly.

6. 12 is $\frac{2}{3}$ of what number? Ans. 18.

7. $\frac{1}{2}$ is $\frac{3}{4}$ of what number? Ans. $\frac{2}{3}$.

8. Change $9\frac{1}{4}$ to an improper fraction, the denominator of which shall be 16? Ans. $\frac{148}{16}$.

9. Now change this fraction to its simplest form. Ans. $\frac{37}{4}$.

10. A man had $\frac{1}{16}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{3}{2}$ of a dollar; how much money had he? Ans. $2\frac{3}{16}$ dollars.

11. Change .15 to a common fraction, and change this fraction to its simplest form. Ans. $\frac{3}{20}$.

12. $9\frac{4}{5}$ is $\frac{7}{8}$ of what number? Ans. $11\frac{1}{5}$.

13. $11\frac{2}{3}$ is $5\frac{3}{4}$ times what number? Ans. $2\frac{2}{69}$.

LESSON 84.

1. If you have $9\frac{3}{4}$ dollars and spend $7\frac{15}{16}$ dollars, how much will you have left? Ans. $1\frac{13}{16}$ dollar.

2. A gentleman made a will, giving $\frac{1}{2}$ of his estate to his wife, $\frac{1}{3}$ of it to his son, and the remainder, amounting to 2,000 dollars, to his daughter; how much was he worth? Ans. 12,000 dollars.

3. If you give $\frac{3}{8}$ of a dollar for $\frac{3}{4}$ of a bushel of wheat, what quantity can you get for 1 dollar? Ans. 2 bushels.

4. A man sold $\frac{2}{3}$ of $\frac{3}{8}$ of a ship for 3,000 dollars; what was the whole vessel worth at this rate? Ans. 12,000 dollars.

5. *What is .1667 of 2,472?* Ans. 412.

6. A teacher stated that $\frac{2}{3}$ of his scholars learned to read and write, that $\frac{3}{4}$ of the remainder learned geography and grammar, and that the rest, amounting to 5, learned arithmetic ; how many scholars did he have ? Ans. 60.

7. $\frac{3}{7}$ of a house is worth 516 dollars ; what is the value of the whole house ? Ans. 1,204 dollars.

8. If you give $\frac{3}{4}$ of a dollar for 1 bushel of corn, what must you give for $12\frac{1}{3}$ bushels ? Ans. $9\frac{1}{4}$ dollars.

9. .375 of a quantity of goods worth 4,000 dollars was destroyed during a fire ; what sum will a man lose who owned .12 of the whole ? Ans. 180 dollars.

10. A man sold .875 of a firkin of butter for 7 dollars ; what was the whole worth at this rate ? Ans. 8 dollars.

11. If you give .625 of a dollar for 1 gallon of molasses, what must you give for .8 of a gallon ? Ans. .5 of a dollar.

LESSON 85.

To be performed in the mind.

1. 15 ounces of honey were given to James and Henry, James receiving 3 ounces, and Henry 12 ounces ; what part of Henry's share was James' ? What part of James' share was Henry's ?

Finding what part of 12 ounces 3 ounces are, we call finding the ratio of 3 to 12, and finding what part of 3 ounces 12 ounces are, we call finding the ratio of 12 to 3.

Therefore, to find the ratio of one number to another,

Find what part of the second number the first is.

2. What is the ratio of James' share of the honey to Henry's ? Of Henry's share to James' ?

3. What is the ratio of 1 to 2 ? Of 2 to 4 ? Of 9 to 15 ? Of 100 to 3 ? Of 8 to 32 ? Of 32 to 8 ? Of 60 to 12 ?

4. In example 1, what part or proportion of the whole 15 ounces did James get, and what proportion of the whole did Henry get ?

Proportion is often used in the same sense as part, as in the preceding question. We also say that James and

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Finding what part of 12 ounces 3 ounces are, we call what ? Finding what part of 3 ounces 12 ounces are, we call what ?

How do we find the ratio of one number to another ?

In what sense is proportion often used ?

Henry shared the honey in the proportion of 3 to 12, and that Henry and James shared it in the proportion of 12 to 3, meaning that James had $\frac{3}{12}$ or $\frac{1}{3}$ as much as Henry, and Henry $\frac{12}{3}$ or 4 times as much as James.

5. A farmer mixed some rye and corn together in the proportion of 3 to 2; what part of the amount of rye was the amount of corn? What part of the amount of corn was the amount of rye?

6. If the mixture had been composed of $\frac{3}{4}$ as much corn as rye, in what proportion would the corn have been to the rye? In what proportion would the rye have been to the corn?

7. A man in his will divided his estate between his son and daughter in the proportion of 7 to 5, giving the son 3,500 dollars; what sum did the daughter get?

For the Slate.

8. A man's estate was divided among his two sons in the proportion of 12 to 17; the share of the first being 1,800 dollars, what was the share of the second?

Ans. 2,550 dollars.

Explanation. What part of the first one's share was that of the second?

9. Four men were paid a certain sum in the proportions of 2, 3, 5, and 7, the first receiving 8 dollars; what was given to each of the others?

Ans. the second had 12, the third 20, and the fourth 28 dollars.

Explanation. The shares of the first and second were in the proportion of 2 to 3, the shares of the second and third were in the proportion of 3 to 5, &c.

10. Four men, A, B, C, and D were weighed; A weighed 135 pounds, and B weighed 150; the weights of C and D were in the same proportion, but that of C was 180 pounds; what did D weigh? Ans. 200 pounds.

11. A is worth 4,500 dollars, and B 6,000 dollars; what part of B's property is that of A, the fraction being changed to its simplest form? Ans. A's property is $\frac{3}{4}$ of B's.

In what proportion do we say that James and Henry shared the honey? In what proportion do we say that Henry and James shared it? *What do we mean by this?*

LESSON 86.

1. A gardener raised 80 bushels of potatoes, and a neighboring farmer 640 bushels ; what part of the gardener's quantity was the farmer's ? Ans. $\frac{640}{80}$.

2. How many times the gardener's quantity was the farmer's ? Ans. 8 times.

3. 80 dollars were contributed to relieve a poor woman; what proportion of the whole did a man give who put in 5 dollars ? Ans. $\frac{1}{16}$.

4. What part of $\frac{1}{2}$ is $\frac{1}{3}$? Ans. $\frac{2}{3}$.

Explanation. To obtain the answer by the rule, we make $\frac{1}{2}$ the denominator, thus, $\frac{\frac{1}{3}}{\frac{1}{2}}$; now dividing the numerator by the denominator, we get $\frac{2}{3}$.

5. $6\frac{3}{4}$ dollars are given to John, Samuel, and William ; John received $2\frac{1}{4}$ dollars, Samuel $1\frac{1}{16}$ dollars, and William $2\frac{13}{16}$ dollars; what proportion of the whole did each receive ?

Ans. John received $\frac{1}{3}$, Samuel $\frac{1}{4}$, and William $\frac{5}{12}$.

6. A owns .0625 of a ship, and B .25 of her ; what part of B's share is A's ? Ans. $\frac{1}{4}$, or .25.

7. A man raised 16 bushels of wheat on one acre of land, and 19 bushels on another acre ; what was the average number of bushels he raised to the acre on the two pieces ? Ans. $17\frac{1}{2}$ bushels.

Explanation. Average signifies equal division ; to average means to divide equally.

8. The wages of 5 laborers are as follows ; the first receives $\frac{3}{4}$ of a dollar a day, the second $\frac{7}{8}$ of a dollar, the third 1 dollar, the fourth $1\frac{1}{2}$ dollar, and the fifth $1\frac{1}{2}$ dollar ; what wages do they get on an average ? Ans. $1\frac{1}{8}$ dollar.

9. A man measured a certain distance 3 times, the first time he made the distance 4,712.25 feet, the second time 4,710.85 feet, and third time 4,713.11 feet ; how far did he make it on an average ? Ans. 4,712.07 feet.

10. If you sell 11 bushels of corn for $9\frac{5}{8}$ dollars, 7 bushels for $6\frac{9}{16}$ dollars, and 7 bushels for $7\frac{13}{16}$ dollars, what is the average price that you get a bushel for the whole ?

Ans. $\frac{24}{25}$ of a dollar.

11. What is the average of the following numbers, 1, 2, 5, 17, and 34 ? Ans. 11.8.

What does average signify ? What does to average mean ?

FEDERAL MONEY.

LESSON 87.

In the United States money is reckoned in dollars, dimes, cents, and mills. The dollars are considered as whole numbers, and the dimes, cents, and mills as decimals ; the dimes being tenths, the cents hundredths, and the mills thousandths of a dollar.

10 mills make 1 cent.
10 cents make 1 dime.
10 dimes or 100 cents make 1 dollar.

The dollar mark $, placed before any figures, shows that they express this money, which is called Federal Money ; thus,

$5.257 signifies 5 dollars, 2 dimes, 5 cents, and 7 mills. It is not customary, however, to use the word dime, but dimes are expressed in cents ; thus,

$5.257 is read, 5 dollars, 25 cents, and 7 mills.
$5.20 is read, 5 dollars and 20 cents.
$35.05 is read, 35 dollars and 5 cents.
$435.207 is read, 435 dollars, 20 cents, and 7 mills.

Read the following sums from your slate.

1. ...$ 4.08
2. ...$ .234
3. ...$ .40
4. ...$28.004
5. ...$13.50
6. ...$ 500.55
7. ...$ 85.307
8. ...$6,334.093
9. ...$ 999.999
10. ...$ 7.001
11. ...$ 233.00
12. ...$ .005
13. ...$ .265
14. ...$13,760.08
15. ...$ .06

Write the following sums on your slate in figures.

1. Thirty-five cents.
2. Seventeen dollars three cents and five mills.

In what is money reckoned in the United States? What are considered as whole numbers in this money? What are considered as decimals? What are the dimes? Cents? Mills?

How many mills make one cent? Cents one dime? Dimes or cents one dollar?

What shows that figures express this money? What is this money called?

What does $5.257 signify? Is it customary to use the word dime? How are dimes expressed?

How do you read $5.257? $5.20? $35.05? $435.207?

3. Two hundred thousand dollars and twelve cents.

4. Seven hundred and fifteen dollars sixteen cents and five mills.

5. Eighteen dollars and seven mills.

6. Twenty-five dollars and fifty cents.

7. Eighty cents.

8. Five hundred and seven dollars thirty cents and one mill.

9. Thirteen cents and two mills.

10. Six cents.

11. Eighty dollars.

12. Two mills.

13. Ninety dollars thirty-seven cents and one mill.

14. Five cents and three mills.

15. Sixteen hundred and twelve dollars.

16. Five thousand one hundred and fifty dollars and fifty cents.

LESSON 88.

To be performed in the mind.

1. How many cents are there in $ 1? In $ 5? In $ 25? In $ 28? In $ 43?

2. A trader had 600 cents; how many dollars were they worth? How many dollars are 400 cents worth? 1,200 cents? 300 cents? 250 cents?

3. How many mills are there in $ .035? In $ .20? In $ 13? In $ 7? In $ 24?

For the Slate.

4. A man exchanged $ 42 for dimes, or ten cent pieces; how many did he receive? How many cents could he have got for the $ 42? How many mills were there in the $ 42? How many dimes are there in $ 42.20? How many cents? Mills? How many dimes are there in $ 42.25? How many cents? Mills? How many dimes are there in $ 42.259? How many cents? Mills?

5. How many cents are there in 1,600 mills? How many dimes? Dollars? How many cents are there in 1,650 mills? How many dimes? Dollars?

6. A man received 2,317 cents for a debt; how many dollars did he get?

7. How many dimes, or ten cent pieces, are 15 ten dollar bills worth? How many cents are they worth?

8. Change $ 5,827.37 to cents.
9. Change 83,254 mills to dollars.
10. A grocer wishes to get $ 250, in cents, for change; how many cents can he obtain for that sum?

LESSON 89.

For the Slate.

As Federal Money is composed of dollars and decimals of a dollar, we must add, subtract, multiply, and divide in it, as in Decimal Fractions.

1. A farmer owes the following sums to different persons; $ 1,325.043, $ 2,875, $ 835, $ 17.50, and $ .375; what is the whole amount of his debts? Ans. $ 2,180.793.
2. If a man gives you his note for $ 180, and afterwards pays you $ 2.50, how much will he then owe you? Ans. $ 177.50.
3. A tailor bought 27⅘ yards of broadcloth at $ 4.50 a yard; how much must he pay for it? Ans. $ 125.10.
Explanation. First change ⅘ to decimals.
4. A man paid $ 45.25 for 35 bushels of wheat; what price did he pay a bushel? Ans. $ 1.293, nearly.
5. A farmer sold 4 cows at $ 13.75 apiece, 3 calves at $ 4.33⅓ apiece, 33 bushels of potatoes at $ .25 a bushel, and 211⅔ pounds of cheese at $ .125 a pound; how much money did he sell the whole for? Ans. $ 102.708, about.
Explanation. Observe that $ .25 is ¼ of a dollar, and $ .125, ⅛ of a dollar.
6. A laborer earns $ 1.25 a day, and spends $ .41⅔ a day; how much will he save of what he obtains for 42 days' labor? Ans. $ 35.
7. 6 men owned equal shares in 13 barrels of pork, which they sold at $ 26 a barrel; what sum out of the proceeds must each receive? Ans. $ 56.333, about.
8. If salt be worth $ .75 a bushel, and corn $ 1.12½ a bushel, how much corn must I give for 83 bushels of salt? Ans. 55⅓ bushels.
9. Add $ 3,777.04, $ 12.057, and $ .12½. Ans. 3,789.222.
10. A trader owes $ 4,327.17, he has a house which will bring $ 2,500, $ 1,304.07 in cash, 4 hogsheads of molasses,

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How do we add, subtract, multiply, and divide in Federal Money? Why?

each of which contains 100 gallons, worth $.31¼ a gallon, sundry other goods worth $245, and various persons owe him $1,250; how much property will he have after paying his debts? Ans. $1,096.90.

Lesson 90.

The coins used in the United States, are the *eagle*, or ten-dollar piece, the *half eagle*, and the *quarter eagle*, which are of gold; the *dollar*, the *half dollar*, the *quarter of a dollar*, the *eighth of a dollar*, the *sixteenth of a dollar*, the *dime* or ten-cent piece, and the *half dime* or five-cent piece, which are of silver; and the *cent*, which is of copper. All of these pieces are American coins, except the dollar and quarter of a dollar, which are most commonly Spanish, and the eighth and sixteenth of a dollar, which are always Spanish coins.

The eighth of a dollar, or 12½ cent piece, is called a *ninepence* in New England, a *shilling* in New York and some other States, and a *levy* or an *elevenpenny bit*, or simply a *bit*, in Pennsylvania and in some other States.

The sixteenth of a dollar, or 6¼ cent piece, is called a *fourpence halfpenny* in New England, a *sixpence* in New York and in some other States, a *fip* or a *fivepenny bit* in Pennsylvania and in some other States, and a *pécune* or a *picayune* in some of the southwestern States.

The custom of dividing things into eighths and sixteenths, and the prevalence of eighths and sixteenths of a dollar, render it convenient to be familiar with the value of different numbers of these pieces.

Recite the following Table.

1 eighth of a dollar is	12½ cents.
2 eighths of a dollar are ¼ of a dollar or ..	25 cents.
3 eighths of a dollar are	37½ cents.

What coins are used in the United States, and of what metals are they composed? Which are American coins, and which Spanish? An eagle being worth $10, what is a half eagle worth? A quarter eagle? How many cents is a half dollar worth? A quarter of a dollar? An eighth of a dollar? A sixteenth of a dollar?

By what names is the eighth of a dollar, or 12½ cent piece called in different places? By what names is the sixteenth of a dollar, or 6¼ cent piece called in different places?

What is said of the custom of dividing things into eighths and sixteenths?

4 eighths of a dollar are ½ of a dollar or ..	50 cents.
5 eighths of a dollar are	62½ cents.
6 eighths of a dollar are ¾ of a dollar or ..	75 cents.
7 eighths of a dollar are	87½ cents.
8 eighths of a dollar are 1 dollar or	100 cents.

To be performed in the mind.

1. How many eighths of a dollar are 2 sixteenths? What then is the value, in cents, of 2 sixteenths of a dollar?

2. What is the value, in cents, of 3 sixteenths of a dollar?

3. How many eighths of a dollar are 4 sixteenths? What part of a dollar are 4 sixteenths?

4. What is the value, in cents, of 5 sixteenths of a dollar?

5. How many eighths of a dollar, and how many cents are 6 sixteenths worth?

6. How many cents are 7 sixteenths worth?

7. How many eighths of a dollar are 8 sixteenths worth? What part of a dollar do 8 sixteenths make?

8. How many cents are 9 sixteenths worth?

9. How many eighths, and how many cents are 10 sixteenths worth?

10. How many cents are 11 sixteenths worth?

11. How many eighths are 12 sixteenths worth? What part of a dollar do 12 sixteenths make?

12. How many cents are 13 sixteenths worth?

13. How many eighths, and how many cents are 14 sixteenths worth?

14. How many cents are 15 sixteenths worth?

15. How many eighths are 16 sixteenths worth? What part of a dollar do 16 sixteenths make?

LESSON 91.

To be performed in the mind.

1. James bought 5 pears at 3 cents apiece, and paid 1 eighth and 1 sixteenth for them; how much change must he receive back? If he had paid 2 eighths, how much change should he have received back?

2. If you buy a penknife for $.62½, and have 1 half dollar and 4 sixteenths in your purse, how will you contrive to pay or make change?

3. A man gave 1 dollar in payment for a book, and received 3 eighths back; what was the price of the book, *in cents?*

4. If you buy a quire of paper for $.25, some quills for 1 sixteenth, and an inkstand for 3 eighths, what sum must you pay for the whole ? How many sixteenths ?

5. At an auction, A bid six eighths for a book, and B 13 sixteenths ; what number of cents did each bid ? How much more did B bid than A ?

6. If you should buy a handkerchief in New York for 4 shillings, how many New England ninepences would you give in payment ? What sum would you have given if the price had been 4 shillings and sixpence ?

7. If a hackman demand 6 shillings for carrying you from the foot of Barclay street, in New York city, to the Astor House, how many cents should you pay him ?

8. If you buy articles of a trader in Philadelphia to the amount of 3 levies and 1 fip, how many cents should you pay him ?

9. A man was charged 5 picayunes for some luncheon in New Orleans, and paid with a 3-dollar bill ; how many dollars and cents should he receive back ?

10. If you carry a pocket full of ninepences and fourpence-ha'pennies from Boston to New York, what will you call them there ? What will you call them in Philadelphia ?

LESSON 92.

In New England 16⅔ cents are called a shilling, 8⅓ cents, or ½ of a shilling, are called sixpence, and 4⅙ cents, or ½ of a sixpence, are called threepence. There are no coins of these values, but prices are often named in these shillings, sixpences, and threepences.

Recite the following Table.

1 shilling is	16⅔ cents.
1 shilling and sixpence are..	25 cents.
2 shillings are	33⅓ cents.
2 shillings and sixpence are.	41⅔ cents.
3 shillings are	50 cents.
3 shillings and sixpence are.	58⅓ cents.
4 shillings are	66⅔ cents.
4 shillings and sixpence are .	75 cents.

What in New England are called a shilling ? Sixpence ? Threepence ? *Are there* any coins of these values ? What is often named in *these shillings, &c. ?*

5 shillings are	83⅓ cents.
5 shillings and sixpence are .	91⅔ cents.
6 shillings are	100 cents or 1 dollar.

To be performed in the mind.

1. If you buy 3 yards of calico in Boston, at 1 shilling a yard, what part of a dollar must you give for your purchase?

2. A man bought a knife in Providence for 4 shillings and sixpence ; how many cents did he give for it ? How many ninepences ? How many dimes ?

3. A man pays 15 shillings a week for his board in Portsmouth ; how many dollars and cents is that price ? *Explanation.* As 6 shillings make 1 dollar, find how many times 6 shillings there are in 15 shillings, and how many shillings over.

4. A laborer in Boston has 7 shillings and sixpence a day ; how many dollars and cents will he get in 4 days ?

5. If a laborer gets 6 shillings and ninepence for a day's work, and pays 2 shillings and sixpence out of it for a handkerchief, how many cents will he have left ?

6. A young man bought a pair of gloves in Lowell for 9 shillings ; how many dollars and cents did he pay ? If he had paid 8 shillings for them, how many dollars and cents would that sum have been ?

7. If you buy a pair of boots in Portland for 16 shillings, how many dollars and cents must you pay for them ?

8. If you buy a pair of stockings for 2 shillings and sixpence, and give 4 ninepences in payment, how much change must you receive back ?

9. A farmer bought 3 gallons of molasses at 2 shillings and threepence a gallon; what must he pay for it in dollars and cents ?

10. What number of cents are 5 shillings and threepence ? 7 shillings and threepence ? 9 shillings and threepence ? 12 shillings and threepence ?

COMPOUND NUMBERS.

Lesson 93.

Quantities are divided into parts of different sizes, for the purposes of traffic and convenience ; thus we measure salt in bushels, pecks, and quarts, 8 quarts making 1 peck, and 4 pecks 1 bushel. The parts or divisions are called *denominations*. Numbers expressing a quantity in different denominations are called *compound numbers*. Those heretofore used may be called *simple numbers*.

Recite the following Tables.

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is the common weight, and is used in weighing all common and coarse articles.

16 drams, sign dr......make 1 ounce, ...sign, oz.
16 ounces............make 1 pound, ...sign, lb.
28 pounds............make 1 quarter, ...sign, qr.
4 quarters, or 112 lbs. make 1 hundred weight, sign, cwt.
20 hundred weight ....make 1 ton,...........sign, T.

This is the old manner of weighing. At present it is usual to buy and sell by the 100 pounds, and when the term ton is employed, it generally means 2,000 pounds.

TROY WEIGHT.

Troy Weight is used in weighing gold and silver.

24 grains, sign gr......make 1 pennyweight, ..sign, pwt.
20 pennyweights.......make 1 ounce,........sign, oz.
12 ounces ....make 1 pound,........sign, lb.

How are quantities divided? For what purposes? Give an example. What are called denominations? Compound numbers? Simple numbers?

What is Avoirdupois Weight, and for what is it used?

What is it usual to buy and sell by, at present, in Avoirdupois Weight? What does the term ton generally mean?

For what is Troy Weight used?

APOTHECARIES' WEIGHT.

Apothecaries' Weight is used in compounding medicines, but not in selling them. They are sold by Avoirdupois Weight.

20 grains, sign gr.......... make 1 scruple, ...sign, ℈.
3 scruples .... make 1 dram, ...sign, ʒ.
8 drams.................. make 1 ounce, ...sign, ℥.
12 ounces .... make 1 pound, ...sign, ℔.

The pound and ounce in Apothecaries' Weight are the same as the pound and ounce in Troy Weight. 175 pounds Troy Weight, are equal to 144 pounds Avoirdupois Weight. There are 7,000 grains in 1 pound Avoirdupois, and 5,760 grains in 1 pound Troy.

LESSON 94.

LONG MEASURE.

Long Measure is used in measuring distances.

12 inches, sign in.. make 1 foot,..............sign, ft.
3 feet...........make 1 yard, ...sign, yd.
16½ feet..........make 1 rod, perch, or pole, sign, rod.
40 rods ....make 1 furlong, ...sign, fur.
8 furlongs ....make 1 mile,..............sign, m.

The following measures are used at sea.

6 feet............make 1 fathom, ...sign, fath.
3 miles ....make 1 league, ...sign, lea.

For what is Apothecaries' Weight used?

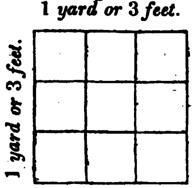
What is the difference between the pound and ounce Apothecaries' Weight, and the pound and ounce Troy Weight? How many pounds, Troy Weight, are equal to 144 pounds, Avoirdupois Weight? How many grains are there in 1 pound Avoirdupois Weight? In 1 pound Troy Weight?

For what is Long Measure used?

LAND, OR SQUARE MEASURE.

Land, or Square Measure, is used in measuring land or any surface.

Figure 1.



Explanation of this measure. A yard is 3 feet long, but a *square yard* is a surface 3 feet long and 3 feet wide, as figure 1. By examining this figure, we see that a space 1 foot wide and 3 feet long, contains 3 square feet; that a space 2 feet wide and 3 feet long, contains 2 times 3, or 6 square feet; and that a space 3 feet wide and 3 feet long, contains 3 times 3, or 9 square feet.

It is also evident from this, that a square foot, being 12 inches long and 12 inches wide, contains 12 times 12, or 144 square inches, &c.

144 square inches make 1 square foot,.....sign, sq. ft.
9 square feet...make 1 square yard, ...sign, sq. yd.
272¼ square feet..make 1 square rod, ...sign, sq. rod.
40 square rods ..make 1 quarter of an acre,sign, qr.
4 quarters.....make 1 acre,...........sign, A.
640 acres........make 1 square mile, ...sign, sq. mile.

CUBIC, OR SOLID MEASURE.

Cubic, or Solid Measure, is used in measuring bodies, or in finding the capacity of rooms, boxes, &c.

Explanation of this measure. A cubic yard is a body 3 feet long, 3 feet wide, and 3 feet high, as figure 2.

For what is Land, or Square Measure used?

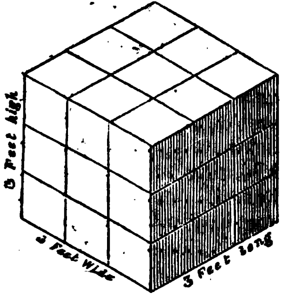
What is a square yard? Explain why a square yard contains 9 square feet.

How many square inches then does a square foot contain?

For what is Cubic, or Solid Measure used?

What is a cubic yard?

Figure 2.



By examining this figure, we see that the surface of the top contains 9 square feet; so if we take a piece off from the top 1 foot thick, we shall get 9 cubic feet; if we take a piece off from the top 2 feet thick, we shall get 2 times 9, or 18 cubic feet; and if we take up the whole, which is 3 feet thick, we shall get 3 times 9, or 27 cubic feet.

It is plain from this, that a cubic foot contains 12 times 144, or 1728 cubic inches.

1728 cubic inches......make 1 cubic foot, sign, cubic ft.
27 cubic feet..........make 1 cubic yard, sign, cubic yd.
16 cubic feet..........make 1 foot of wood, sign, ft.
8 feet of wood ....make 1 cord of wood, sign, C.
50 cubic feet of timber, whether hewed or round } make 1 ton,.........sign, T.
40 cubic feet of round timber, when $\frac{1}{5}$ is allowed for waste from knots, crooks, &c. make 1 ton, sign, T.

LESSON 95.

DRY MEASURE.

Dry Measure is used in measuring grain, fruit, potatoes, salt, coal, and other dry articles.

2 pints, sign pt. .....make 1 quart,........sign, qt.
8 quarts............make 1 peck,........sign, pk.
4 pecks ....make 1 bushel,.......sign, bu.
8 bushels ....make 1 quarter,......sign, qr.
36 bushels ....make 1 chaldron,.....sign, chal.

A bushel contains 2,150.4 cubic inches.

Explain why a cubic yard contains 27 cubic feet.
How many cubic inches then does a cubic foot contain?
When do 40 cubic feet make 1 ton?
For what is Dry Measure used?
How many cubic inches does a bushel contain?

BEER MEASURE.

Beer Measure is used in measuring ale, beer, and milk.

2 pints, sign pt.......make 1 quart, ...sign, qt.
4 quarts............make 1 gallon,........sign, gal.
9 gallons...........make 1 firkin, ...sign, fir.
2 firkins............make 1 kilderkin,......sign, kil.
36 gallons...........make 1 barrel,........sign, bl.
54 gallons...........make 1 hogshead......sign, hhd.
2 hogsheads........make 1 butt, ...sign, bt.

WINE MEASURE.

Wine Measure is used in measuring all liquors except ale, beer, and milk.

4 gills, sign gi........make 1 pint,..........sign, pt.
2 pints..............make 1 quart, ...sign, qt.
4 quarts............make 1 gallon,........sign, gal.
31½ gallons..........make 1 barrel,........sign, bl.
2 barrels, or 63 gals..make 1 hogshead,.....sign, hhd.
2 hogsheads.........make 1 pipe, ...sign, p.
2 pipes..............make 1 tun,..........sign, T.
42 gallons...........make 1 tierce, ...sign, tier.
2 tierces, or 84 gals. .make 1 puncheon,.....sign, pun.

In the United States ½ of a peck, sometimes called the Dry gallon, contains 268.8 cubic inches; the Beer gallon contains 282 cubic inches, and the Wine gallon 231 cubic inches. In Great Britain, since 1826, the *Imperial gallon*, containing 277.274 cubic inches, has been used in place of the Dry, Beer, and Wine gallons.

The *casks*, called hogsheads, are of various capacities, but usually contain more than 100 gallons.

For what is Beer Measure used?

For what is Wine Measure used?

How many cubic inches does each of the different kinds of gallons used in the United States, contain? What is now used in Great Britain in place of the Dry, Beer, and Wine gallons?

How much do the casks, called hogsheads, usually contain?

LESSON 96.

OF TIME.

60 seconds, sign sec.	make 1 minute,	sign, min.
60 minutes	make 1 hour,	sign, h.
24 hours	make 1 day,	sign, d.
7 days	make 1 week,	sign, w.
365 days	make 1 year,	sign, yr.

There are 12 calendar months in a year, each of which contains 31 days, except April, June, September, and November, which have 30 days, and February, which has 28. There are really 365 days, 5 hours, 48 minutes, and 49.7 seconds in a year, or nearly 365¼ days; so one year in four, we give to February 29 days, thereby making the year consist of 366 days; such a year is called *leap year.* As this allowance is a little too much, we omit 3 leap years in 400 years. Any year, at the end of a century, that can be divided by 400 without a remainder, is leap year, as 1200, 1600, 2000. Any other year that can be divided by 4 without a remainder, is also leap year, as 1836, 1840, 1844.

4 weeks are sometimes called a month.

DIVISION OF CIRCLES.

The circumference of every circle, whether great or small, is considered to be divided into 360 equal parts, called degrees.

60 seconds, sign ″	make 1 minute,	sign, ′.
60 minutes	make 1 degree,	sign, °.
360 degrees	make 1 circumference,	sign, circum.

MISCELLANEOUS TABLE.

12 things	make 1 dozen,	sign, doz.
12 dozen	make 1 gross.	
12 gross	make 1 great gross.	

How many calendar months are there in a year? How many days does each month contain? What is the real length of the year? What then is done one year in four? What is such a year called? How many leap years do we omit in 400 years? Why? What years are leap years? What are 4 weeks sometimes called?

How is the circumference of every circle, whether great or small, *considered to be divided?*

20 things ....make 1 score.

24 sheets of paper make 1 quire.
20 quires ....make 1 ream.

6 points ....make 1 line, used in measuring
12 lines ....make 1 inch, clock pendulums.
4 inches ....make 1 hand, used in measuring the height of horses.
112 pounds........make 1 quintal of fish.
200 pounds........make 1 barrel of beef or pork.
196 pounds........make 1 barrel of flour.

REDUCTION, OR CHANGE OF FORM OF COMPOUND NUMBERS.

LESSON 97.

To be performed in the mind.

1. How many ounces of butter are there in 2 pounds 3 ounces ?
2. How many yards, and how many odd feet are there in a pole 10 feet long ? 13 feet long ?
3. How many quarts are there in 2 bushels ? In 1 bushel 2 pecks ?
4. A grocer on the 4th of July, retailed 10 quarts 1 pint of wine ; how many gallons and odd quarts and pints did he sell ?
5. If you are 3 hours and 5 minutes getting your lesson, how many minutes are you ?
6. How many weeks are there in 28 days ? In 32 days ? In 37 days ?
7. At 1 dollar an ounce, how much are 2 pounds of old silver worth ? 3 pounds ? 5 pounds ? 1 pound 3 ounces ?
8. How many gallons of molasses are there in 16 pints ? In 76 pints ? In 15 quarts ? In 35 quarts ;
9. How many square inches are there in 2 square feet 12 square inches ?
10. A man bought 48 pecks of oats at different times, and gave $6 in payment ; what price a bushel did he give ?

For the Slate.

11. A farmer had 5 cwt. 3 qrs. 12 lbs. of cheese ; how *many pounds had he ?*

OPERATION.

```
     cwt.  qrs.  lbs.
      5     3    12
      4
     ---
     2 0
Add  3 qrs.
     ---
     2 3
     2 8
    -----
    1 8 4
    4 6
Add   1 2 lbs.
    -----
    6 5 6 lbs. Ans.
```

Explanation. There are 4 qrs. in 1 cwt., so to get the number of qrs. we multiply the 5 cwt. by 4, and add the 3 qrs. to the product. There are 28 lbs. in a qr., so to get the number of lbs. we multiply the 23 qrs. by 28, and add the 12 lbs. to the product.

Therefore, to change or reduce a quantity to a lower denomination,

Multiply the highest denomination in it by so many of the next lower as make one of this highest, and add to the product the number in the lower denomination ; multiply the result in the same way, and so on until the quantity be brought into a denomination as low as desired.

12. A farmer having 656 lbs. of cheese, wished to know how many cwt. he had; how many had he ?

OPERATION.

```
            4
2 8 ) 6 5 6 ( 2 3 ( 5 cwts.
      5 6     2 0
     -----   ---
      9 6     3 qrs.
      8 4
     -----
      1 2 lbs.
5 cwts. 3 qrs. 12 lbs. Ans.
```

Explanation. There being 28 lbs. in 1 qr., we divide 656 lbs. by 28, and get 23 qrs. and 12 lbs. over. There being 4 qrs. in 1 cwt., we divide 23 qrs. by 4, and get 5 cwt. and 3 qrs. over. So 656 lbs. are 5 cwt. 3qrs. 12 lbs.

Therefore, to change or reduce a quantity to a higher denomination,

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Explain how example 11, lesson 97, is performed.
How do we change or reduce a quantity to a lower denomination?
Explain how example 12, lesson 97, is performed.

Divide by as many as it takes to make one of the next higher denomination; divide the quotient in the same way, and so on until the quantity be brought into a denomination as high as desired.

The operations under either of the two preceding rules are proved by reversing them. Thus the operation in example 11 is reversed in example 12, and the operation in example 12 is reversed in example 11

Note. Each example should now be proved.

LESSON 98.

For the Slate.

1. How many tons are there in 36,000 ounces? Ans. 1 T. 10 lbs.

2. How many pounds are there in 5 tons? Ans. 11,200.

3. A jeweller has 4,312 pennyweights of gold, in various pieces; how many pounds has he? Ans. 17 lbs. 11 oz. 12 pwts.

4. How many grains of silver are there in 6 ounces? Ans. 2,880.

5. How many grains of ipecacuanha are there in 2℔ 4℥ 2ʒ 0℈ 1 gr.? Ans. 13,561.

6. How many ounces of calomel are there in 640 grains? Ans. 1℥ 2ʒ 2℈.

7. A surveyor has a chain containing 100 links, each link being 7.92 inches long; how many rods are there in the chain? Ans. 4.

8. How many fathoms are there in 2 leagues? Ans. 5,280.

9. A man sold a house lot 6 rods long and 4 rods wide, at $.0625 a square foot; what sum did it bring? Ans. $408.375. See lesson 82, Decimal Fractions.

10. How many square yards of carpeting will cover a floor 18 feet long, and 16 feet wide? Ans. 32.

LESSON 99.

1. How many cubic inches are there in a block of wood 3 feet long, 2 feet wide, and 1 foot thick? Ans. 10,368.

How do we change or reduce a quantity to a higher denomination? *How are the operations* under either of the two preceding rules proved? *Give some examples.*

2. How many cords are there in a pile of wood 37 feet long, 4 feet wide, and 5 feet high ?
Ans. 5 C. 6 ft. 4 cubic ft.

3. A trader bought some beans at $.50 a peck ; what sum must he pay for 12 bushels and 3 pecks ? Ans. $25.50.

4. How many chaldrons are there in 112 bushels of Richmond coal ? Ans. 3 chal. 4 bu.

5. How many pints of milk are there in 12 gallons 3 quarts 1 pint, and how much is the whole worth, at 2 cents a pint ? Ans. 103 pts., and it is worth $2.06.

6. How many firkins of beer are there in 1,313 quarts ?
Ans. 36 fir. 4 gals. 1 qt.

7. If a man drinks 2 quarts of wine a day, how long will 2 tierces 5 gallons and 2 quarts last him ? Ans. 179 days.

8. How many hogsheads of molasses are there in 2,217 quarts ? Ans. 8 hhds. 50 gals. 1 qt.

9. How many weeks are there in 1,000,000 seconds ?
Ans. 1 w. 4 d. 13 h. 46 m. 40 sec.

10. 20 cubic feet of water run over a mill-dam in a second ; how many cubic feet will pass over the dam in 1 week 4 days 7 hours 0 minutes 48 seconds ?
Ans. 19,512,960.

Lesson 100.

1. In 17° 26′ 14″ how many seconds ? Ans. 62,774.

2. How many degrees are there in 5,700″ ? Ans. 1° 35′.

3. How many days were there between the time of the Declaration of Independence, July 4th, 1776, and the time of the settlement of a general peace, January 20th, 1783 ?
Ans. 2,391.

Explanation. 1780 can be divided by 4 without a remainder.

4. 4 weeks are often called a month ; how many of such months are there in 67 days ? Ans. 2 mo. 1 w. 4 d.

5. A man dug a cellar 36 feet long, 24 feet wide, and 6 feet deep ; how many cubic yards of earth did he take out of it ? Ans. 192.

6. A farmer sold some oak wood at $1 a foot ; how much did he get for 3 cords and 7 feet ? Ans. $31.

7. How many rods are there in 1 mile and 33 rods ?
Ans. 353.

8. How many leagues and fathoms are there in 17 miles and 45 *fathoms ?* Ans. 5 lea. 1,805 fath.

9. How many square yards of cloth are there in a piece 28 yards long, and 6 quarters, that is, $\frac{6}{4}$ wide ? Ans. 42.

10. A gold beater has 453,778 square inches of gold leaf ; how many square yards are there in that quantity ?
Ans. 350 sq. yds. 1 sq. ft. 34 sq. in.

Lesson 101.

1. How many tons of flour are there in 3,600 barrels and 425 half barrels ? Ans. 333 T. 11 cwt. 3 qrs. 14 lbs.

2. What sum are 3 tons 2 quarters and 17 pounds of butter worth at $16⅔ a pound ? Ans. $ 1,132.16⅔.

3. A silversmith has 7 pounds of silver in an ingot; how many pennyweights are there in the ingot ? Ans. 1,680.

4. A retailer sells beans at $.12½ a quart; what sum will he get for 5 bushels ? Ans. $ 20.

5. How many bushels will a box 5 feet square hold ?
Ans. 100.446 bushels, about.

Explanation. Find the number of cubic inches in the box, and divide by the number of cubic inches in a bushel.

6. A man retailed 35,217 pints of beer ; how many hogsheads was that quantity ? Ans. 81 hhds. 28 gals. 1 pt.

7. How much are 4 hogsheads of beer worth at $.50 a gallon ? Ans. $ 108.

8. How many gallons of molasses are there in 16 casks, called hogsheads, each of which holds 108 gallons, and in 5 barrels ? Ans. 1,885½.

9. How much are 5 puncheons of rum worth at $ 1 a gallon ? Ans. $ 420.

10. A number of silver spoons weigh 428 pennyweights; how many pounds of silver do they contain ?
Ans. 1 lb. 9 oz. 8 pwts.

Lesson 102.

To be performed in the mind.

1. What part of a bushel are 3 pecks? 2 pecks ? 1 peck ?

2. A man bought .5 of a pound of butter ; how many ounces did he get ? How many ounces would he have got had he bought .25 of a pound ? 1.5 pound ?

3. Two boys had a stick of candy 1 foot in length ; they broke it so that the largest boy got a piece 8 inches long ; what part of the whole was that ?

4. What part of a week are 3 days? 5 days? 11 days?

5. How many hours are there in $\frac{5}{6}$ of a day? In $\frac{1}{4}$ of a day? In $\frac{3}{8}$ of a day?

6. What part of a gallon are 1 quart and 1 pint?

7. If I buy .6 of a ton of hewed timber, how many cubic feet do I get?

8. How many feet of wood are there in $\frac{3}{4}$ of a cord? In $\frac{5}{8}$ of a cord? In .5 of a cord? In .25 of a cord?

9. What part of a pound of silver are 3 ounces? What must I give for 3 ounces of silver at $ 12 a pound?

10. What part of a day are 3 hours? 4 hours? 20 hours?

For the Slate.

11. What part of a bushel, in common and in decimal fractions, are 3 pecks, 4 quarts, 1 pint?

OPERATION in common fractions.	OPERATION in decimals.
$2\,)\,1$ pt.	$2\,)\,1.0$ pt.
$8\,)\,4\frac{1}{2}$ qts.	$8\,)\,4.5$ qts.
$4\,)\,3\frac{9}{16}$ pks.	$4\,)\,3.5625$ pks.
$\frac{57}{64}$ of a bushel. Ans. in common fractions.	.890625 of a bushel. Ans. in decimal fractions.

Explanation. We first find what part of a quart, or of 2 pints, the 1 pint makes; then what part of a peck, or of 8 quarts, the 4 quarts and part of a quart make; and finally, what part of a bushel, or of 4 pecks, the 3 pecks and part of a peck make. See Fractions, lesson 61. *We divide, in fact, just as we do to change pints to quarts, quarts to pecks, and pecks to bushels. Pursue a similar course in like cases.*

Note. The whole of the operation in common fractions is not shown, since $4\frac{1}{2}$ and $3\frac{9}{16}$ are changed to improper fractions before dividing them. See Fractions, lesson 72, example 7.

12. How many pecks, quarts, &c., are there in $\frac{57}{64}$ of a bushel, and in .890625 of a bushel?

Explain how example 11, lesson 102, is performed.
How, in fact, do we divide? What must we do in like cases?

OPERATIONS.

57
4

64)228(3 pks.
192

36 remainder, $\frac{36}{64}$ of a pk.
8

64)288(4 qts.
256

32 remainder, $\frac{32}{64}$ of a qt.
2

64)64(1 pt.
64

.890625
4

pks. 3.562500
8

qts. 4.500000
2

pt. 1.000000

Ans. 3 pks. 4 qts. 1 pt.

Explanation. We first find how many pecks there are in $\frac{57}{64}$ and .890625 of a bushel, or of 4 pecks; then how many quarts there are in the fractions of a peck, or of 8 quarts; and finally, how many pints there are in the fractions of a quart, or of 2 pints. *We multiply, in fact, just as we do to change bushels to pecks, pecks to quarts, and quarts to pints. Pursue a similar course in like cases.*

LESSON 103.

For the Slate.

1. A blacksmith sold 3 quarters 16 pounds of iron; what part of a ton did he sell, and what was it worth at $ 60 a ton? Ans. $\frac{5}{112}$ of a ton, and it was worth $ 2.68, nearly.

2. How many pounds are there in .25 of a quarter? Ans. 7.

3. How many ounces, pennyweights, and grains are there in $\frac{14}{35}$ of a pound of silver? Ans. 4 oz. 16 pwts.

4. What part of an ounce of gold, in decimals, are 13 pennyweights 3 grains? Ans. .65625 of an ounce.

Explain how example 12, lesson 102, is performed.
How, in fact, do we multiply? What must we do in like cases?

5. $\frac{1}{16}$ of a yard is sometimes called a nail; now what part of a yard are 3 quarters of a yard and 3 nails?
Ans. $\frac{15}{16}$.

6. What is the value of .1325 of a mile in smaller denominations? Ans. 1 fur. 2 rods 6 ft. 7.2 in.

7. $\frac{1}{16}$ of a square mile is how many acres, and how much is it worth at $ 10 an acre?
Ans. 40 A., and it is worth $400.

8. What part of a square rod, in decimals, are 187 square feet? Ans. .68687, nearly.

9. .8 of a cord of wood contains how many feet of wood; and what is it worth at $ 1 a foot?
Ans. 6.4 ft. of wood, and it is worth $ 6.40.

10. What part of a ton of hewed timber are 44 cubic feet and 86.4 cubic inches? Ans. .881.

LESSON 104.

1. If Sidney coal is worth $ 9 a chaldron, what is the value of 14 bushels 3 pecks? Ans. $ 3.69, nearly.

Explanation. First change 14 bushels 3 pecks to decimals of a chaldron.

2. If a grocer sells filberts at 4 cents a quart, how much will he get for $\frac{2}{3}$ of a bushel? Ans. $ .85, about.

Explanation. First change $\frac{2}{3}$ of a bushel to quarts and decimals of a quart.

3. A man sells beer at 6¼ cents a quart; how much at this rate will $\frac{4}{5}$ of a hogshead be worth? Ans. $ 10.80.

4. What part of a gallon of milk, in common fractions, are 2 quarts 1 pint? Ans. $\frac{5}{8}$.

5. A man has 1 pipe 1 hogshead and 20 gallons of wine; how many hogsheads and decimals of a hogshead has he?
Ans. 3.3175, nearly.

6. If molasses is worth $.33⅓ a gallon, what is ¾ of a hogshead worth? Ans. $ 15.75.

7. If a clock ticks 172,800 times a day, how many times will it tick in 6 hours, 3 minutes, 4 seconds?
Ans. 43,568 times.

8. How many hours are there in $\frac{1}{6}$ of a day? Ans. 4.

9. What part of the circumference of a circle are 7 degrees and 12 minutes? Ans. .02.

10. How many degrees is $\frac{1}{8}$ of the circumference of a circle? Ans. 45°.

Lesson 105.

1. 2 tons 3 hundred weight 1 quarter of cheese sold for $250; what did 1 ton bring? Ans. $115.61, nearly.

Explanation. What quantity in tons and decimals of a ton sold for $250?

2. If 8 pennyweights 8 grains of silver make 1 silver spoon, how many of such spoons can be made from 5 ounces of silver? Ans. 12.

3. A man travelled 10 miles 2 furlongs 20 rods in 3.3 hours; how many minutes and seconds was he travelling 1 furlong? Ans. 2 min. 24 sec.

4. A man bought 1 square mile and 360 acres of land for $1,000; how much did it cost him a square mile? Ans. $640.

5. I bought 2 tons 12 cubic feet of hewed timber for $21; how much did I give a ton? Ans. $9.33⅓.

6. If 4 bushels 3 pecks of potatoes weigh 332½ pounds, how many pounds will 1 bushel weigh? Ans. 70.

7. 6 gallons 2 quarts 1 pint of milk were sold for $1.59; what was the price of a quart? Ans. 6 cents.

8. A vessel sailed 10 leagues in 3 hours 20 minutes; how far did she sail in 1 hour? Ans. 3 lea.

9. A man rode 4 miles in 20 minutes; how far would he go in 1 hour at this rate? Ans. 12 miles.

10. A vessel on the equator sailing west, passed over 39 degrees 45 minutes 12 seconds of the circumference of the earth in 21 days; how long was she in sailing over 1 degree? Ans. 12 h. 40 min. 41 sec., about.

Lesson 106.

1. What part of a pound is $\frac{1}{5000}$ of a ton? Ans. $\frac{56}{125}$.

2. What part of a pound is .0001 of a ton? Ans. .224.

Explanation. Proceed the same as if you were asked how many pounds were in $\frac{1}{5000}$ of a ton, and in .0001 of a ton?

3. What part of a ton is $\frac{1}{2}$ of a pound? Ans. $\frac{1}{4480}$.

Explanation. Proceed the same as you would to find what part of a ton 1 pound is.

4. What part of a foot in decimals is $\frac{1}{2}$ of an inch? or which is the same thing, change $\frac{1}{2}$ of an inch to the decimal of a foot. Ans. .041667, nearly.

5. Change .01 of a foot to the decimal of an inch. Ans. .12 of an in.

6. A man sold $\frac{1}{320}$ of an acre of land for $ 12; at this rate how much is a square rod worth? Ans. $ 24.

7. Change .75 of 1 quarter of an acre to the decimal of an acre. Ans. .1875 of an A.

8. What part of 1 foot of wood are .2 of a cord; and what is 1 foot of wood worth if .2 of a cord be worth $ 1.60? Ans. .2 of a C. is 1.6 ft., and 1 ft. is worth $ 1.

9. What part of a cubic foot, in common fractions, is $\frac{1}{3}$ of a cubic inch? Ans. $\frac{1}{5184}$.

10. Change $\frac{1}{50}$ of a bushel to the decimal of a pint. Ans. 1.28 pt.

11. Change $\frac{1}{5}$ of a peck to the decimal of a bushel. Ans. .05 of a bu.

12. What part of a year of 365 days is $\frac{1}{2}$ of a day? Ans. $\frac{1}{730}$.

LESSON 107.

1. If 1 bu. of wheat purchase 2 bu. 3 pks. of potatoes, how many bu. of potatoes will 5 bu. 1 pk. of wheat purchase? Ans. 14 bu. $1\frac{3}{4}$ pk.

Explanation. First change 2 bu. 3 pks., and 5 bu. 1 pk. to bushels and decimals.

2. A silversmith gave 1 oz. of silver for 2 cwt. of hay; how much hay could he have obtained for 5 oz. 15 pwts. of silver? Ans. 11 cwt. 2 qrs.

3. If I give 5 lbs. 8 oz. of sugar for 1 ft. of wood, how much sugar must I give for 4 C. 7 ft. of wood? Ans. 214.5 lbs.

4. A vessel sailed 3 lea. in 1 hour; how far would she sail in 2 d. 7 h. at the same rate? Ans. 165 lea.

5. What number of sq. rods are there in a field 16 rods 12 ft. long, and 12 rods 5 ft. wide? Ans. 205.796, about.

6. How many sq. ft. are there in a mat 7 ft. 3 in. long, and 3 ft. 8 in. wide? Ans. 26.58, about.

Note. When feet and inches are to be changed to the same denomination, it is usually better to change them to feet and decimals, than to inches. It is easy to change feet and inches to feet and decimals, if we understand the table in lesson 92, Federal Money, for there are as many

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What is said of *changing* feet and inches to the same denomination?
Why is it easy to change feet and inches to feet and decimals.

hundredths of a foot in an inch as there are cents or hundredths of a dollar in a New England sixpence. Thus sixpence is $.08⅓, and 1 inch .08⅓ of a foot. 1 shilling is $.16⅔, and 2 inches .16⅔ of a foot. 1 shilling and sixpence, or 3 sixpences, are $.25, and 3 inches .25 of a foot, &c.

Engineers and some surveyors of wood and lumber, employ measures divided into feet and decimals; that is, divided into feet, tenths and hundredths of a foot.

7. There is a cistern of water which is 13 ft. 4 in. long, 6 ft. 9 in. wide, and 4 ft. 6 in. deep; how many cubic ft. of water are there in it? Ans. 405.

8. How many cords of wood are there in a pile 25 ft. 6 in. long, 4 ft. wide, and 8 ft. 11 in. high.
Ans. 7 C. 13.5 cubic ft.

9. What part of 1 C. of wood is there in a pile 8 ft. long, 4 ft. wide, and 3 ft. 3 in. high? Ans. .8125.

10. How many acres are there in a tract of land 1 mile 264 ft. long, and ½ of a mile wide? Ans. 336.

Explanation. How many feet are there in a mile? How long is the tract in miles and decimals?

LESSON 108.

1. If 3 cwt. 2 qrs. of bread last a family 1 year, how long, in years and days, will 1 T. 2 cwt. 1 qr. 16.8 lbs. last them? Ans. 6 yrs. 146 d.

2. A farmer exchanged 2 T. 4 cwt. of hay for 2 hhds. 17 gals. of molasses; how many gallons of molasses could he have obtained for 1 T. of hay? Ans. 65.

3. A horse ran 1 mile in 3 min. 15 sec.; how far could he run in 1 h. 20 m. 36 sec. at the same rate?
Ans. 24 m. 6 fur. 16 rods.

4. How many rings, each weighing 2 pwts. 4 grs. can be made from 1 lb. 5 oz. 8 pwts. 20 grs. of gold? Ans. 161.

5. A piece of ground in the shape of an oblong square, containing 1 A. 3 qrs. 30 sq. rods, is 15 rods 8 ft. 3 in. wide; how long is it? Ans. 20 rods.

6. There is a door 3 ft. 3 in. wide, the surface of which is 23 sq. ft. 108 sq. in.; what is its height in feet and inches? Ans. 7 ft. 3.7 in., nearly.

Explanation. First get its height in feet and decimals.

Give some examples.

What measures do engineers and some surveyors of wood and lumber employ?

7. How high in feet and inches must you pile a load of wood which is 8 ft. long and 3 ft. 9 in. wide, so that it may contain a cord ? Ans. 4 ft. 3.2 in.

Explanation. The number of cubic feet in a cord, divided by the surface of the bottom of the load, will evidently give the height. See Division, lesson 51.

8. If a box contain 27 cubic ft., and the surface of one end be 4 sq. ft., what is its length ? Ans. 6 ft. 9 in.

9. A cart containing 30 cubic ft., is 2 ft. 3 in. high; what is the surface of the bottom ? Ans. 13 sq. ft. 48 sq. in.

10. The surface of the end of a stick of hewed timber, containing 1 T. 12 cubic ft., is 2¼ sq. ft. ; what is its length ? Ans. 27 ft. 6⅔ in.

LESSON 109.

Compound numbers can be used in every respect as simple numbers, if they are first changed to the same denomination.

1. A trader sold to three men the following quantities of wine. To the first 2 gals. 1 qt.; to the second 3 qts. 1 pt.; and to the third 2 qts. ; how much did he sell all of them ? Ans. 3 gals. 2 qts. 1 pt.

Explanation. Before adding, we change all the quantities to pints, or to quarts and decimals of a quart, or to gallons and decimals of a gallon. After adding, we change the sum to any denomination we please.

2. Add 15 bu. 2 pks. 5 qts., 7 bu. 1 pk. 3 qts., and 23 bu. 2 pks. Ans. 46 bu. 2 pks.

3. If you are allowed to play 4 h. 20 min., how long can you remain after having played 2 h. 40 min. ? Ans. 1 h. 40 min.

4. A man took 1 C. 7 ft. of wood from a heap that contained 6 C. 3 ft. ; how many cords were left ? Ans. 4 C. 4 ft.

5. A milkman left 2 gals. 3 qts. 1 pt. of milk at a boarding house every morning for 6 days ; what quantity did he leave in that time ? Ans. 17 gals. 1 qt.

6. If you have 3 loads of hay, each weighing 18 cwt. 3 qrs. 20 lbs., how many tons have you ? Ans. 2 T. 16 cwt. 3 qrs. 4 lbs.

When can compound numbers be used in every respect as simple *numbers ?*

7. Multiply 33 ft. 7 in. by 4. Ans. 134 ft. 4 in.

8. Divide 15 lbs. 9 oz. by 6. Ans. 2 lbs. 9 oz. 8 dr.

9. 5 men owned 1 hhd. 1 bl. 16 gals. of wine in equal shares; how much was each one's portion? Ans. 22 gals. 3.2 gi.

10. How many square feet are there in 3 carpets, each of which contains 186 sq. ft. 66 sq. in.? Ans. 559.375.

It will be well to recollect that compound numbers can be used in this way, but we can add and subtract in these numbers, and can often multiply and divide them without the trouble of changing them to the same denomination as we shall now proceed to show.

ADDITION OF COMPOUND NUMBERS.

LESSON 110.

To be performed in the mind.

1. A man bought at different times the following quantities of cinnamon; 3 oz., 5 oz., and 9 oz.; how many ounces did he buy? How many pounds, &c.?

2. How many pecks of corn will you have after harvesting 2 pks., 3 pks., and 5 pks.? How many bushels, &c., will you have? How many bushels will you have after harvesting 1 bushel 3 pecks, and 3 bushels 2 pecks? 4 bushels 1 peck, 3 pecks, and 2 pecks?

3. A laborer worked for a merchant 4 h. 30 min. at one time, 5 h. at another, and 12 h. 30. min. at another; how many hours did he work?

4. What is the sum of 4 gals. 2 qts. 1 pt., 1 qt. 2 pts., and 3 pts.

5. How many fathoms are there in 2 ft., 4 ft., 5 ft., and 5 ft.?

For the Slate.

6. A farmer sold at different times the following quantities of butter; 3 qrs. 12 lbs. 4 oz., 9 lbs. 14 oz., and 1 qr. 15 oz.; what was the whole quantity he sold?

What else is said of compound numbers?

OPERATION.

cwt.	qrs.	lbs.	oz.
	3	12	4
		9	14
	1	0	15
1	0	23	1

Ans. 1 cwt. 0 qrs. 23 lbs. 1 oz.

Explanation. We first place oz. under oz., lbs. under lbs., &c. Adding up the oz. we find there are 33, or 2 lbs., 1 oz.; we place the 1 oz. under the column of oz., and add the 2 lbs. with the lbs. There not being pounds enough to make 1 qr., we write down the whole number, and then adding up the qrs., we get 4 qrs. or 1 cwt.; so we put 0 in the place of qrs. and 1 in the place of cwts.

Therefore, to add compound numbers,

Write the numbers so that the quantities in the same denomination may stand directly under each other. Add the quantities in the lowest denomination first; change the sum to the next higher denomination, and the result carry, and add with the next higher denomination, having first put down the remainder, and so on.

The work is proved as in simple numbers.

Note. **Each example should now be proved.**

Numbers to add.
(7.)

℔	℥	ʒ	℈	grs.
5	11	5	2	16
		2	1	16
27	0	0	0	18

Numbers to add.
(8.)

chal.	bu.	pks.
4	16	3
12	0	3
2	3	1
6	18	1

Numbers to add.
(9.)

miles.	fur.	rods.	ft.
2	0	27	0
1	0	18	3
		36	10
			5

10. A farmer sold four loads of hay, weighing as follows; the first 1 T., the second 16 cwt. 2 qrs. 18 lbs., the third 1 T. 2 cwt. 10 lbs., and the fourth 18 cwt.; what was the weight of the whole? Ans. 3 T. 16 cwt. 3 qrs.

11. If I buy the following quantities of oats, 12 bu. 3 pks. 5 qts., 2 bu. 2 pks. 2 qts. 1 pt., 3 bu. 7 qts., 6 qts., and 1 pk. 4 qts., what is the whole amount purchased?

Ans. 19 bu. 1 pk. 1 pt.

12. Add the following quantities of silver; 5 lbs. 11 oz. 7 pwts. 3 grs., 6 lbs., 2 lbs. 15 grs., and 20 grs.

Ans. 13 lbs. 11 oz. 8 pwts. 14 grs.

Explain how example 6, lesson 110, is performed.
How do we add compound numbers?
How is the work proved?

LESSON 111.

For the Slate.

1. How much beer is there in the following quantities ; 2 hhds. 1 bl., 1 hhd. 2 bls. 6 gals., and 1 bl. 3 gals.?

Ans. 5 hhds. 1 bl. 1 fir.

2. The first day a ship left port, she sailed 25 lea. 2,520 fath., the second day, 13 lea. 180 fath., the third day, 17 lea. 1,820 fath., and the fourth day, 3 lea. 2,260 fath.; how far did she sail during the four days ?

Ans. 60 lea. 1,500 fath.

Explanation. How many feet are there in a mile ? In a league ? How many fathoms then are there in a league ?

3. How much land is there in three pieces, the first of which contains 17 A. 3 qrs. 12 sq. rods, the second 25 A., the third, 4 A. 6 sq. rods, and the fourth, 25 sq. rods ?

Ans. 47 A. 3 sq. rods.

4. How many cords of wood are there in ½ of a C., 2⅔ C., 3 C. 12 ft., 1 C. 10 ft., and 13 ft. ?

Ans. 11 C. 4⅓ ft.

Explanation. First change ½ of a C. to feet, and 2⅔ C. to cords and feet.

5. A grocer sold, at different times, the following quantities of molasses ; 4 hhds. 24 gals., 2 hhds. 2 bls. 4 gals. 3 qts., and 24 hhds. ; what was the whole quantity sold ?

Ans. 30 hhds. 2 bls. 28 gals. 3 qts.

6. Add 24 yrs. 55 d. 17 h., 2 yrs. 4¼ d., and 5 d. 2 h.

Ans. 26 yrs. 65 d. 1 h.

7. Add 12° 04′ 13″, 5° 12′ 55″, and 02′ 07″.

Ans. 17° 19′ 15″.

8. There is a bin of wheat, 8 ft. square on the bottom, and 4.5 ft. high, and another bin containing 17 bu. 4 pks. 6 qts. ; how much wheat is there in both bins ?

Ans. 249 bu. 2 pks. 3 qts. 1 pt., about.

Explanation. First find how many cubic inches of wheat there are in the first bin, and recollect that there are 2150.4 cubic inches in a bushel.

9. A man has 4 pieces of rope ; the first is 2 yds. 2 ft. 7 in. long; the second 24 yds., the third 7½ yds., and the fourth 3 yds. 1 ft. 11 in. long; what is the length of the four pieces ? Ans. 38 yds.

10. A man has a farm of 120 A. 3 qrs. 21 sq. rods, and another piece of land 27 rods long, and 17 rods wide; how much land has he ? Ans. 123 A. 3 qrs.

SUBTRACTION OF COMPOUND NUMBERS.

LESSON 112.

To be performed in the mind.

1. If a grocer has 12 lbs. 9 oz. of butter, and sells 5 lbs. 4 oz., how much will he have left?

2. A man having a stick 2 ft. 3 in. long, cut 1 ft. 2 in. off from it; how long was the piece left?

3. If 1 gal. 3 qts. of wine leak out of a cask that contained 3 gals., what quantity will be left? What quantity will be left if 3 qts. 1 pt. leak out of the cask?

4. A vessel employed 4 w. 1 d. in a voyage from Boston to Liverpool, having stopped 1 w. 2 d. at Cork; how long was she at sea?

5. I sold 10 sq. rods from a piece of land containing 2 quarters; what quantity of land was left in the piece?

For the Slate.

6. A coal dealer having 112 chal. 4 bu. 3 pks. of coal, sold 38 chal. 17 bu. 2 pks.; what quantity was there left?

OPERATION.

chal.	bu.	pks.
112	4	3
38	17	2
73	23	1

Ans. 73 chal. 23 bu. 1 pk.

Explanation. We first take the 2 pks. from 3 pks.; then, being unable to take the 17 bu. from 4 bu., we add 1 chal., or 36 bu. to 4 bu., and take 17 from 40. As we have added 1 chal. to the greater number, to balance it, we now add 1 chal. to 38 chal. in the smaller number, and take 39 from 112.

Therefore, to subtract one compound number from another,

Subtract the quantity in the lowest denomination of the smaller number from that above, and set down the remainder, and so on. When the quantity we are to subtract from is the smallest, add as many to it as make one of the next higher denomination, subtract, and then carry 1 *to the next higher denomination of the smaller number.*

The work is proved as in simple numbers.

Note. Each example should now be proved.

Explain how example 6, lesson 112, is performed.
How do we subtract one compound number from another?
How is the work proved?

(7.)

	T.	cwt.	qrs.	lbs.
From	4	13	2	19
take	2	17	3	20

(8.)

	℔	℥	ʒ	℈	grs.
From	3	10	4	1	12
take	1	0	0	1	15

(9.)

	h.	min.	sec.
From	17	21	12
take	10	17	5

10. A trader having 215 bu. of beans, sold 38 bu. 3 pks. 4 qts.; what quantity had he left ? Ans. 176 bu. 4 qts.

11. A grocer bought 4 bls. 8 gals. 2 qts. of beer, but lost 1 bl. 7 gals. 3 qts. by leakage; how much did he have left ? Ans. 3 bls. 3 qts.

12. Subtract 7 lbs. 10 oz. 15 pwts. 17 grs., from 9 lbs. 11 oz. 17 pwts. 21 grs. Ans. 2 lbs. 1 oz. 2 pwts. 4 grs.

LESSON 113.

For the Slate.

1. A merchant having $\frac{2}{3}$ of a T. of rice, sold 3 qrs. 17 lbs.; how much had he left ? Ans. 12 cwt. 1 qr. 20$\frac{1}{3}$ lbs

Explanation. First change $\frac{2}{3}$ of a T. to smaller denominations.

2. A landholder who owned 3 sq. miles, sold 712 A. 2 qrs. 35 sq. rods; how much did he retain ? Ans. 1,207 A. 1 qr. 5 sq. rods.

3. There are two cities 98 miles 5 furlongs 3 rods apart; how far is a man travelling between these cities from one of them if he is 12 miles 6$\frac{1}{10}$ furlongs from the other ? Ans. 85 miles 6 furlongs 39 rods.

4. A man bought some hewed timber for 7 T. 12 cubic ft., but on measuring it, the quantity fell short 30 cubic ft. 1,200 cubic in.; how much timber was there ? Ans. 6 T. 31 cubic ft. 528 cubic in.

5. Take 1 pt. from 2 bu. Ans. 1 bu. 3 pks. 7 qts. 1 pt.

6. Take 10 cubic in. from 3 T. round timber, when $\frac{1}{5}$ is allowed for waste. Ans. 2 T. 39 cubic ft. 1,718 cubic in.

7. Take 2 in. from 3 yds. Ans. 2 yds. 2 ft. 10 in.

8. Take 1 pt. from 17 gals. 2 gi. Ans. 16 gals. 3 qts. 1 pt. 2 gi.

9. Take 4 grs. from 1 lb. Ans. 11 oz. 19 pwts. 20 grs.

10. Take 1 d. from 2 w. 10 sec. Ans. 1 w. 6 d. 10 sec.

11. A wine dealer having 2 tier. 2 bls. and 12 gals. of Madeira wine, sold 100 gals. ; how many gallons had he left ? Ans. 59.

12. From 7 w. 3 d. 16 h. 5 min. 28 sec., take 1 w. 4 d. *17 h. 16 min. 39* sec. Ans. 5 w. 5 d. 22 h. 48 min. 49 sec.

13. A certain city is 71° 18′ 45″ west of Greenwich, and another 89° 36′; how many degrees, minutes, &c. is one of these cities west of the other? Ans. 18° 17′ 15″.

14. A farmer made 185¾ bls. of cider, and sold 123 bls. 5 gals. 1 pt.; how much had he left?

Ans. 62 bls. 18 gals. 2 qts.

15. A goldsmith having 2 lbs. 4 oz. 15 grs. of gold, used 1 lb. 17 pwts. in manufacturing some articles; what quantity had he left? Ans. 1 lb. 3 oz. 3 pwts. 15 grs.

MULTIPLICATION OF COMPOUND NUMBERS.

Lesson 114.

To be performed in the mind.

1. There is a basket which holds 1 bu. 1 pk. 2 qts.; how much corn is there in a heap that contains 3 such baskets full? 4 such baskets full? 5? 6?

2. A man bought of a farmer 6 kegs full of cider; what quantity, in gallons, &c., did he get if the kegs held 5 qts. each? What if they held 6 qts. each? 7 qts. each? 1 qt. 1 pt. each?

3. If you study arithmetic 1 h. 15 min. each day, how much time will you occupy in that study in 2 days? In 3 days? In 4 days? In 5 days?

4. What is the product of 4 yds. 2 ft. by 2? By 3? By 5? By 8?

For the Slate.

5. A butcher sold 4 loads of beef, each of which contained 6 cwt. 2 qrs. 21 lbs.; what was the whole quantity sold?

OPERATION.

T.	cwt.	qrs.	lbs.
	6	2	21
			4
1	6	3	0

Ans. 1 T. 6 cwt. 3 qrs.

Explanation. The 21 lbs. multiplied by 4, give 84 lbs., or 3 qrs., 0 lbs.; so we put down 0 in the lbs. place, and add the 3 qrs. to the product of the 2 qrs. by 4. These 11 qrs. are equal to 2 cwt. 3 qrs.; so we put down the 3 qrs., and add the 2 cwt. to the product of the 6 cwt. by 4. These 26 cwt. are equal to 1 T. 6 cwt., which we put down.

Therefore, to multiply a compound number by a whole number,

Explain how example 5, lesson 114, is performed.

Multiply each of the denominations, beginning with the lowest, and carry as in addition of compound numbers.

The work is proved by dividing the product by the multiplier; if the quotient be equal to the multiplicand, the work will generally be right. See Division, lesson 51.

Note. Each example should now be proved.

6. Multiply 4 chal. 17 bu. 2 pks. by 8.
7. Multiply 2° 15′ by 90.
8. A wagon is loaded with 12 bales of cotton, each of which weighs 4 cwt. 27 lbs. 5 oz.; what is the weight of the load? Ans. 2 T. 10 cwt. 3 qrs. 19 lbs. 12 oz.

Explanation. Each bale is 4 cwt. 0 qrs. 27 lbs. 5 oz.

9. What is the weight of 28 ingots of silver, each of which contains 1 lb. 17 grs.? Ans. 28 lbs. 19 pwts. 20 grs.
10. How much are 5 times 2℔ 7℥ 2ʒ 17 grs.?
Ans. 13℔ 3ʒ 1℈ 5 grs.
11. Multiply 2 miles 275 rods by 37.
Ans. 105 miles 255 rods.

Explanation. Find how many rods there are in a mile.

12. A speculator divided some land into 6 house lots, each of which contained 1 A. 4 sq. rods 120 sq. ft.; what quantity of land was there in all of the lots?
Ans. 6 A. 26 sq. rods 175½ sq. ft.
13. What is the amount of 2 times 5 C. 5 ft. 178 cubic in.? Ans. 11 C. 2 ft. 356 cubic in.
14. Multiply 2 hhds. 1 kil. 1 fir. by 3.
Ans. 7 hhds. 1 kil. 1 fir.
15. What is the product of 5 qts. 1 pt. 2 gi. by 125?
Ans. 179 gals. 2 qts. 1 pt. 2 gi.
16. What is the product of 1 d. 18 h. 10 sec. by 5?
Ans. 1 w. 1 d. 18 h. 50 sec.

DIVISION OF COMPOUND NUMBERS.

LESSON 115.

To be performed in the mind.

1. 3 soldiers obtained 3 lbs. 9 oz. of silver from the plunder of a city; if they divide it equally, what will each one's share be? What would each one's share have been had they obtained 4 lbs.? 4 lbs. 6 oz.? 4 lbs. 3 oz.?

How do we multiply a compound number by a whole number?
How is the work proved?

2. If 2 men have equal shares in 3 bushels of potatoes, what is the portion of each in bushels and pecks ?

3. What is the quotient of 6 w. 3 d. 12 h. divided by 3 ? Of 1 d. 12 h. divided by 4 ?

4. If you have 2 qts. 1 pt. of chestnuts to divide among 2 boys, what quantity must you give to each ?

For the Slate.

Way of proceeding when the divisor is not more than 10.

5. A farmer wishes to carry 34 bu. 2 pks. 5 qts. of salt in 3 equal loads; how much must he put in each load ?

OPERATION.

	bu.	pks.	qts.	pts.
3)	34	2	5	
	11	2	1	1⅓

Ans. 11 bu. 2 pks. 1 qt. 1⅓ pt.

Explanation. Dividing 34 bu. by 3, we get 11 bu. and 1 bu. over. Changing this 1 bu. to pks., adding them to 2 pks. and dividing the sum, 6 pks., by 3, we get 2 pks. Dividing the 5 qts. by 3, we get 1 qt. and 2 qts. over. Changing the 2 qts. to pts., and dividing, we get 1⅓ pt.

Way of proceeding when the divisor is more than 10.

6. What is the quotient of 1 w. 3 d. 54 min. divided by 18 ?

OPERATION.

```
      w.  d.  h.  min. w.  d.   h.  min.
18 ) 1   3   0   54 ( 0   0    13   23 Ans.
     7
     —
     7(0 w.          6  brought up for
Add 3 days.          60     room.
     —               ——
    10(0 d.          360
    24           Add 54 min.
    ——               ——
    40               414(23 min.
    20               36
    ——               ——
   240(13 h.          54
    18                54
    ——                ——
    60
    54
    ——
     6 carried up.
```

Explanation. We proceed as before, but for the sake of ease write down all of the work. As 18 is not contained in the 1 week, nor in the 10 days, we get no weeks nor days in the answer.

Explain how example 5, lesson 115, is performed.
Explain how example 6, lesson 115, is performed.

Therefore, to divide a compound number by a whole number,

Divide the highest denomination first, and the remainder, if any, change to the next lower denomination, add it to the number in that denomination, and divide the result as before, and so on.

The work is proved as in simple numbers.

Note. Each example should now be proved.

7. Divide 5 lbs. 11 oz. 12 grs. by 6.
8. Divide 7℥ 6ʒ 1℈ by 3.
9. If I buy 1 T. 5 cwt. 2 qrs. of hay for $ 25, how much can I get for $ 1 at the same rate? Ans. 1 cwt. $2\frac{6}{25}$ lbs.
10. Divide 4 miles 55 rods by 12. Ans. $111\frac{1}{4}$ rods.
11. Divide $17\frac{2}{3}$ A. by 15, and get the answer in acres, quarters, and square rods.
Ans. 1 A. 28.444 sq. rods, about.
12. If 11 C. 5 ft. 10 cubic ft. of wood cost $ 60, what quantity can I get for $ 1 at the same rate?
Ans. 1 ft. $8.9\frac{2}{3}$ cubic in.
13. Divide 4 chal. 0 bu. 3 pks. by 7. Ans. 20 bu. $2\frac{4}{7}$ pks.
14. If 2 fir. of beer be divided equally among 50 men, what quantity will each receive? Ans. 1 qt. $3\frac{26}{50}$ gi.,
15. Divide 5 pipes of wine into 50 equal parts.
Ans. 12 gals. 2 qts. $3\frac{1}{5}$ gi.
16. What is the quotient of 14° divided by 27?
Ans. $31'\ 6\frac{2}{3}''$.

MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS BY FRACTIONS AND MIXED NUMBERS.

LESSON 116.

Note. If there are decimals in the numbers by which we are to multiply or divide, they must first be changed to common fractions, unless we proceed as directed in lesson 109, Reduction of Compound Numbers.

For the Slate.

1. If you are to have $\frac{2}{3}$ of a load of hay that weighs 1 T 2 qrs. 19 lbs., what quantity will that be?
Ans. 13 cwt. 3 qrs. $3\frac{1}{3}$ lbs.

How do we divide a compound number by a whole number?
How is the work proved?
If there are decimals in the numbers by which we are to multiply or *divide, what is to* be done with them?

Explanation. Observe the rule in Fractions, lesson 70, and the two preceding rules for multiplying and dividing compound numbers.

2. A man gave \$ $8\frac{3}{4}$ for 1 chal. 7 bu. 3 pks. of Sidney coal ; how much could he have obtained for \$ 1, at the same rate ? Ans. 5 bu.

Explanation. Observe the rule in Fractions, lesson 73, and the two preceding rules for multiplying and dividing compound numbers.

3. Multiply 6 lbs. 2 oz. 10 grs. by $\frac{1}{8}$.
Ans. 9 oz. 5 pwts. $1\frac{1}{4}$ gr.

4. Multiply 10℥ 2ʒ by $4\frac{1}{3}$. Ans. 3℔ 8℥ 3ʒ 1℈.

5. Divide 10 sq. rods 2.25 sq. ft. by .6.
Ans. 16 sq. rods 185.25 sq. ft.

6. If you buy 4 C. 9 ft. of wood for \$ $24\frac{3}{4}$, how much can you buy for \$ 1, at the same rate ?
Ans. 1 ft. $10\frac{50}{99}$ cubic ft., or 1 ft. $10\frac{1}{2}$ cubic ft., about.

7. A trader sold $\frac{5}{12}$ of a bin of wheat for \$ 12.50 ; the bin, on measurement, was found to contain 18 bu. 3 pks.; how much did he sell for \$ 12.50, and how much for \$ 1 ?
Ans. 7 bu. 3 pks. 2 qts. for \$ 12.50, and 2 pks. 4 qts. for \$ 1.

8. How much is $4\frac{3}{4}$ times 2 hhds. 1 bl. 7 gals. of wine ?
Ans. 12 hhds. 25.375 gals.

9. A laborer being 8 h. 45 min. doing a piece of work, another man agreed to perform a similar job in $\frac{5}{6}$ of the time ; how long was that ? Ans. 7 h. 17 min. 30 sec.

10. A portion of the circumference of a circle, containing 11° 34′ 06″, is to be divided into $5\frac{1}{2}$ equal parts ; how large will each portion be ? Ans. 2° 06′ 12″.

PROMISCUOUS QUESTIONS

IN

FEDERAL MONEY AND COMPOUND NUMBERS.

LESSON 117.

To be performed in the mind.

1. A piece of land is divided into two house lots, one of which is 25 ft. 8 in. wide, and the other 30 ft. 6 in. wide ; what is the width of the whole piece ?

2. If the piece had been divided into two lots of equal widths, how wide would each have been ?

3. If I buy 3 C. 6 ft. of pine wood for $ 10, how much do I get for $ 1 ? What do I give a foot for it ?

4. A trader had 16 gals. 2 qts. of vinegar in a cask; how much was there left after selling 3 qts. ? 1 gal. 2 qts. ? 3 gals. 3 qts. ?

5. A girl in Boston bought a silver pencil case for 75 cents, a sheet of drawing paper for 1 shilling, and four quills for ninepence ; she gave a 2-dollar bill in payment ; what sum did she receive back ?

6. What quantity of silver is there in 4 silver pitchers, each of which weighs 1 lb. 5 oz. ?

7. A man in Albany paid 3 shillings and sixpence for a cane, and $ 3¼ for an umbrella ; what sum did he give for both ?

8. How many square yards are there in a piece of ground 15 ft. square ?

9. What must I give for 16 yds. of cloth at $ 2¼ a yard ?

10. How many hundred weight are there in ¼ of a ton of anthracite coal ? How many pounds ?

11. How will you contrive to pay a man 70 cents, in Pittsburg, if you have in your purse, ½ of a dollar, ¼ of a dollar, 2 levies, 3 fips, and 4 cents ? How near can you pay him the exact sum ?

12. A man bought ⅛ of a load of hay containing 1 T. 4 cwt. ; what quantity did he obtain ?

LESSON 118.

For the Slate.

1. A man paid $ .90 for $\frac{1}{25}$ of a quantity of hay, estimated to be 2 T. ; what price a hundred weight did he give ? Ans. $ .56¼.

2. If a boatman in New York charge you 6 shillings for rowing you over the Hudson River and back again, how many cents must you pay him ? Ans. 75 cts.

3. A man owns one farm containing 115 A. 2 qrs. 13 sq. rods ; another containing 37 A. 30 sq. rods, and a third containing 18 A. 3 qrs. ; how much land has he ?
Ans. 171 A. 2 qrs. 3 sq. rods.

4. A trader sold 2 bu. 1 pk. of beans for $ 5.94 ; what price did he get a peck ? Ans. $ .66.

5. If you retail 1 quintal of cod fish at 4 cents a pound, 2 bls. of beef at 10 cents a pound, and 1 bl. of flour at $3\frac{3}{4}$ cents a pound, what sum will you get for the whole ?
Ans. $ 51.83.

6. What part of a pint, wine measure, is $\frac{1}{1008}$ of a hogshead ? Ans. $\frac{1}{2}$.

7. A young man is 19 years 86 days old, and his sister 17 years 119 days old; what is the difference of their ages ?
Ans. 1 yr. 332 d.

8. A man boarded in Northampton, Massachusetts, 12 weeks, at 15 shillings a week; what sum in Federal money must he pay his landlord ? Ans. $ 30.

9. A man sold 1 A. 2 qrs. 8 sq. rods of land at 8 cents a square foot ; what sum did it bring him ?
Ans. $ 5,401.44.

10. What part of a cord of wood, in common fractions, are 48 cubic ft. ? Ans. $\frac{3}{8}$.

Lesson 119.

1. The distance between two headlands on the coast of Maine, was measured, and found to be 6,530 fathoms ; what is this distance expressed in miles and rods ?
Ans. 7 m. $134\frac{6}{11}$ rods.

2. If a man pump 27 gals. 3 qts. 1 pt. of water in 1 minute, how much can he pump in 9 h. 4 min. 30 sec. ?
Ans. 15,177 gals. 3 qts. 1 pt. 2 gi.

3. What must I pay for 2 T. 7 cwt. 3 qrs. of hay, at the rate of $ 17 a ton ? Ans. $ $40.58\frac{3}{4}$.

4. How many square feet are there in a garden 5 rods 12 ft. 6 in. long, and 2 rods 8 ft. wide ? Ans. 3,895 sq. ft.

5. How many hours and minutes are there in $\frac{8}{15}$ of a day ? Ans. 12 h. 48 min.

6. 6 silver tea spoons weigh 2 oz. 9 pwts. 12 grs. ; what is the weight of each ? Ans. 8 pwts. 6 grs.

7. 2 T. 3 cwt. 2 qrs. of American bar iron was sold for $ 130.50 ; what price a ton did it bring ? Ans. $ 60.

8. How many pieces of iron, each weighing 1 lb. 4 oz., can be cut off from a bar weighing 2 cwt. 1 qr. 8 lbs. ?
Ans. 208 pieces.

9. If you sell $3\frac{1}{2}$ tubs of butter, each of which weighs 56 lbs. 13 oz. what is the whole quantity disposed of ?
Ans. 198 lbs. $13\frac{1}{2}$ oz.

10. There is a block of wood 4 ft. long, and 2 ft. 6 in. high, containing 12.5 cubic ft.; how wide is it?

Ans. 1 ft. 3 in.

Explanation. See Division, lesson 51.

PERCENTAGE.

LESSON 120.

Per cent. is a contraction of the Latin *per centum*, and signifies per hundred.

1 per cent. of any number is $\frac{1}{100}$ of it, or .01 of it; 2 per cent. is .02 of it, 3 per cent. is .03 of it, 25 per cent. is .25 of it, 120 per cent. is 1.20 of it, and 100 per cent. of any number is just equal to it, &c. Also, $\frac{1}{2}$ per cent., or $\frac{1}{2}$ of 1 per cent. of any number, is $\frac{1}{2}$ of .01, or .005 of it; $\frac{1}{4}$ per cent. is .0025 of it, and $2\frac{1}{2}$ per cent. is .025 of it, &c.

Any per cent. of a number, then, is so many hundredths of that number.

Write on your slate answers to the following questions.

What is 6 per cent. of any number?
What is $\frac{1}{8}$ per cent. of any number?
What is $5\frac{1}{4}$ per cent. of any number?
What is 200 per cent. of any number?
What is 17 per cent. of any number?
What is $2\frac{1}{3}$ per cent. of any number?
What is 525 per cent. of any number?
What is $33\frac{1}{3}$ per cent. of any number?
What is 1012 per cent. of any number?
What is 20 per cent. of any number?
What is $4\frac{1}{5}$ per cent. of any number?
What is 100 per cent. of any number?

What is per cent. a contraction of, and what does it signify?

What is 1 per cent. of any number? 2 per cent.? 3 per cent.? 25 per cent.? 120 per cent.? 100 per cent.? $\frac{1}{2}$ per cent.? $\frac{1}{4}$ per cent.? $2\frac{1}{2}$ per cent.?

What is any per cent. of a number?

What per cent. of any number, that is, how many hundredths of any number will .005 of that number be?
will ½ of that number be?
will .185 of that number be?
will .12125 of that number be?
will ⅛ of that number be?
will .0025 of that number be?
will .44 of that number be?
will ⅕ of that number be?
will 2¼ times that number be?
will 5 times that number be?
will 12⅛ times that number be?
will 7⅙ times that number be?

The number of which we are to get a certain per cent. is called the *principal*, and the per cent., considered as hundredths, is called the *rate*.

Lesson 121.

1. A trader intrusted a man with $225, 4 per cent. of which he was directed to invest in books, and the rest in West India goods; what sum must he lay out in books?

OPERATION.
$2 2 5
.0 4
$9.0 0 Ans.

2. A man let a house in Baltimore for a rent of $950 a year, with a condition that 8½ per cent. of the rent should be expended in repairs on the house; how many dollars a year must be laid out in repairs? Ans. $80.75.

3. A merchant having $2,507.75 in a bank, drew out 20 per cent. of it; how much was that? Ans. $501.55.

4. What is 435 per cent. of $1,000? Ans. $4,350.

5. A merchant had 3 bales of cotton, the first of which contained 4 cwt. 1 qr. 27 lbs., the second 5 cwt. 7 lbs., and the third 3 cwt. 3 qrs. 8 lbs.; they were so much damaged during a fire that 18 per cent. of the cotton was destroyed; what quantity was that?
Ans. 2 cwt. 1 qr. 17 lbs. 10.24 oz.

What is called the *principal*? The rate?

Since we multiply the principal by the rate to get the amount of the per cent.,

We must divide the amount of the per cent. by the rate to get the principal, and by the principal to get the rate.

6. A merchant having $ 4,000 deposited in a bank, drew out a certain per cent. of it, the amount of which was $ 20; what per cent. of the deposit was that? Ans. ½ per cent.

7. The owner of an iron mine allowed some laborers to work it for one year, on condition of paying him 10 per cent. of the proceeds; at the end of the year they paid him $ 250; what were the proceeds? Ans. $ 2,500.

8. 17 bu. 2 pks. are 7½ per cent. of a certain quantity; what is that quantity? Ans. 233 bu. 1⅓ pk.

9. If I pay a man $ 8, and have $ 1,992 left, what per cent. of my money do I pay him? Ans. $\frac{4}{10}$ of 1 per cent.

Explanation. $ 8 added to $ 1,992 is the sum which I had, and $ 8 is the amount of the per cent. I pay.

10. If a man paid me $ 114, which was 5 per cent. less than what he owed me, how much did he owe me? Ans. $ 120.

Explanation. He paid 95 per cent. of what was due.

11. If a trader sends you 75 yards of black broadcloth, 20 per cent. more than you ordered, what quantity did you order? Ans. 62½ yds

COMMISSION.

Lesson 122.

Commission is a reward paid to an agent, factor, broker, or correspondent, of so much per cent. on the amount of the purchases or sales made by him.

1. What amount of commission must I pay my factor for selling $ 3,525.16⅔ worth of rice, at 2¼ per cent.? Ans. $ 79.32.

How do we get the principal from the amount of the per cent. and rate, and the rate from the amount of the per cent. and principal? *Why?*

What is commission?

2. If I buy 375 chal. 35 bu. 3 pks. of Newcastle coal at $ 12 a chal., and receive 2 per cent. commission, what will be the amount of my commission ? Ans. $ 90.24.

3. The agent of a landholder sold 117 A. 2 qrs. 4 sq. rods of land, at $ 18 an acre, and charged $1\frac{1}{3}$ per cent. commission ; what sum must be paid to the landholder after deducting the amount of the commission ?

Ans. $ 2,087.24.

4. A broker charged $ 35 for assisting me in purchasing a farm for $ 5,000 ; what per cent. commission or brokerage did he demand ? Ans. $\frac{7}{10}$ of 1 per cent.

5. The expenses of selling a quantity of copper, at 2 per cent. commission, were $ 253.42; what was the sum it sold for ? Ans. $ 12,671.

6. My correspondent sold 2,525 bu. of potatoes for me, at a commission of 3 per cent., and sent me $ 979.70 as the balance due after deducting the amount of his commission ; how much did he sell them all for, and how much did he get a bushel ?

Ans. he sold all for $1,010, and got 40 cents a bu.

7. If a man sends you $ 2,050 to lay out in goods for him, and directs you to retain $2\frac{1}{2}$ per cent. on the amount of purchases for commission, what sum must you retain ?

Ans. $ 50.

STOCKS.

Lesson 123.

Stock is the name of the funds of government, and of the capital of banks, insurance offices, factories, canals, railroads, and like companies. It is owned in shares.

When a share of any stock sells at its original cost, it is said to be at *par ;* when it sells at more than its original cost, it is said to be *above par*, and at so much per cent. *advance ;* when it sells at less than its original cost, it is said to be *below par*, and at so much per cent. *discount.*

What is stock ? How is it owned ?

When is stock said to be at par ? Above par ? Below par ? In advance ? At a discount ?

1. A man bought 23 shares of the Utica and Schenectady railroad stock at 21½ per cent. advance, the par value being $ 100 a share; what did he give for them?
Ans. $ 2,794.50.

2. If I buy several shares of bank stock at $ 54 a share, the par value being $ 60, at what per cent. discount do I obtain them? Ans. 10 per cent.

3. A merchant bought a share of the stock of the Locks and Canals Company in Lowell at 270 per cent. advance; what did it cost, the par value being $ 500? Ans. $ 1,850.

4. A broker bought 5 shares of the Massachusetts Bank for me, at 1 per cent. advance; what did they cost me, the par value being $ 250 a share, and the broker charging ¾ per cent. commission? Ans. $ 1,271.97.

5. If you buy canal stock at 40 per cent. discount, and give $ 30 a share, what is the par value?
Ans. $ 50 a share.

6. What is the par value of a share of factory stock which sold for $ 560 at 12 per cent. advance?
Ans. $ 500 a share.

BANKRUPTCY.

It sometimes happens that a merchant, through miscalculation or misfortune, becomes unable to pay the full amount of his debts, in which case he is called a *bankrupt*, and his property is distributed among his creditors in proportion to what is due to each.

7. A man failed for $ 3,528, having property to the amount of $ 2,963.52; what per cent. of his debts can he pay; that is, what per cent. of $ 3,528 is $ 2,963.52?
Ans. 84 per cent.

8. How much will he pay on a debt of $ 100? Ans. $ 84.

9. Samuel Jackson failed, owing as follows; to John Smith $ 1,800, to Charles Brown $ 8,350, to John Williams $ 2,511.16⅔, and to James Thompson $ 5,000; if he has property to the amount of $ 10,596.70, what per cent. of his debts can he pay, and what can he pay each of his

---

What sometimes happens to a merchant? In that case, what is he *called, and what is done* with his property?

creditors? Ans. he can pay 60 per cent. of his debts; to John Smith $ 1,080, Charles Brown $ 5,010, John Williams $ 1,506.70, and James Thompson $ 3,000.

10. A bankrupt who pays $33\frac{1}{3}$ per cent. of his debts, owes me $ 180; how much can I get of it? Ans. $ 60.

11. A bankrupt who has property to the amount of $ 16,525, owes A. $ 10,000, B. $ 5,000, C. $ 3,828, and D. $ 6,500; if the above are all his debts, what per cent. of them can he pay, and what sum will each of his creditors receive? Ans. he can pay $65\frac{244}{1000}$ per cent. of his debts, nearly; to A. $ 6,524.40, B. $ 3,262.20, C. $ 2,497.54, and D. $ 4,240.86.

LOSS AND GAIN.

Lesson 124.

1. A merchant bought a quantity of molasses for $ 3,500; for what sum must he sell it so as to gain 10 per cent.? Ans. for $ 3,850.

2. If I buy 5 pieces of broadcloth, containing 27 yards each, at $ 4 a yard, and sell the whole for $ 583.20, what per cent. do I gain? Ans. 8 per cent.

Explanation. The first cost is the principal, and the gain is the amount of the per cent.

3. Having bought a quantity of wine for $ 873.25, I lost so much by leakage and other accidents, that I was content to sell it for 18 per cent. less than the first cost; what did I get for it? Ans. $ $716.06\frac{1}{2}$.

4. A merchant bought $ 2,000 worth of goods, and marked the prices $33\frac{1}{3}$ per cent. higher than the cost, but in selling the goods he took 25 per cent. less than was marked; what sum did he gain or lose?
Ans. he sold the goods at first cost.

5. What per cent. would he have gained or lost had he taken from the price marked 25 per cent. of the first cost?
Ans. he would have gained $8\frac{1}{3}$ per cent.

6. A man bought 35 T. 18 cwt. of hay for $ 600; at what price a ton must he sell it so as to gain $12\frac{1}{2}$ per cent.?
Ans. $ 18.80.

7. *If your* agent buys one thousand bushels of corn

$750, and charges 2 per cent. commission, at what per cent. in advance of the cost, which includes commission, must you sell the corn to gain $350?

Ans. $45\frac{3}{4}$ per cent., about.

8. A merchant bought a quantity of pork at $25 a barrel, and put the price 30 per cent. more than the cost; what sum must he take off from the price of each barrel to gain 10 per cent.? Ans. $5.

9. Sold 700 barrels of flour for $4,550, gaining 4 per cent. on it; what did it cost me a barrel? Ans. $6.25.

10. A merchant bought 65 hogsheads of molasses in Matanzas, at $21.50 a hogshead; after paying $50.59 for duties, $79.33 for freight, $39 for insurance to Portland, $12 for truckage, and $4.50 for sundry other expenses, he finally sold it at $30 a hogshead; what per cent. did he gain? Ans. $23\frac{18}{100}$ per cent., about.

11. A man sold a ship for $11,500, gaining 15 per cent. on what she cost him; what per cent. would he have gained by selling the ship for $10,800? Ans. 8 per cent.

DRAFT AND TARE.

Lesson 125.

Draft is a small allowance on the weight of an article so that the quantity may hold out when retailed.

The following table shows the allowances usually made for draft.

Note. It is not necessary to commit it to memory.

The allowance for draft on a parcel weighing

	112 lbs.		is 1 lb.
above	112 lbs.	and not over 224 lbs.	is 2 lbs.
above	224 lbs.	and not over 336 lbs.	is 3 lbs.
above	336 lbs.	and not over 1,120 lbs.	is 4 lbs.
above	1,120 lbs.	and not over 2,016 lbs.	is 7 lbs
above	2,016 lbs.		is 9 lbs.

Tare is an allowance for the weight of the box, cask,

What is draft? What is tare?

bag, &c., containing the goods. It is reckoned on what is left after the draft has been deducted. Draft and tare are the only allowances now made by merchants.

The following table shows the allowances usually made for tare on several important articles of merchandise.

Note. It is not necessary to commit it to memory.

The allowance for tare on

Almonds in bags, is	3 per cent.
Alum in casks,	12 per cent.
Bristles from Cronstadt,	12 per cent.
Bristles from Archangel,	14 per cent.
Beef, jerked, in hogsheads,	112 lbs. a hhd.
Beef, jerked, in drums,	70 lbs. a drum.
Cordage in mats,	1¾ per cent.
Camphor, crude, in tubs,	35 per cent.
Candles in boxes,	8 per cent.
Cinnamon in chests,	16 lbs. a chest.
Cinnamon in mats,	8 per cent.
Cloves in casks,	15 per cent.
Cocoa in bags,	1 per cent.
Cocoa in casks,	10 per cent.
Cocoa in serons,	10 per cent.
Chocolate in boxes,	10 per cent.
Coffee in bags from the West Indies,	2 per cent.
Coffee in grass bags from the East Indies,	2 lbs. a bag.
Coffee in bales,	3 per cent.
Coffee in casks,	12 per cent.
Cotton in bales,	2 per cent.
Cotton in serons,	6 per cent.
Currants in casks,	12 per cent.
Cheese in hampers or baskets,	10 per cent.
Cheese in boxes,	20 per cent.
Copperas in casks,	12 per cent.
Figs in boxes of 60 lbs.,	9 lbs. a box.
Figs in half boxes of 30 lbs.,	5¼ lbs. a box.
Figs in quarter boxes of 15 lbs.,	3¼ lbs. a box.
Figs in drums,	10 per cent.
Glue, from Russia, in bales,	5 lbs. a bale.
Glauber salts in casks,	8 per cent.

How is tare *reckoned?* What are the only allowances now made by *merchants.*

Indigo in bags or mats,	3 per cent.
Indigo in serons,	10 per cent.
Indigo in barrels,	12 per cent.
Indigo in other casks,	15 per cent.
Indigo in cases,	20 per cent.
Mace in casks or kegs,	33 per cent.
Nails in casks,	8 per cent.
Ochre, French, in casks,	12 per cent.
Pepper in bags,	2 per cent.
Pepper in bales,	5 per cent.
Pepper in casks,	12 per cent.
Pimento in bags,	3 per cent.
Pimento in bales,	5 per cent.
Pimento in casks,	16 per cent.
Prunes in boxes.	7 lbs. a box.
Raisins, Malaga, in boxes,	6 lbs. a box.
Raisins, Malaga, in jars,	5 lbs. a jar.
Raisins, Malaga, in casks,	12 lbs. a cask.
Raisins, Smyrna,	12 per cent.
Sugar, Java, in willow baskets,	60 lbs. a basket.
Sugar in bags or mats,	5 per cent.
Sugar in casks,	12 per cent.
Sugar in boxes,	15 per cent.
Sugar in canisters,	35 per cent.
Sugar candy in baskets,	5 per cent.
Sugar candy in boxes,	10 per cent.
Soap in boxes,	10 per cent.
Soap, Marseilles, in boxes,	12 per cent.
Shot in casks,	3 per cent.
Steel in bundles,	2 lbs. a bundle.
Steel in cases,	60 lbs. a case.
Tea, bohea, in chests,	70 lbs. a chest.
Tea, bohea, in half chests,	36 lbs. a piece.
Tea, bohea, in quarter chests,	20 lbs. apiece.
Twine in casks,	12 per cent.
Twine in bales,	3 per cent.
Tallow in serons,	10 per cent.
Tallow in casks,	12 per cent.
Wool, Smyrna, in bales,	10 lbs. a bale.
Wool, Hamburg, in bales,	3 per cent.
Wool, South American, in bales,	15 lbs. a bale.

The weight of the envelopes of articles not named in the table is estimated, and the tare fixed accordingly.

How is the tare fixed on articles not named in the table.

Note. Tare is always expressed in whole pounds; less than half a pound is omitted; more than half a pound is reckoned a pound.

The whole weight of any parcel of goods, including the box, cask, bag, &c., containing the goods, is called the *gross weight*. The weight of any parcel of goods after the draft and tare have been deducted, is called the *neat weight*.

Leakage. After the exact quantity of liquid in a cask is found, by gauging, it is usual to deduct 2 per cent. for leakage.

1. What is the neat weight of 20 casks of Malaga raisins, the gross weight of each being 128 lbs., the draft and tare as in the tables? Ans. 2,280 lbs.

2. A merchant bought 3 bales of cotton; the gross weight of the first was 5 cwt. 23 lbs.; of the second, 4 cwt. 3 qrs.; and of the third, 3 cwt. 3 qrs. 26 lbs.; how much cotton did he get? Ans. 13 cwt. 2 qrs. 6 lbs.

3. If I buy 10 bags of West India coffee, the gross weight of each of which is 1 cwt., how much coffee do I obtain? Ans. 1,088 lbs.

4. The gross weight of 17 bags of pepper is 17 cwt.; how much is the neat weight? Ans. 16 cwt. 2 qrs. 1 lb.

5. What is the neat weight, in pounds, of 27 hhds. of sugar, the average gross weight of each being 7 cwt. 1 qr. 6 lbs.? Ans 19,341 lbs.

6. What amount of leakage must be allowed on three hhds. of rum, which being gauged were found to contain the following quantities; the first, 68 gals. 3 qts., the second 60 gals., and the third, 65 gals. 5 qts.? Ans. 3.88 gals., say 4 gals.

7. What is the draft and tare on 4 casks of nails, the average gross weight of each being 217 lbs.? Ans. the draft is 8 lbs., and the tare 69 lbs.

8. The gross weight of a firkin of butter is 50 lbs., and the tare is reckoned 6 lbs.; what per cent of the whole is butter? Ans. 88 per cent.

In what is tare always expressed? What is done with less than half a pound? With more than half a pound?
What is called gross weight? Neat weight?
What is said of leakage?

DUTIES.

LESSON 126.

Duties are taxes on many kinds of goods imported into the United States from foreign countries, and are collected by Custom House Officers, appointed by government.

The duty on some kinds of goods is a certain per cent. of their legal value, and is called an *ad valorem* duty. The legal value of goods subject to an ad valorem duty, is their original cost, which includes all the original charges, except insurance.

The duty on other kinds of goods is so much a ton, hundred weight, pound, gallon, yard, &c. and is called a *specific* duty; allowances for draft, tare, and leakage are made before calculating it.

Debenture is a remission of the duty on foreign goods re-exported.

Drawback is the amount of such a remission.

Bounty is a reward paid to the exporter of certain domestic goods.

Note. When the duties on any goods are less than $ 50, the re-exporter is not entitled to debenture.

1. What is the duty on a quantity of woollen goods that cost in England $ 10,275, at 44 per cent. ad valorem ? Ans. $ 4,521.

2. If I import 18 boxes of chocolate, the gross weight of each being 1 cwt., what sum must I pay for duty, at 4 cents a pound ? Ans. $ 71.92.

3. What will the bounty on 1,637 bls. of American shad exported to Messina, amount to at 20 cents a bl. ? Ans. $ 327,40.

4. What is the duty on 7 pipes of Port wine, containing on an average 130 gallons apiece, at 15 cents a gallon ? Ans. $ 133.80.

What are duties, and how collected ?

What is called an ad valorem duty ? What is the legal value of goods subject to an ad valorem duty ? What does the original cost include ?

What is called a specific duty ? What allowances are made before calculating it ?

What is debenture ? Drawback ? Bounty ?

When is the re-exporter not entitled to debenture ?

Explanation. Remember the leakage.

5. What will the drawback amount to on a quantity of silk goods that cost $ 8,237, the duty being 10 per cent. ad valorem ? Ans. $ 823.70.

6. In a catalogue of goods with their prices, &c., called an invoice of goods, from Matanzas, I find 16 boxes of brown sugar, the average gross weight of each of which is 7 cwt. 2 qrs. 6 lbs., the tare being stated at 17 per cent.; if the invoice tare be allowed, what will the duty be at 2½ cents a pound ? Ans. $ 279.55.

7. What is the duty on a cargo of iron imported from Newport, in Wales, containing 275 T., at $ 30 a ton, no allowance being made for draft or tare ? Ans. $ 8,250.

8. How much drawback am I entitled to on exporting 7 puncheons of rum, containing, according to measurement, 800 gallons, the duty being 42 cents a gallon ? Ans. $ 329.28.

9. A merchant of Boston, exported 230 hhds. of New England rum, averaging 112 gals. apiece, to Smyrna; what sum must he receive for bounty, the law allowing him 4 cents a gal. ? Ans. $ 1,009.80.

INTEREST,

OR THE REWARD PAID FOR THE USE OF MONEY.

SIMPLE INTEREST.

LESSON 127.

Simple interest is a reward of so much a year for the money lent. A certain per cent. of the money is generally paid for its use one year.

The money lent is called the *principal*; the per cent. paid for the use of the money one year, is called the *rate*; and the principal and interest added together are called the *amount*.

What is simple interest? What is generally paid for the use of money one year?

What is called the principal? Rate? Amount?

Note 1. In speaking of interest, when we say 1 per cent., 2 per cent., 3 per cent., &c., we mean 1 per cent., 2 per cent., 3 per cent., &c., *a year*, unless some other time is stated.

Note 2. The law in most of the states allows the creditor to receive only 6 per cent.; but there is nothing to hinder an agreement at a lower rate.

1. What is the interest of $125 for 1 year, at 6 per cent.?

OPERATION.

$1 2 5 principal.
. 0 6 rate.

$7. 5 0 interest. Ans.

2. If you lend a man $2,750.25 for 1 year, at 5 per cent., what interest must he pay you at the end of that time? Ans. $137.51.

3. How much will the principal and interest both amount to; that is, what will be the amount? Ans. $2,887.76.

4. What is the interest of $1,723.33 for 1 year, at $5\frac{3}{4}$ per cent.? Ans. $99.09.

5. A man borrowed $1,800 for 3 years, at 6 per cent.; what will be the interest for that time? Ans. $324.

Explanation. How many times the interest for 1 year will the interest for 3 years be?

6. What is the interest of $1,000, at $4\frac{7}{8}$ per cent., for 12 years? Ans. $585.

7. If I lend $12,230 for 5 years, at 6 per cent., what amount must I be paid at the end of that time? Ans. $15,899.

8. What is the greatest amount of interest a man can get lawfully in 6 years for $1,250, in the state of New York, where the legal rate is 7 per cent.? Ans. $525.

9. What is the amount of the principal and interest on $1,605.05 for $6\frac{1}{2}$ years, at $6\frac{2}{3}$ per cent.? Ans. $2,300.57.

In speaking of interest, when we say 1 per cent., 2 per cent., 3 per cent., &c., what do we mean?

What is the highest per cent. the law allows the creditor to receive in most of the states? Can an agreement be made at a lower rate?

10. What sum must I give for the use of \$250 during 7 years, at 6 per cent.? Ans. \$105.

11. What will \$16.25 amount to in $2\frac{1}{4}$ years, at 6 per cent.? Ans. \$18.69.

Lesson 128.

1. What is the interest of \$100.33 from January 1st, 1835, to January 16th, 1837, at 6 per cent.?

OPERATION.

\$1 0 0 . 3 3 principal,	\$6 . 0 2 nearly, interest for 1 yr.
. 0 6 rate,	1 5
6 . 0 1 9 8 interest for 1 yr.	3 0 1 0
$2\frac{15}{365}$ years, time.	6 0 2
1 2 . 0 3 9 6	3 6 5) 9 0 3 0 (\$.2 5 nearly, interest for 15 d.
. 2 5 interest for 15 days.	7 3 0
\$1 2 . 2 9 interest for 2 years 15 days. Ans.	1 7 3 0
	1 8 2 5

Explanation. We find the interest for one year, and as 15 days are $\frac{15}{365}$ of a year, we multiply the interest of 1 year by $2\frac{15}{365}$ years, the time. We have cast the interest to the nearest cent.

From the preceding we derive the following

RULE FOR SIMPLE INTEREST.

Multiply the principal by the rate, and the product by the time, in years and 365ths of a year.

2. A merchant borrowed \$2,000 the 20th of January, 1821, at 6 per cent., and paid the debt the 12th of February, 1822; how much was the interest for that time? Ans. \$127.56.

The table on the following page will enable us to find part of a year in days, with great ease.

Explain how example 1, lesson 128, is performed.
What is the rule for Simple Interest?

TABLE

Showing the number of days between any two months in one year.

From	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
To Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
March	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

The use of this table is explained by the following examples ;

How many days are there between Feb. 5th and June 12th ? Look in the column under Feb., and opposite June you find 120, the number of days between Feb. 5th and June 5th ; add the 7 days between June 5th and June 12th, and you get 127 days, the answer.

How many days are there between Sept. 20th and March 8th ? Look in the column under Sept., and opposite March you find 181, the number of days between Sept. 20th and March 20th ; subtract the 12 days between March 8th and March 20th, and you get 169 days, the answer.

In leap years, if the end of Feb. be in the time, 1 day must be added to the number found in the table.

How do you find the number of days between Feb. 5th and June 12th by the table ?

How do you find the number of days between Sept. 20th and March *8th by the table ?*

How is the table used in leap years ?

Note. Let the scholar compute the time in each example of this lesson, and compare it with the time found by the table.

3. A farmer gave his note July 17th, 1827, for $116.50 at 7 per cent. interest, and the 4th of March, 1828, he paid principal and interest; what sum was the interest?
Ans. $5.16.

Explanation. How many days were there in February, 1828?

4. If a man gave his note May 7th, 1830, for $1,800, at 6 per cent., what sum was necessary to discharge the debt June 21st, 1834? Ans. $2,245.32.

5. A man borrowed $605.25 for 1 year and 20 days, at $6\frac{3}{4}$ per cent.; what interest was due at the time of payment? Ans. $43.09.

6. If a note was given for $1,000, at 7 per cent., to be paid in 2 years and 10 days, what interest was due at the end of that time? Ans. $141.92.

7. What sum will a note given September 1st, 1836, for $2,800, at $5\frac{2}{3}$ per cent., amount to the 1st of the next July? Ans. $2,931.72.

8. Cast the interest on a note for $71.30, at 6 per cent., given December 18th, 1830, and paid August 7th, 1837.
Ans. $28.39.

9. What is the interest on $2,500.50 for 28 days, at 10 per cent.? Ans. $19.18.

10. What will be the amount of $5,000 for 211 days, at 6 per cent.? Ans. $5,173.42.

11. Cast the interest on the following note, at 7 per cent., paid January 12th, 1827.

$800. BUFFALO, May 1st, 1819.
For value received, I promise to pay John Howard, or order, eight hundred dollars, on demand, with interest.
Edward Jones.
Ans. $431.28.

LESSON 129.

Another common but incorrect method.

To obtain the interest for part of a year many persons

Take as many 12ths of the interest for 1 year as there are months, and as many 30ths of the interest for 1 month as there are odd days.

How do many persons obtain the interest for part of a year?

The time from the 1st of one month to the 1st of the next, is called a month, also from the 2d of one month to the 2d of the next, from the 3d of one month to the 3d of the next, and so on.

Note. This method may be employed with propriety when the time in a note or agreement is expressed in months. It may often be abridged by considering 2 months as $\frac{1}{6}$ of a year, 3 months as $\frac{1}{4}$, 4 months as $\frac{1}{3}$; 6 months as $\frac{1}{2}$; 8 months as $\frac{2}{3}$, and 9 months as $\frac{3}{4}$ of a year.

1. What is the interest by the preceding method on a note for $88, at 7 per cent., given July 15th, 1817, and paid December 28th, 1818 ?

OPERATION.

$88
.07

6.16 interest for 1 yr.
2.57 interest for 5 mo.
.22 interest for 13 d.

$8.95 in. for 1 yr. 5 mo. 13 d. Ans.

$
12) 6.16 (.513 about, in. for 1 mo.
6 0 5 mo.

16 2.565 in. for 5 mo.
12

40
36

$
30) .513 (.017 about, interest for 1 d.
30 13

213 51
210 17

.221 in. for 13 d.

Explanation. The time being 1 year 5 months and 13 days, we first get 1 year's interest, which divided by 12 gives the interest for 1 month, and this multiplied by 5 gives the interest for 5 months. The interest for 1 month divided by 30, gives 1 day's interest, which multiplied by 13 gives the interest for 13 days. Finally, adding the interest for 1 year 5 months and 13 days together, we get the whole interest.

2. A man lent $850.25 for 8 months, at 5 per cent. ; what was the interest for that time ? Ans. $28.34.

What is called a month?
When may this method be employed with propriety? How may it *often be abridged?*
Explain how example 1, lesson 129, is performed.

3. What is the interest on $75.50 for 4 months 12 days, at 8 per cent.? Ans. $2.21.

4. What interest was due on a note given March 22d, 1831, for $1,738, at 6 per cent., and paid June 6th, 1836, by the preceding rule? Ans. $543.12½.

5. A merchant borrowed $100 the 20th of May, 1835, at 2 per cent. a month, and paid principal and interest the 4th of the next August; what did the interest amount to? Ans. $5.

6. If I borrow $1,500, at 2½ per cent. a month, and pay the debt in 5 months and 18 days, what will be its amount? Ans. $1,710.

7. What is the interest on $2,000 for 90 days, at 6 per cent.? Ans. $30.

Explanation. When money is lent for 30, 60, or 90 days, 30 days are called a month.

8. If you lend $500 at 1 per cent. a month for 60 days, and are paid at the end of 70 days, what amount must you receive? Ans. $511.66⅔.

9. A man borrowed $150 for 30 days, at 10 per cent.; what will be the amount at the end of that time? Ans. $151.25.

10. What was the interest by the preceding rule on a note for $18.25, at 8 per cent., given April 3d, 1830, and paid June 6th, 1832? Ans. $3.18.

LESSON 130.

We have seen that the interest is obtained by multiplying the principal, rate, and time together;

Therefore, when we know any two of these three things, the principal, rate, and time, by dividing the interest by their product, we get the other. See Division, lesson 51.

Note. The principal and interest added together, compose the amount; so the principal subtracted from the amount leaves the interest, and the interest subtracted from the amount leaves the principal.

1. A man paid $5.35 for the use of $55.20 during 1

When we know any two of these three things, the principal, rate, and time, how do we find the other?

What things compose the amount? How then do you get the interest from the amount and principal? The principal from the amount and interest?

year 7 months and 12 days; what rate per cent. did he give ? Ans. 6 per cent.

Explanation. First change the days and months to fractions of a year.

2. If I receive $163.12½ as the amount of $150 for 1 year 91 days, what rate per cent. do I get ? Ans. 7 per cent.

3. How long must $2,515.31¼ remain at interest, at 6 per cent., so that it may amount to $2,615.37½ ? Ans. 242 d.

4. I owe a man $17.50 ; how long must I let him use $87, at 6 per cent., so that the interest may pay the debt ? Ans. 3 yrs. 129 d, about.

5. My agent paid me $37.25 as the interest due on money lent for 2 years 17 days, at 7 per cent. ; what was the sum lent ? Ans. $260.02.

The rate, time, and amount given to find the principal.

6. What sum will amount to $37.50 in 2½ years, at 6 per cent. ?

OPERATION.

	$
$1	1.15) 37.50 (32.61 nearly. Ans.
06 rate.	34 5
.06 interest of $1 for 1 yr.	300
2½	230
.12	700
3	690
.15 interest of $1 for 2½ yrs.	100
1	115
$1.15 amount of $1 for 2½ yrs.	

Explanation. We find the amount of $1 for the rate and time, and divide the amount, $37.50, by it. It will evidently be contained in $37.50 as many times as there are dollars in the principal.

Therefore, to obtain the principal when we know the rate, time, and amount,

Divide the amount by the amount of $1.

7. What principal, at 5 per cent., will amount to $100 in 1 year 105 days ? Ans. $93.95.

Explain how example 6, lesson 130, is performed.

How do we obtain the principal when we know the rate, time, and amount ?

8. What principal, at 7 per cent., will amount to $25.12½ in 4 years 3 months ? Ans. $19.36.

9. A certain sum lent at 6 per cent. produced $250 between July 5th, 1830, and December 26th, 1831 ; what was that sum ? Ans. $229.65.

10. What sum will amount to $10 in 1 year, at 6 per cent. ? Ans. $9.43.

LESSON 131.

To cast the interest on notes, bonds, &c., when partial payments have been made.

The rule for this purpose adopted by the Supreme Court of the United States, and by the Courts in most of the States is as follows ;

Cast the interest to the time when the money paid shall at least be equal to the interest, then discharge the interest from the money paid, subtract the excess, if any, from the principal, and cast the interest on the new principal as before, and so on.

1.

$327.

BOSTON, June 4th, 1829.

For value received, I promise to pay John Smith, or order, three hundred and twenty-seven dollars, on demand, with interest. George Brown.

On this note were the following endorsements ;

April 16th, 1831, received	$100.
July 4th, 1831, received	$3.
January 12th, 1832, received	$18.25.
October 1st, 1832, received	$55.16.

What was due January 1st, 1833 ?

OPERATION.

Payment April 16th, 1831,	$100.
Subtract interest on $327 from June 4th, 1829 to April 16th, 1831,	36,61
Excess,	63,39.

Recite the rule generally employed in the United States to obtain the interest on notes, bonds, &c., when partial payments have been made.

From principal		$327.00.
Subtract excess,		63.39.
Remainder,		263.61.
Payment July 4th, 1831,	$3.00.	
Interest on $263.61 from April 16th, 1831, to July 4th, 1831, $3.42, which being more than the payment, we must cast the interest to the time when the payments amount to as much or more than the interest.		
Payment January 12th, 1832.	18.25.	
Sum of the two last payments,	21.25.	
Subtract interest on $263.61 from April 16th, 1831, to January 12th, 1832,	11.74.	
Excess,	9.51.	
From remainder subtract excess,		9.51.
Remainder,		254.10.
Payment October 1st, 1832,	55.16.	
Subtract interest on $254.10 from January 12th, 1832, to October 1st, 1832,	10.99.	
Excess,	44.17.	
From last remainder subtract excess,		44.17.
Remainder,		209.93.
Add interest on $209.93 from October 1st, 1832, to January 1st, 1833,		3.17.
Ans. due January 1st, 1833,		$213.10.

2. $5,255.50. NATCHEZ, June 12th, 1820.

For value received, I promise to pay James Waldron, or order, five thousand two hundred and fifty-five dollars and fifty cents, in one year, with interest afterwards.

Charles Laval

On the back of this note were written the following receipts ;

August 2d, 1822, received	$600.
December 14th, 1822, received	$1,000.
March 1st, 1823, received	$2,260.37.

What was due July 1st, 1825, 6 per cent. being allowed ?
Ans. $2,555.88.

3. $625. Schenectady, December 1st, 1830.

For value received, we, jointly and severally, promise to pay Peter Vanderheyden, or order, six hundred and twenty-five dollars, in four years from date, with interest till paid.
Hermann Van Pelt.
Walter Suydam.

On this note were the following endorsements ;
February 1st, 1831, received $180.
January 2d, 1832, received $5.50.
February 20th, 1832, received $2.
March 15th, 1833, received $200.
How much must the creditor receive when the note becomes due ? Ans. $349.35.

Explanation. Schenectady is in New York, where 7 per cent. is the legal interest.

4. $210.14. Wheeling, August 13th, 1828.

For value received, I promise to pay Alexander Stevens, or order, two hundred and ten dollars and fourteen cents, on demand, with interest.
William Doyle.
Witness, Samuel White.

On this note were the following endorsements ;
December 12th, 1828, received $3.
May 2d, 1829, received $5.
June 1st, 1829, received $180.25.
What was due July 29th, 1829, reckoning 6 per cent. interest ? Ans. $32.28.

5. Suppose I give a man in Boston a bond for $600, dated June 1st, 1835, and drawing interest, by which I am required to pay $100 at the beginning of every month for 5 months, and at the end of 6 months to pay the balance ; what will the balance be if I pay promptly ?
Ans. $110.68.

6. What is due July 1st, 1836, on a note given August 3d, 1835, for $25.50, drawing 6 per cent. interest, on the back of which were the following endorsements ;
January 13th, 1836, received $18.12.
February 1st, 1836, received $2. Ans. $6.24.

Lesson 132.

Another common but incorrect method.

To obtain what is due on a note or bond when partial payments have been made, many persons,

Find the amount of the principal at the time of settlement, also the amount of each payment at the time of settlement, and then subtract the sum of the amounts of the payments from the amount of the principal.

This method is easy but illegal, and is considered unjust towards the lender; for if you lend me $100 at 6 per cent., and I pay you the interest, or $6, at the end of every year, in 25 years the amounts of the payments will be more than the amount of the principal.

Experiment, however, shows it may be employed without much error, when a settlement is made within a year of the commencement of interest on the note or bond. It may be used with propriety to find the balance due on an account at interest.

Perform the following examples by this rule.

1. Example 4, lesson 131. Ans. $32.08.
2. Example 5, lesson 131. Ans. $110.50.
3. Example 6, lesson 131. Ans. $6.22.
4. What is the balance due on the following account, settled January 1st, 1835; each item drawing interest at 6 per cent.?

Samuel Jay's account with David Sibley,

Dr.					Cr.
1834.			1834.		
March 1.	To Goods	$200	June 2.	By Cash	$100
May 5.	" Goods	50	Aug. 12.	" Cash	85
Sept. 8.	" Pork	130	Dec. 20.	" Cash	125

Ans. $78.77, due David Sibley.

Explanation. The easiest course is to multiply each item on the Dr., or debtor side by the time in days, multiply the sum of the products by the rate, or .06, and divide this product by 365. The quotient is plainly the whole

---

How do many persons obtain what is due on a note or bond, when partial payments have been made?

What is said of the ease, legality, and justice of this method?

When may it be employed without much error? When also may it *be used?*

interest on the Dr. side. Proceed the same way to get the interest on the Cr., or creditor side.

5. Find the sum due on the following account at interest, settled August 11th, 1838, 7 per cent. being reckoned.

R. Leach, his account current with J. Barr.

Dr.					Cr.
1838.			1837.		
Feb. 21.	To Cash	$75	Oct. 10.	By Hay	$250
June 11.	" Cash	200	1838		
July 6.	" Cash	175	Jan. 18.	" Corn	$75
			Feb. 6.	" Oats	60
			April 11.	" Potatoes	100

Ans. $51.04 due R. Leach.

6. What is due on the following account at interest, settled July 1st, 1830, money being worth 5 per cent. ?

John Marsh's account current with James Colburn.

Dr.					Cr.
1830.			1830.		
Jan. 2.	To Goods	$40	Feb. 12.	By Beef	$50
June. 2.	" Goods	100	March 1.	" Pork	60
			May 4.	" Hay	20

Ans. $9.27 due James Colburn.

Note. The rule employed in the Courts of New Jersey, is but very little different from that given at the beginning of lesson 131. The rules employed in the Courts of Connecticut and Vermont, produce results a little different from this rule.

The following is the rule established by the Supreme Court of the State of Connecticut in 1804, and should be studied by residents in that State. It may be omitted by others.

CONNECTICUT RULE.

Compute the interest to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due, to the next payment, and then deduct the payment as above; and in like manner from one payment to another till all the payments are absorbed; provided the time between one payment and another be one year or more. *But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum*

What is said of the rule employed in the Courts of New Jersey? Of the rules employed in the Courts of Connecticut and Vermont?

Recite the Connecticut Rule.

from the principal and interest, added as above. However, if the year extends beyond the time of settlement, find the amount of the principal remaining unpaid, up to the time of such settlement, also the amounts of the payment or payments up to the same time, and deduct their sum from the amount of the principal.

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.

Example 1, lesson 131, is performed by this rule, thus;

Principal,	$327.00.
Add interest from June 4th, 1829, to April 16th, 1831,	36.61.
	363.61.
Deduct first payment,	100.00.
	263.61.
Add interest of $263.61 for 1 year,	15.82.
	279.43.
Deduct amount of second and third payments, at the end of this year, or April 16th, 1832,	21,58.
	257.85.
Add interest on $257.85 to the time of settlement,	11.02.
	268.87.
Deduct amount of fourth payment at time of settlement,	55.99.
Due January 1st, 1833,	$212.88.

22 cents less than the answer found by the first method given.

Explanation. We found the amount of the second and third payments from the time the *third* payment was made; for when the second payment was made, the interest on the principal sum amounted to more than the payment.

COMPOUND INTEREST.

LESSON 133.

Compound Interest is a reward paid for the principal, and also for the interest after it becomes due.

To calculate compound interest, therefore,

Make the amount at the time interest is due a new principal, on which cast the interest to the time when interest is again due, and so on, and finally subtract the first principal from the last amount.

What is compound interest?
How do we calculate compound interest?

1. Suppose I give a note, on demand, for $100, at 6 per cent., and at the end of each year give a new note for the principal and interest; what will all the interest come to in 3 years?

OPERATION.

	$100	principal.
	.06	
	6.00	
	100.	
	106.	amount, principal for the second year.
	.06	
	6.36	interest for the second year.
	106	
	112.36	amount, principal for the third year.
	.06	
	6.7416	interest for the third year.
	112.36	
	119.1016	amount at the end of 3 years.
subtract	100	first principal.
	$19.10	interest for 3 years. Ans.

Note. Instead of multiplying by .06, and adding the principal to the product for the amount, it is plain that we can obtain each amount by multiplying by 1.06.

2. What will $255.16⅔ amount to in 5¼ years, at 6 per cent., compound interest; the interest being added to the principal in a new note at the end of each year?

Ans. $346.59.

3. A man lent $1,000 at 7 per cent., on condition of receiving the interest every 90 days; what will be the whole amount in 1 year 40 days, if all the interest proceeding from this sum be lent on the same terms? Ans. $1,081.24.

4. What will a note for $1,843.12½ amount to in 6 years, at 6 per cent., the interest being added to the principal in a new note at the end of each year?

Ans. $2,614.51.

In example 1, instead of multiplying by .06 and adding the principal to the product for the amount, what course can we take?

5. What will the interest on a note for $1,200 amount to in 3 years, at 6 per cent., the note being renewed every 90 days, and the interest included in it? Ans. $238.33.

The following table will enable us to perform examples in compound interest with great ease.

TABLE,

Showing the amount of $1 at 5, 6, and 7 per cent., compound interest, from one year to 40.

Years.	At 5 per cent.	At 6 per cent.	At 7 per cent.
1	$1.050000.	$1.060000.	$1.070000.
2	1.102500.	1.123600.	1.144900.
3	1.157625.	1.191016.	1.225043.
4	1.215506.	1.262477.	1.310796.
5	1.276282.	1.338226.	1.402552.
6	1.340096.	1.418519.	1.500730.
7	1.407100.	1.503630.	1.605781.
8	1.477455.	1.593848.	1.718186.
9	1.551328.	1.689479.	1.838459.
10	1.628895.	1.790848.	1.967151.
11	1.710339.	1.898299.	2.104852.
12	1.795856.	2.012196.	2.252192.
13	1.885649.	2.132928.	2.409845.
14	1.979932.	2.260904.	2.578534.
15	2.078928.	2.396558.	2.759032.
16	2.182875.	2.540352.	2.952164.
17	2.292018.	2.692773.	3.158815.
18	2.406619.	2.854339.	3.379932.
19	2.526950.	3.025600.	3.616528.
20	2.653298.	3.207135.	3.869684.
21	2.785963.	3.399564.	4.140562.
22	2.925261.	3.603537.	4.430402.
23	3.071524.	3.819750.	4.740530.
24	3.225100.	4.048935.	5.072367.
25	3.386355.	4.291871.	5.427433.
26	3.555673.	4.549383.	5.807353.
27	3.733456.	4.822346.	6.213868.
28	3.920129.	5.111687.	6.648838.
29	4.116136.	5.418388.	7.114257.
30	4.321942.	5.743491.	7.612255.

What does the table show?

Years.	At 5 per cent.	At 6 per cent.	At 7 per cent.
31	4.538039.	6.088101.	8.145113.
32	4.764941.	6.453387.	8.715271.
33	5.003189.	6.840590.	9.325340.
34	5.253348.	7.251025.	9.978114.
35	5.516015.	7.686087.	10.676581.
36	5.791816.	8.147252.	11.423942.
37	6.081407.	8.636087.	12.223618.
38	6.385477.	9.154252.	13.079271.
39	6.704751.	9.703507.	13.994820.
40	7.039989.	10.285718.	14.974458.

A table like this can be calculated by finding the amount of $1, at 5, 6 and 7 per cent., compound interest, for 1, 2, 3, &c. years; that is, by multiplying $1 by 1.05, 1.06, 1.07, and the product by the same multipliers, and so on. See note to example 1.

6. What is the compound interest on $50 for 30 years, at 7 per cent.? Ans. $330.61.

Explanation. What will $1 amount to in 30 years, at 7 per cent., by the table? Then what will $50 amount to?

7. What will the compound interest on $216 come to in 19 years, at 6 per cent.? Ans. $437.53.

8. What will $18 amount to in 29 years 3 months, at 5 per cent., compound interest? Ans. $75.02.

Explanation. What does $1 amount to in 29 years? What does this sum amount to in 3 months?

9. What will $11 amount to in 39 years, at 6 per cent., the interest being added to the principal in a new note at the end of every year? Ans. $106.74.

10. What will the compound interest on $20.25 come to in 12 yrs. 2 mo. and 14 d., at 6 per cent.? Ans. $21.

DISCOUNT.

LESSON 134.

1. A dealer in carriages has a chaise marked $195, but offers to make a discount or deduction of $23; what sum will purchase the chaise? Ans. $172.

How can a table like this be calculated?

2. A bookseller demanded $17.50 for Irving's works handsomely bound, but concludes to discount 12 per cent., for immediate payment; what is his cash price for the works? Ans. $15.40.

3. If you owe a man $500, to be paid in 2 years, *without interest*, what must you pay now to cancel the debt, if money is worth 7 per cent.? Ans. $438.60.

Explanation. It is plain that you must pay a sum, which at 7 per cent. interest will amount to $500 in 2 years. The sum to be paid now may be considered as the principal, and $500 as the amount. You have therefore the rate, time, and amount given to find the principal. See rule in Interest, middle of lesson 130.

The principal which will amount to a debt, not on interest, when it is payable, is called the *present worth* of that debt. The difference between the debt and present worth, is the proper *discount* to be made for immediate payment.

4. What is the present worth of a note for $385 to be paid in 3½ years without interest, money being worth 6 per cent.? Ans. $318.18.

5. A country trader bought $1,850.37½ worth of goods of a merchant of Philadelphia, on 6 months' credit; what discount should the merchant make for immediate payment, if money is worth 6 per cent.? Ans. $53.89.

Explanation. Get the present worth first.

6. What is the present worth of a note for $9,825 to be paid in 276 days, if money is worth 7 per cent.?
Ans. $9,331.09.

7. If you give a man one note for $400, not bearing interest, to be paid in 1 year, and another for $2,537.55, not bearing interest, to be paid in 2 years and 8 months, what sum must you pay down to cancel both of these notes, 6 per cent. being the legal rate? Ans. $2,564.90.

8. A merchant paid $875 down for goods, and sold them the same day for $983.33⅓, on 9 months' credit; what did he gain, money being worth 10 per cent.? Ans. $39.73.

9. If you are offered $3,000 down for a house, or $3,300 to be paid in 2 years, without interest, which will be the best bargain, money being worth 6 per cent.?
Ans. $3,000 down is the best bargain.

Explain how example 3 in Discount should be performed.

What is called the present worth of a debt? What is the proper *discount to be made* for immediate payment?

10. A hardware dealer sold me a stove for $28.50, discounting 5 per cent. from the ordinary price for ready money; what was the ordinary price? Ans. $30.

Explanation. See Percentage, example 10, lesson 121.

BANKING.

Lesson 135.

A bank is an institution which trades in money. It is usually owned in shares by persons called *stockholders*, who choose a President and Directors to manage its concerns. The principal object of a bank is to make and lend notes, called *bank bills*, as money.

When money is borrowed from a bank, the usual manner of proceeding is as follows. If A wants $1,000 for a certain time, say 90 days, and his friend B is willing to become his surety, he writes a note promising to pay B. at the bank $1,000 in 90 days; B now indorses the note, that is, writes his name on the back, thereby making himself a security for the payment. A then proceeds to the bank, and the officers, if they choose, discount the note, that is, they take it, cast the interest on $1,000 for 3 days more than the time, or 90 days, and deducting it from $1,000, hand A the balance in bills. The 3 days are called *days of grace*, and the bank does not require the $1,000 to be paid until the end of 93 days. Interest paid in this way is improperly called *discount*. It is plain, then, that banks take interest for larger sums than they lend, and by calling 30 days 1 month, 60 days 2 months, 90 days 3 months, or $\frac{1}{4}$ of a year, &c., obtain interest for a longer time than the borrower has the money.

1. Interest being 6 per cent., what sum do I get on a note for $683, payable in 60 days, which a bank discounts for me? Ans. $675.83.

Explanation. Remember the 3 days of grace.

What is a bank? How is it usually owned? What is the principal object of a bank?

When money is borrowed from a bank what is the usual manner of proceeding? What are the 3 days called, and when does the bank require the note to be paid? What in banking is called discount? What is plain?

2. What is the amount of the bank discount on the following note, interest being reckoned at 7 per cent.?

$3,327.40. New York, April 20th, 1837.

Ninety days from date I promise to pay James Carver or order, at the Mechanics' Bank, three thousand three hundred and twenty-seven dollars and $\frac{40}{100}$, for value received George Stilman.

Ans. $60.17.

3. A merchant bought $10,000 worth of cotton at $50 a bale, and sold it immediately at $60 a bale, obtaining a note for the amount of the sale, payable in 90 days, without interest; what will be the amount of his gain, if he gets the note discounted at a bank, 6 per cent. being the rate of interest? Ans. $1,814.

4. How much did a man receive on a note for $982, payable in 70 days, which he had discounted at a bank, where the rate was 7 per cent.? Ans. $968.06.

5. What is the bank discount on a note for $2,500, payable in 4 months, the rate of interest being 6 per cent.? Ans. $51.25.

6. What is the bank discount on a note for $700, payable in 90 days, where the legal rate of interest is 7 per cent.? Ans. $12.66.

7. How much do I receive on a note for $2,000, payable in 30 days, which I get discounted at a bank, the rate of interest being 6 per cent.? Ans. $1,989.

8. I have a note due me for $100, payable in 60 days, which the officers of a bank are willing to discount if I indorse it; what will the discount amount to, 6 per cent. being the rate of interest? Ans. $1.05.

EQUATION OF PAYMENTS.

LESSON 136.

1. A merchant owes a trading company the following notes. One for $8,480, due in 1 year, without interest, one for $1,526, due in 18 months, without interest, and one for $1,326, due in 21 months without interest; in what time

can these debts be paid at once, and neither party sustain loss, reckoning any per cent., say 6 ?

Ans. 1 yr. 1 mo. 24 d., about.

Explanation. We first find the present worth of each of these notes, and then find in what time the sum of these present worths amounts to the sum of the debts. This time is evidently the answer.

Therefore, to find the time when several debts, due at different times, can be paid at once without loss to either debtor or creditor,

Find the present worth of each of the debts, and then find in what time the sum of these present worths will amount to the sum of the debts. This time will be the answer.

2. A merchant sold a country trader a quantity of goods, which were to be paid for as follows ; $2,500 were to be paid in 4 months, $350 in 6 months, and $1,000 in 8 months ; in what time can all these sums be paid at once, without loss to either party ? Ans. in 5 mo. 6 d., about.

3. I bought goods to the amount of $10,067, agreeing to pay $2,300 at the end of 3 months, $2,575 at the end of 6 months, $2,600 at the end of 8 months, and $2,862 at the end of 12 months ; it was afterwards thought best that the whole amount should be paid at once ; how long after the purchase should the payment be made ?

Ans. 7 mo. 17 d., about.

Other Questions concerning the Payment of Debts.

4. If you owe a man a note for $80, on demand, with interest, and three other notes not drawing interest, one for $16, due in 2 months, one for $25, due in 3 months, and one for $40, due in 6 months, what sum must you pay at the end of 4 months to discharge the whole, 7 per cent. being the legal interest ? Ans. $162.74.

Explanation. A note not on interest is considered as drawing interest as soon as it becomes due; this being understood, we find the amount of $80 for 4 months, of $16 for 2 months, of $25 for 1 month, and the present worth of $40

Explain how example 1 in Equation of Payments is performed.

How do we find the time when several debts, due at different times, can be paid at once without loss to either debtor or creditor?

When is a *note, not* on interest, considered as drawing interest?

Explain how example 4 in Equation of Payments is performed.

paid 2 months before due. These four sums added together will evidently give the answer.

5. I owe a man $600, due in 1 year without interest, $800 due in 2 years without interest, and he has my note for $2,000, on demand, with interest; what sum should I pay to cancel these debts at the end of 4 months, the rate being 6 per cent.? Ans. $3,344.19.

6. If I owe you $1,200, to be paid at the end of 6 months, but to accommodate you pay $400 down, how long must I keep the remainder so that neither of us shall lose interest? Ans. 9 mo. 1 d., about.

Explanation. What is the amount of $400 on interest for 6 months? This amount being subtracted from $1,200, how long will it take the remainder to amount to $800?

7. A farmer had $200 due him in 1 year, but the debtor chose to pay $100 down, and $30 more at the end of 8 months; what time should he be allowed to keep the remaining $70? Ans. 1 yr. 11 mo. 25 d., about.

Another common but incorrect method.

To find the time when several debts, due at different times, can be paid at once, without loss to either debtor or creditor, merchants usually

Multiply each debt by the time to elapse before it is due, and divide the sum of the products by the sum of the debts.

Though this method is inaccurate it is easy, and experiment shows it may be employed without much error, to find the mean time for the payment of a number of debts, the first and last of which are due within a year of each other. The reasoning employed in many arithmetics to show its accuracy is absurd.

Perform the following examples by this rule.

8. Example 2, lesson 136. Ans. 5 mo. 7 d., nearly.

9. Example 3, lesson 136. Ans. 7 mo. 19 d., nearly.

10. A. B. sells the following articles, on 6 months' credit to C. D., who on May 13th, 1831, settles with A. B. by giving his note, not on interest, for the whole sum due;

What course do merchants usually take to find the time when several debts, due at different times, can be paid at once without loss to either debtor or creditor?

When may this method be employed? What is said of the reasoning *concerning it?*

when must the note be made payable so that no interest will be gained or lost by either party ?

Dr. C. D.

1831.		
Jan. 4.	To 17 bls. Flour, at $8 a bl.	$136.
Feb. 28.	" 108 gals. Molasses, at 25 cents a gal.	27.
April 7.	" 200 lbs. Malaga Raisins, at $6\frac{1}{4}$ cents a lb.	12.50
" 19.	" 6 bls. Mackerel, at $12 a bl.	72.

Ans. in 94 d., or August 15th, 1831.

PROMISCUOUS QUESTIONS

IN

PERCENTAGE, COMMISSION, STOCKS, AND THE SIMILAR RULES.

LESSON 137.

To be performed in the mind.

1. What is the interest on $200 for 2 years, at 5 per cent. ? At 6 per cent. ? At 8 per cent. ? At 12 per cent. ?

2. I borrowed $100 for one year, at 6 per cent., but kept the sum 1 year and 3 months ; what interest was then due ? What interest would have been due had I kept the money 1 year and 4 months ? 1 year and 6 months ? 1 year and 10 months ?

3. A merchant bought some St. Domingo hides for $450, and immediately sold them so as to gain 3 per cent. ; how much did he gain ?

4. What is the bank discount on a note for $80 payable in 60 days, the rate of interest being 6 per cent. ?

Explanation. What is the interest of $80 for 60 days, and for the 3 days of grace, being $\frac{1}{20}$ of 60 days ?

5. A bankrupt who pays only 25 per cent. of his debts owes me $41, what sum shall I get ?

Explanation. See Decimal Fractions, lesson 82.

6. Calling 30 days a month, what will the interest of $100 amount to in 15 days, at 6 per cent. ? In 20 days ? In 25 days ?

7. What duty must I pay on 75 tons of English iron, at *$30 a ton ?*

8. My agent bought me a farm for $2,000; if I pay him 3 per cent. commission for his services, what sum must he receive ?

9. The shares in a certain manufacturing company, are worth $500 a piece at par; what must I give for 2 shares at 4 per cent. discount ?

10. What is the interest of $90 for 3 months, at 5 per cent. ?

11. What is 8 per cent. of $4,000 ? 7 per cent. ?

12. What is the compound interest on $50 for 2 years, at 6 per cent. ?

Lesson 138.

For the Slate.

1. What is the interest on a note for $100 dated July 18th, 1830, and paid November 3d, 1831, at 5¾ per cent. ? Ans. $7.45.

2. A man had $600 due him in 3 months, without interest, and $375 due in 1 year 133 days, without interest; what sum must be paid in 2 months to cancel these debts, 6 per cent. being allowed ? Ans. $946.82.

3. If 8 per cent. be deducted from the amount of a bill, and the creditor be paid $276, what is the amount of the bill ? Ans. $300

4. Received by the brig Nestor, 30 pipes of Port wine, found by the gauges to contain 3,900 gallons; what is the amount of duty to be paid, at 15 cents a gallon, allowing for leakage ? Ans. $573.30.

5. A man paid $12.64 for the use of $200 during 1 year 3 months and 5 days; what was the rate given ? Ans. 5 per cent.

6. An agent received 3 per cent. for selling 15 lbs. 10 oz. of silver in ingots, at $13 a pound; what did his commission amount to ? Ans. $6.17.

7. $1,200 amounted to $1,216.80 in a certain time, at 7 per cent.; what was the time ? Ans. 73 days.

8. 8 shares in the Boston and Worcester railroad sold for $26.66⅔ less than the par value, which is $100 a share; at what per cent. below par did they sell ? Ans. 3⅓ per cent.

9. What principal will amount to $81.20 in 2 years 8 months, at 6 per cent. ? Ans. $70.

10. What sum shall I get at a bank for a note for $800 *payable in 90* days, at 6 per cent. ? Ans. $787.60.

Lesson 139.

1. A trader bought a quantity of goods for $2,000, and marked the price 25 per cent. more than the cost; what per cent. must he deduct from the price marked, in order to gain 10 per cent.? Ans. 12 per cent.

2. $350. STEUBENVILLE, May 4th, 1827.
For value received, I promise to pay John Darnell, or order, three hundred and fifty dollars, on demand, with interest. William Tolman.

On the back of this note were the following endorsements;

January 11th, 1828, received $50.
June 13th, 1828, received $25.

What was due November 12th, 1828, 6 per cent. being allowed? Ans. $304.89.

3. If you owe a man $400, due in 9 months, without interest, and pay him $150 down, how long should he wait for the remainder? Ans. 14 mo. 16 d., about.

4. A merchant imported manufactured articles from Liverpool, which cost at that place $4,000; what will the duties amount to, at 20 per cent. ad valorem.
Ans. $800.

5. What will $700 amount to in 35 years, at 7 per cent., compound interest? Ans. $7,473.61.

6. I owe the following notes, not bearing interest; one for $250.25, due in 6 months, one for $11.30, due in 9 months, one for $65, due in 15 months, and one for $110, due in 18 months; in what time must I pay all these notes so that neither I nor my creditor shall lose?
Ans. in 10 mo. 9 d., about.

7. If I owe a debt of $530, due in 1 year 3 months, without interest, what sum should I pay now to cancel the debt, money being worth 6 per cent.? Ans. $539,53.

8. What sum was due on the following account at interest, at 6 per cent., settled January 1st, 1835.

George Draper's cash account with S. Pierce.

Dr.					
1834.			1834.		Cr.
May 15.	To Cash,	$400	April 3.	By Cash	$800
Nov. 2.	" Cash,	1,000	Aug. 31.	" Cash	900

Ans. $329.05 due George Draper.

RULE OF THREE.

So called because questions in this rule usually have *three* numbers given to find another, or the answer. This rule embraces many questions of so different a nature that no general rule can be given for calculating them.

Lesson 140.

To be performed in the mind.

1. John gave 10 cents for 1 pound of sugar; how many cents must he give for 3 pounds? For 5 pounds? For 8 pounds? For 9 pounds? For 12 pounds? For 15 pounds?

2. A teamster hauled 24 cwt. of pork in 4 equal loads; how much did he haul in 1 load? If he had hauled 28 cwt. in 4 equal loads, how much would there have been in 1 load? If he had hauled 30 cwt. in 5 equal loads, how much would there have been in each load?

3. If you get 6 apples for 3 cents, how many can you obtain for 18 cents?

Explanation. How many apples can you obtain for 1 cent? How many, then, can you get for 18 cents?

4. If you pay 9 cents for 3 oranges, how many cents must you pay for 8 oranges?

Explanation. How many cents do you pay for 1 orange? How many cents, then, must you pay for 8 oranges?

5. A farmer bought 2 gallons of molasses for 60 cents, what would 5 gallons have cost?

6. If you buy 4 bushels of potatoes for 2 dollars, how many bushels can you get for $30?

7. A woman bought 3 yards of cotton cloth for 60 cents; how many yards could she have bought for $2, or 200 cents?

8. Suppose a man travels 40 miles in 5 days, how far can he travel, at the same rate, in a week, or 7 days?

9. If 16 tons of hay keep 8 cows over winter, how many tons will keep 5 cows?

10. A man gives $45 for 3 loads of hay; how much must he give for 7 such loads?

Why is the Rule of Three so called? - What does this rule embrace?

11. If 4 barrels of pork weigh 8 hundred pounds, how much will 9 barrels weigh?

12. A man bought 8 yards of cloth for $4; how many yards can he buy, at the same rate, for $25?

Lesson 141.

1. A certain load of hay will last 6 oxen 5 days; how long will it last 1 ox?

2. If 1 pipe will empty a cistern in 12 hours, how long will it take 3 such pipes to empty it?

3. If 1 pipe will empty a cistern in 12 hours, how many of such pipes will empty it in 2 hours?

Explanation. How many will empty it in 1 hour? How many, then, will empty it in 2 hours?

4. If 1 man can do a piece of work in 16 days, how many men will it take to do it in 4 days?

5. 8 men have provisions enough to last them 9 days; how long will the provisions last 12 men?

Explanation. How long will the provisions last 1 man? How long, then, will they last 12 men?

6. 6 men have provisions enough to last them 8 days; how many men will these provisions last 12 days?

Explanation. How many men will these provisions last 1 day? How many, then, will they last 12 days?

7. A teamster can carry a certain number of barrels of beef in 4 loads if he takes 6 at a load; how many loads must he make if he takes 8 barrels at a load?

8. If a certain quantity of oats last 4 horses 3 days, how long will it last 3 horses?

9. 2 oxen can haul away a pile of lumber in 20 loads; what number can haul it away in 5 loads?

10. A garrison of 200 men have provisions enough for 12 months; if the number be increased to 600, how long will the provisions last?

11. If 6 men can do a piece of work in 2 days, how many men will be necessary to complete it in 4 days?

12. If 6 equal sized pipes will empty a cistern in 2 hours, how many pipes of the same magnitude will empty it in 3 hours?

LESSON 142.

1. If 4 apples cost 3 cents, what will 12 apples cost?

2. What will 5 pounds of raisins cost, if 2 pounds cost 15 cents?

3. If 3 oranges cost 6 cents, how many will 24 cents buy?

4. A man gave $4 for ½ of a yard of broadcloth; how much would ¾ of a yard have cost?

Explanation. How much would 1 yard have cost? Then how much would ¾ of a yard have cost?

5. James gave 6 cents for ½ of a pound of raisins; how many pounds could he have bought for 36 cents?

6. What will 5 gallons of molasses cost, if 1 quart, or ¼ of a gallon is worth 10 cents?

7. 12 men dug a ditch in ⅓ of a day; in what time could 2 men have dug it?

8. 8 men can do a piece of work in ¼ of a day; how many men will it take to do it in ⅔ of a day?

9. If ⅔ of a vessel cost $4,000, how much will ½ of it cost?

10. A man bought 4 yards of shirting for 48 cents; how much would ¾ of a yard have cost at this rate?

11. If 3 quarts of beans cost 37½ cents, what will 2 quarts cost?

12. A boy gave 13 cents for 2 picture books; how many books of the kind could he have purchased for 65 cents?

LESSON 143.

For the Slate.

1. If 18 tons of hay cost me $324, what will 23 tons cost, at the same rate? Ans. $414.

2. 4 horses being found to consume thirty bushels of oats in 6 weeks, how many horses will consume the same quantity in 4 weeks? Ans. 6.

3. If I give $2 for .4 of a cord of wood, how much must I give for ½ of a cord? Ans. $2.50.

4. What will 19 barrels of flour cost, if 4 barrels cost $26? Ans. $123.50.

5. If 19 barrels of flour cost $123.50, what will 4 barrels cost? Ans. $26.

6. A man gave $125 for 5 barrels of pork; how many barrels could he have got for $275. Ans. 11.

7. A stone wall was built in 12 days by a number of workmen who labored only 6 hours in a day ; how long would it have taken them if they had labored 8 hours in a day ? Ans. 9 days.

8. A merchant, who owned $\frac{2}{3}$ of a ship, sold $\frac{4}{5}$ of his part for $7,500 ; what was the whole ship worth at this rate ? Ans. $14,062.50.

Explanation. What part of the ship did he sell ?

9. A. and B. hired a pasture for $35.20, in which A. pastured 15 oxen and B. 7 ; what should each one pay ?
Ans. A. should pay $24, and B. $11.20.

Explanation. If the pasturing of 22 oxen cost $35.20, what will the pasturing of 15 cost, and of 7 cost ?

10. If a man pays 28.12\frac{1}{2}$ for 18 bu. 3 pks. of wheat, what can he get 10 bu. for, at the same rate ? Ans. $15.

LESSON 144.

1. If I give $15 for 7 yards of cloth, what must I give for 1,235 yards, at the same rate ?

OPERATION.

7) 1 5

2.1 4 2 8 6 price of 1 yard.
1 2 3 5 yards.

1 0 7 1 4 3 0
6 4 2 8 5 8
4 2 8 5 7 2
2 1 4 2 8 6

$2 6 4 6.4 3 2 1 0 price of 1,235 yards. Ans.

OPERATION BY AN EASIER METHOD.

$$\$\tfrac{15}{7}$$ price of 1 yard.
1 2 3 5 yards.
1 5

6 1 7 5
1 2 3 5

7) 1 8 5 2 5

$2 6 4 6.4 3 Ans.

Explanation. To obtain the answer we get the price of 1 yard, and multiply it by 1,235, the number of yards; now if we get the price of 1 yard in dollars, cents, mills, &c., by dividing $15 by 7, and then multiply this price by 1,235, the operation, as we see, is tedious; but if we get the price of 1 yard in the form of a fraction, by writing 7 under $15, according to Common Fractions, lesson 60, and then multiply $\$\frac{15}{7}$ and 1,235 together, the operation is much easier.

We should always get a quotient in the form of a fraction whenever an operation by that course becomes easier. The

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Explain how example 1, lesson 144, can be performed.
When should we get a quotient in the form of a fraction?

other method is the best when the number of figures in the quotient will be less than the number of figures in both divisor and dividend.

2. The discount on a note for \$53, due in one year, without interest, being \$3, what will be the discount on a note for \$18,232, due in the same time, without interest? Ans. \$1,032.

3. If 5 yards of cloth cost \$$6\frac{3}{8}$, what will $12\frac{2}{3}$ yards cost? Ans. \$16.15.

4. A grocer gave \$20 for 6 barrels of apples; how many barrels could he have obtained for \$70? Ans. 21.

5. 16 men can dig a ditch in 9 days; how long will it take 6 men to dig it? Ans. 24 days.

6. If 12 soldiers can subsist on 17 pounds of provisions 1 day, what quantity will suffice for 2,300 men for the same time? Ans. $3,258\frac{1}{3}$ lbs.

7. A blacksmith gave \$5.60 for 1 cwt. of Swedes iron; how much must he pay for 15 cwt. 2 qrs. $8\frac{1}{4}$ lbs., at the same rate? Ans. \$87.21.

8. A farmer gave \$838.56 for 83 A. 3 qrs. 17 sq. rods of land; for what sum can he sell 5 A. 2 qrs. 38 sq. rods, so as neither to gain nor lose? Ans. \$$57.37\frac{1}{2}$.

9. If a trader pays \$18 for a hogshead of molasses, containing 80 gals., what should he pay for 2 hhds. 1 tier. 23 gals.? Ans. \$$42.97\frac{1}{2}$.

10. 16,000 cubic feet of water were found to flow over a mill-dam in 1 h. 23 min. 10 sec.; how much will flow over the dam in 3 days? Ans. $831,102\frac{102}{499}$ cubic ft.

LESSON 145.

Note. Many questions in arithmetic, contain numbers that are of no use in the calculation. In what precedes, we have been careful to write such numbers in words, but shall hereafter express them in figures; and the ingenuity of the learner must be exercised in discovering what numbers must be employed to obtain the answer.

1. 3 men laid 28 rods of stone wall in 5 days; how long then should it take 10 men to lay the same amount? Ans. $1\frac{1}{2}$ day.

2. A merchant who owned $\frac{2}{9}$ of a block of stores, sold $\frac{3}{4}$ of his share to 5 men, for \$12,000; what was the whole block worth, at this rate? Ans. \$72,000.

3. If $4\frac{1}{4}$ yards of broadcloth cost \$17, what will $13\frac{7}{8}$ yards cost? Ans. \$55.50.

When will the other method be the best?

4. A merchant bought 3 tierces of rice, each of which contained 6 cwt. 3 qrs. 27 lbs., for $135; how many pounds could he have purchased for $87.33⅓? Ans. 1,519.6 lbs.

5. 5 tons of hay will last 80 sheep 120 days; how long will it last 150 sheep? Ans. 64 days.

6. A trader hired a man to team 6¼ tons of iron 28 miles, for $10; how many modern tons weighing 2,000 lbs. apiece, can he get teamed 16 miles for the same sum? Ans. 12¼ modern tons.

7. If 20 bushels of wheat can be obtained for 100 bushels of potatoes, how much wheat can be obtained for 575 bushels of potatoes? Ans. 115 bushels.

8. If you lend a person $400 for 9 months, with the promise of having a like favor, and some time after borrow $600 of him, how long should you keep that sum in order to receive full compensation? Ans. 6 months.

9. A trader bought a quantity of cloth estimated at 88 yards, for $169; on measuring it he found there were 97 yards; if he sells it so as to make $25, for what sum must he sell 5 yards? Ans. $10.

10. A grocer sells coffee at $.33⅓ a pound, and gains 25 per cent.; how much did 11 pounds cost him? Ans. $2.93⅓.

LESSON 146.

1. A man gave $5.30 for 6.25 cwt. of hay; how much would 1 ton have cost at the same rate? Ans. $16.96.

2. A trader having ⅞ of a cwt. of coffee, sold ⅔ of it for $19.37½; at what price a pound did he sell it? Ans. $.29656, nearly.

3. If $13.50 will buy .75 of a boat, what sum will buy .33⅓ of her? Ans. $6.

4. 112½ yards of cloth, 1½ yard wide, will make 25 suits of clothes; how many yards of cloth ¾ of a yard wide, will make the same number of suits? Ans. 225.

5. If 91⅓ yards of cloth, ⅞ of a yard wide, are sufficient to make 10 suits of clothes, what quantity of cloth, 1¾ yard wide, will be required to make the same number of suits? Ans. 45⅔ yds.

6. The moon moves 13° 10′ 35″ towards the east in 1 day; how long is it in performing a revolution round the earth, *that is, how long* is it in passing over 360°? Ans. 27 d. 7 h. 43 min. 6 sec.

7. If .75 of a pound of butter costs $.1875, what quantity will $37 buy ? Ans. 148 lbs.

8. A trader bought $\frac{3}{4}$ of a pipe of wine for $100 ; what sum should he give for 3 tierces of the same quality ? Ans. 133.33\frac{1}{3}$.

9. If 3 C. 5 ft. of oak wood are of the same value as 1 T. 5 cwt. 3 qrs. of hay, what quantity of wood should 1 modern ton of hay purchase ? Ans. 2 C. 4$\frac{1}{3}$ ft.

10. If a bar of iron 7 feet long, 4 inches wide, and 1$\frac{1}{2}$ inch thick, weighs 140 pounds, what will a bar 3 ft. 2 in. long, and 2 in. square weigh ? Ans. 39 lbs. 7$\frac{1}{3}$ oz.

Explanation. How many cubic inches weigh 140 pounds.

LESSON 147.

1. If 4 men can lay a wall 75 feet long, 4 feet thick, and 10 feet high, in 8 days, how long will it take them to lay a wall 97 feet long, 3$\frac{1}{2}$ feet thick, and 9 feet high ? Ans. 8.148 d.

2. If 125 bushels of wheat grow on 4 A. 2 qrs. 4 sq. rods, how much land will be necessary to produce 650 bushels, supposing the crop to be equally good ? Ans. 23 A. 2 qrs. 4.8 sq. rods.

3. Bought 10 gals. 3 qts. 1 pt. of wine for 16.16\frac{2}{3}$; what sum would 27 gals. 2 qts. have cost, at the same rate ? Ans. $40.88.

4. A merchant bought a hogshead of sugar, containing 5 cwt. 3 qrs. 7$\frac{1}{4}$ lbs., for $50 ; what should he give for 2 other hogsheads, at the same rate, one of which contains 6 cwt. 18 lbs., and the other 4 cwt. 3 qrs.? Ans. $93.82.

5. The fore wheels of a wagon being 11 ft. in circumference, turn round 960 times in 2 miles ; how many times do the hind wheels, which are 14 ft. in circumference, turn round in the same distance ? Ans. 754$\frac{2}{7}$ times.

6. A wine merchant bought 64 pipes of wine for $4,000; he paid $58 for freight, $29 duties, $20 for storage, and for truckage and other expenses, $35 ; if he sells it so as to make $500, for what sum can I get 13 pipes ? Ans. $942.91.

7. If a man buys 257 chal. 15 bu. 3 pks. of Sidney coal for $2,000, and sells it so as to make 25 per cent., for what sum should he sell 13$\frac{1}{2}$ chal. ? Ans. $131.10.

8. A trader bought 10 puncheons of rum for $800 ; 2

puncheons were stove, through accident, and he lost 28 gals. besides, from leakage; if he sells the remainder so as to lose 9 per cent. of the whole cost, for what sum can you buy 3 puncheons? Ans. $284.87.

9. If 3,000 lbs. of provisions last 24 men 8 weeks and 4 days, how long will the same quantity last 96 men?
Ans. 2 w. 1 d.

10. If 3.25 lbs. of tobacco cost $.975, what quantity can you purchase for $5.46? Ans. 18.2 lbs.

LESSON 148.

1. A. can reap a field in 6 days, and B. in 10 days; how many days will it take both of them to reap it? Ans. $3\frac{3}{4}$ d.

Explanation. What part of it can A. reap in 1 day? what part of it can B. reap in 1 day? What part of it then can A. and B. both reap in 1 day? How long will it take them both to reap it then?

2. A cistern has 4 pipes; the first will empty it in 12 minutes, the second in 15 minutes, the third in 30 minutes, and the fourth in 45 minutes; in what time will they all empty it? Ans. 4 min. $51\frac{33}{37}$ sec.

3. How long will you be in overtaking a horseman, who has been gone 5 hours, and who rides 8 miles an hour, if you ride 10 miles an hour? Ans. 20 h.

4. If a man who owes $3,000, drawing 6 per cent. interest, receives $4.50 a day, and spends $1.45 a day, how long before he will be worth $1,000?
Ans. 4 yrs. 104 d., about.

5. The hour and minute hands of a watch are together at 12 o'clock; when are they together again?
Ans. in 1 h. 5 min. $27\frac{3}{11}$ sec

Explanation. Observe that the minute hand travels 60 minutes in an hour, and the hour hand 5 minutes in an hour.

6. A. can mow a piece of ground in 11 days; with the assistance of B. he can do it in 7 days; in what time can B. do it alone? Ans. in $19\frac{1}{4}$ d.

7. 2 men, A. and B., being 50 miles apart, commenced travelling from each other; A. proceeds at the rate of 2.75 miles an hour, and B. at the rate of 3.5 miles an hour; how long before they will be 110 miles apart?
Ans. in 9 h. 36 min.

8. A. lives in New York, and B. in Philadelphia; A. *can travel $\frac{1}{5}$ of the* distance between the 2 cities in 4 hours,

and B. can travel $\frac{1}{6}$ of the distance in 5 hours; if they both start to travel towards each other, at the same moment, how long before they will meet? Ans. in 12 h.

9. If 1 T. 2 cwt. 1 qr. of hay will last 3 horses 4 days, what quantity will be necessary to last 7 horses the same length of time? Ans. 2 T. 11 cwt. $3\frac{2}{3}$ qrs.

10. A man owns 4 farms; the first contains 200 A., and the second 250 A.; the other 2 farms bear the same proportion to each other as to size, but the fourth contains only 31 A.; how much land is there in the third farm?

Ans. 24 A. 3 qrs. 8 sq. rods.

RULE OF THREE COMPOUND,

CALLED ALSO

DOUBLE RULE OF THREE, AND COMPOUND PROPORTION.

LESSON 149.

Questions in the Rule of Three Compound, contain two questions in the Rule of Three, and the learner must exercise his judgment in decomposing them, and in obtaining several partial answers from which to deduce the true answer.

1. If it takes 2 years for the interest of $150 to amount to $18, how long will it take for the interest of $675 to amount to $162? Ans. 4 yrs.

Explanation. How long will it take for the interest of $1 to amount to $18? How long will it take for the interest of $1 to amount to $1? How long will it take for the interest of $675 to amount to $1? How long then will it take for the interest of $675 to amount to $162?

Note. There are many ways to decompose a question, and obtain the answer by methods similar to the preceding. Recollect that whenever we have to divide one number by another, we can write the divisor under the dividend, and form a fraction, which can be used as the quotient. When this course renders an operation easier, it should be adopted.

What is said of questions in the Rule of Three Compound?

What ways are there to decompose a question in the Rule of Three Compound, and obtain the answer? What is to be recollected? When should this course be adopted?

2. The interest on \$200 for 4 months being \$4, what will be the interest on \$590 for 1 year and 3 months, that is, for 15 months ? Ans. \$44.25.

3. If 76 bushels of oats last 19 horses 16 days, how long will 12 bushels last 3 horses ? Ans. 16 days.

4. A trader paid \$3 for the transportation of 14 cwt. 2 qrs. 17 lbs. 42 miles ; how much ought he to pay for the transportation of 6 T. 26 miles, at the same rate ?

Ans. \$15.21.

5. 4 men built a wall 33 feet long, 14 feet high, and 5 feet thick, in 13 days ; how long then should it take 7 men to build a wall 40 feet long, $2\frac{1}{2}$ feet thick, and 7 feet high ?

Ans. $2\frac{58}{231}$ d. or $2\frac{1}{4}$ d., about.

6. If 3 lbs. of rice cost 24 cents, and 12 lbs. of rice are worth as much as 5 lbs. of coffee, what are 78 lbs. of coffee worth ? Ans. \$14.98.

Lesson 150.

1. By working 8 hours in a day, 7 men hoed 21 acres in 6 days ; how long then will it take 4 men, working 11 hours in a day, to hoe $16\frac{1}{2}$ acres ? Ans. 6 days.

2. 12 men put $\frac{2}{5}$ of a vessel's cargo on board in $\frac{3}{4}$ of a day ; how long then will it take 5 men to load 3 such vessels ? Ans. $13\frac{1}{2}$ d.

3. If 15 bushels of wheat last 12 men 75 days, how long will 45 bushels last 30 men ? Ans. 90 d.

4. A garrison of 400 men had a supply of provisions for 32 weeks, each individual receiving 32 ounces a day ; at the end of 20 weeks, 50 of the men were killed in an assault ; what quantity of food can each of the remaining men receive a day, so that the provisions may last 30 weeks longer ? Ans. $14\frac{22}{35}$ oz.

5. If $\frac{5}{8}$ of a yard of cloth, $\frac{3}{4}$ of a yard wide, cost $\$2\frac{1}{4}$, what will $3\frac{1}{2}$ yards of cloth of the same quality cost, that is $1\frac{3}{4}$ yard wide ? Ans. \$29.40.

6. A man gave \$15.50 for .875 of a load of hay ; what should he pay for .8 of another load of the same quality, estimated to contain .75 as much hay as the first ?

Ans. \$10.63.

7. If 3 men are hired 6 days for \$21, how many men can be hired 15 days, at the same rate, for \$157.50 ?

Ans. 9

CHAIN RULE,

CALLED ALSO CONJOINED PROPORTION.

LESSON 151.

Questions in the Chain Rule are a continuous chain of questions in the Rule of Three.

1. If 2 lbs. of butter are worth as much as 3 lbs. of cheese, and 8 lbs. of cheese are worth as much as 4 lbs. of tea, how many lbs. of butter can be obtained for 6 lbs. of tea?

Operation and Explanation. It is plain that 1 lb. of cheese is worth $\frac{2}{3}$ of a lb. of butter, and that 1 lb. of tea is worth $\frac{8}{4}$ of a lb. of cheese; 1 lb. of tea, then, is worth $\frac{8}{4}$ of $\frac{2}{3}$ of a lb. of butter, or $\frac{16}{12}$ of a lb. of butter, and 6 lbs. of tea are worth 6 times $\frac{16}{12}$ of a lb. of butter, or $\frac{96}{12}$ of a lb. of butter, or 8 lbs. of butter.

2. If 2 lbs. of butter are worth as much as 3 lbs. of cheese, and 8 lbs. of cheese are worth as much as 4 lbs. of tea, how many pounds of tea can be purchased for 12 lbs. of butter?

Operation and Explanation. It is plain that 1 lb. of butter is worth $\frac{3}{2}$ of a lb. of cheese, and that 1 lb. of cheese is worth $\frac{4}{8}$ of a lb. of tea; 1 lb. of butter, then, is worth $\frac{3}{2}$ of $\frac{4}{8}$ of a lb. of tea, or $\frac{12}{16}$ of a lb. of tea, and 12 lbs. of butter are worth 12 times $\frac{12}{16}$ of a lb. of tea, or $\frac{144}{16}$ of a lb. of tea, or 9 lbs. of tea.

From the preceding, we obtain the following rule.

To find what quantity of the first thing mentioned is equal in value to a certain quantity of the last,

Multiply together the first number in the question, and every alternate one, and divide this product by the product of the other numbers.

To find what quantity of the last thing mentioned is equal in value to a certain quantity of the first,

Multiply together the first number in the question, and

What are questions in the Chain Rule?
State how example 1, lesson 151, is performed.
State how example 2, lesson 151, is performed.
What is the rule we obtain?

every alternate one, except the last, and divide by this product the product of the other numbers.

3. 6 Massachusetts shillings are equal to 8 New York shillings, and 16 New York shillings are equal to 15 Pennsylvania shillings, and 30 Pennsylvania shillings are equal to 20 Canada shillings; now how many Massachusetts shillings are equal to 40 Canada shillings? Ans. 48.

4. If $31 in the United States are equal to 25 milrees in Portugal, and 10 milrees in Portugal are equal to 31 florins in Amsterdam, how many florins in Amsterdam are equal to $16 in the United States? Ans. 40.

5. If 25 boys can do as much as 12 men, and 10 men can do as much as 15 women, how many women will be necessary to do the work of 50 boys? Ans. 36.

Note. The rules in this lesson are not indispensable; the learner should now perform the examples without employing them.

BARTER,

OR THE EXCHANGE OF COMMODITIES.

LESSON 152.

To be performed in the mind.

1. A farmer bought 1 tierce of molasses, at $.40 a gallon, and agreed to pay in corn at $.80 a bushel; what quantity will discharge the debt?

2. How much wood at $8 a cord can I get for 2 tons of hay at $18 a ton?

3. A farmer in Pennsylvania sold a quarter of veal weighing 18 lbs. at 9 cents a lb., and received in payment 10 lbs. of rice at 5 cents a lb., a levy's worth of raisins, and the rest in cash, how much cash did he get?

4. If a trader in Portland sells you a straw bonnet for 15 shillings, and receives in payment 3 pairs of stockings at 2 shillings a pair, 2 pairs of mittens at 15 cents a pair, and the balance in money, how much money do you pay him?

5. How much butter at 1 shilling a pound, must I give a trader in Troy, New York, for a quantity of shad worth $2.50.

For the Slate.

6. What quantity of cod fish, at $3 a quintal, must be given for 19 cords 7 ft. of wood, at $6 a cord ? Ans. 39 quintals 84 lbs.

7. How many bushels of potatoes, at $.41⅔ a bushel, must be given in payment for 12 yards of broadcloth, at $4.31¼ a yard ? Ans. 124.2 bu.

8. What quantity of cheese, at $.12½ a pound, must be given in exchange for 1 hogshead of St. Ubes salt, valued at $4.75 ? Ans. 38 lbs.

9. A farmer exchanging some eggs for a quantity of coffee, found that the grocer demanded 36 cents a pound for coffee that was worth only 30 cents, in cash ; what price ought he to ask for his eggs, which were worth 15 cents a dozen, in cash ? Ans. 18 cts. a doz.

10. If you sell 80 pounds of wool at 42 cents a pound, and in payment receive 1 barrel of mess beef for $15, and the balance in sugar at 8 cents a pound, what quantity of sugar do you get ? Ans. 232½ lbs.

11. A. and B. barter ; A. has wood worth $5 a cord, and B. flour worth $6 a barrel, but for which he expects $7 a barrel in a barter trade ; for what price should B. barter his wood, and how much wood should he give for 12 barrels of flour ?

Ans. B. should barter his wood at $5.83⅓ a C. and give 14 C. 3.2 ft. for 12 barrels of flour.

12. A country trader agreed to exchange 553 lbs. of butter, worth 20 cents a lb., for 1 hhd. of white sugar, containing 7 cwt. 3 qrs. 16.8 lbs. ; how much a lb. was the sugar valued at ? Ans. 12½ cts.

ASSESSMENT OF TAXES.

LESSON 153.

DEFINITIONS. *Real Estate* is composed of houses, lands, and other immovable property.

Personal Estate is composed of money, cattle, horses, and other movable property.

Poll tax is a tax of so much on every able bodied man.

1. In the year 1835, the state tax of a certain town was

What is real estate composed of? Personal estate? *What is a poll tax ?*

$2,368.07 ; the county tax was $939.81, and the town voted to raise $2,500 to defray town charges. The value of the real estate in town, together with that of all the personal estate, was $354,000, and there were 663 polls, which are to pay $\frac{1}{6}$ part of the whole tax. What was A.'s tax, whose real estate was valued at $4,373, personal estate at $813, and who was taxed for 2 polls ? Ans. $73.82.

Explanation. What was the whole tax to be assessed on the polls and estates of the town ? What sum did the tax on the polls, and on each of the polls, amount to ? What was the remaining tax to be assessed on the estates ? As $354,000 paid the remaining tax, what did $1 pay ? What did the sum of A.'s real and personal estate pay ? What was his whole tax then, including the tax for 2 polls ?

2. What was B.'s tax in the same town, the value of whose real estate amounted to $1,000, personal estate to $1,175, and who was taxed for 3 polls ? Ans. $34.12.

Note. It frequently happens that a person is taxed for more property than belongs to him, in which case the assessors reduce his tax; moreover, many persons are unable to pay their taxes. The consequence is, if the exact amount of taxes to be raised in a town be assessed on the polls and estates, the sum actually paid will be less than the tax required. To obviate this difficulty, the assessors usually add 2 or 3 per cent. of a tax to the tax, and consider the amount as the sum to be assessed; by which means the sum of all the taxes paid, will generally exceed the taxes required.

3. The whole amount of the taxes to be paid in a certain town, in the year 1830, was $1,286.40 to which the assessors added 2 per cent. ; the value of the real estate, together with that of all the personal estate, was $117,273, and there were 211 polls, the tax on each of which was fixed at $1.20. What was G.'s tax, who possessed real estate to the amount of $1,583, personal estate to the amount of $275, and who was taxed for 1 poll ? Ans. $17.98.

Assessors usually put the tax paid by various sums, in a table, by means of which any person's tax can be determined with great ease. On the next page is such a table, made from the last example as follows,

$$\begin{array}{rl} \$1{,}286.40 & \text{tax.} \\ .02 & \\ \hline 25.7280 & \text{2 per cent.} \end{array}$$

What frequently happens? What is the consequence? How is this difficulty *obviated?*

What do assessors usually do? Explain how a table is made from *the last example.*

To tax $1,286.40
add 2 per cent. 25.728

$1,312.13 amount to be assessed.
253.20 tax paid by the polls.

$1,058.93 tax to be assessed on the real and personal estates.

211 polls.
$1.20 tax on each poll.

422
211

$253.20 tax paid by the polls.

$
117273) 1058.930 (.0090296 tax on $1, about.
1055 457

347300
234546

1127540
1055457

720830
703638

TABLE.

The tax paid by

$	$	$	$	$	$
1	is .009.	by 20	is .181.	by 200	is 1.806.
2	“ .018.	30	“ .271.	300	“ 2.709.
3	“ .027.	40	“ .361.	400	“ 3.612.
4	“ .036.	50	“ .451.	500	“ 4.515.
5	“ .045.	60	“ .542.	600	“ 5.418.
6	“ .054.	70	“ .632.	700	“ 6.321.
7	“ .063.	80	“ .722.	800	“ 7.224.
8	“ .072.	90	“ .813.	900	“ 8.127.
9	“ .081.	100	“ .903.	1,000	“ 9.03.
10	“ .09.				

G.'s tax is found from the table as follows;
$1,583 G.'s real estate.
275 G.'s personal estate.

$1,858 whole estate.

By the table, the tax on $1,000, is $9.03.
" " " on 800, is 7.224.
" " " on 50, is .451.
" " " on 8, is .072.

16.78
1.20 poll tax.

$17.98 G.'s tax. Ans.

Explanation. The manner of forming this table is very plain; for having obtained the tax on $1, we multiply it by each of the other sums to get the tax which they pay. The table can be enlarged by carrying it up to $10,000, and down to 1 cent, if we choose.

4. In example 3, add 3 per cent. to the amount of the taxes, and then find the tax to be paid by G.; supposing each poll to pay $1.20 as before. Ans. $18.18.

FELLOWSHIP,

OR COMPANY BUSINESS.

LESSON 154.

When two or more persons associate together for the purpose of trade, they are said to *enter into partnership*, and are called a *company* or *firm*. The amount of property that each partner puts into the firm, is called his *capital*, or *stock in trade*, and his share of the gain is called his *dividend*.

SIMPLE FELLOWSHIP.

In Simple Fellowship, the capitals of the different partners are employed during equal times.

What is the manner of forming this table? Can the table be enlarged, and how?

When two or more persons associate together for the purpose of trade, what are they said to do? What are they called? What is the capital, or stock in trade of each partner? His dividend?

How are the capitals of the different partners employed in Simple *Fellowship?*

1. A. and B. entered into partnership for the purpose of trade ; A. put in $1,000, and B. $1,250 ; after trading 1 year, they found a gain had been made of $945 ; what was each one's dividend ? Ans. A.'s dividend $420, B.'s $525.

Explanation. What was the sum put in by both ? As his sum gained $945, what did $1 gain ? What did $1,000 gain ? 1,250 gain ?

To prove Simple Fellowship,

Add together all the shares in the gain or loss, and the sum will evidently be equal to the whole gain or loss, if the work be done right.

2. James Wallace, William Clark, and Samuel Shaw, formed a connection in business under the firm of James Wallace and Co. The capital put in by Wallace was $2,500, by Clark $2,200, and by Shaw $12,000. On settling their business, at the end of 18 months, they found that the profits amounted to $6,500 ; what was each one's share of this sum ?

Ans. Wallace's share $973.05, Clark's $856.29, Shaw's $4,670.66.

3. A. and B. formed a company for trade ; A. put in $950, and B. $800 ; after trading a short time they found a loss had been made of $300; what was each one's share of the loss ? Ans. A.'s share $162.86, B.'s $137.14. How much of his capital did A. save ? How much of his capital did B. save ?

4. A. and B. bought a quantity of land on speculation, B. paying $\frac{5}{9}$ as much as A. They cleared $1,540 by the speculation ; what was each one's dividend of the gain ?

Ans. A.'s $990, B.'s $550.

5. A gentleman who had 1 son and 2 daughters, left by his will $5,000 to the son, $3,000 to the eldest, and $2,500 to the youngest daughter. At his death it was found that the property remaining, after paying his debts, was $14,950; what part of this sum should each of the children take ?

Ans. the son $7,119.05, the eldest daughter $4,271.43, and the youngest daughter $3,559.52.

6. The yearly profits of a cotton factory, valued at $17,000, and owned in 80 shares, amounted to $2,825.16, $275.16 of which it was thought prudent to keep back, to

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To prove Simple Fellowship how do we proceed ?

meet contingent expenses ; the remainder being divided among the owners, what was the amount of the dividend paid to the owner of 2 shares ? Ans. $63.75.

7. If the factory, instead of yielding an income, had cost the proprietors $1,000, what part of this sum would the owner of 2 shares have paid ? Ans. $25.

8. 4 merchants, A. B. C. and D., bought a ship for $14,000, of which A. paid $5,000, B. $2,000, C. $3,000, and D. $4.000. During her first voyage she earned $5,600 ; what was each one's share of the gain ?
Ans. A.'s share $2,000, B.'s $800, C.'s $1,200, D.'s $1,600.

COMPOUND FELLOWSHIP.

Lesson 155.

In Compound Fellowship the capitals of the different partners are employed during unequal times.

1. A. and B. traded in company ; A. had a capital of $400, which was employed 6 months, and B. a capital of $450, which was employed 8 months. They gained $120; what was the share of each ?
Ans. A.'s share $48, B.'s $72.

Explanation. How many dollars should A. have employed 1 month, to be equal to the use of $400, 6 months ? How many dollars should B. have employed 1 month, to be equal to the use of $450, 8 months ? The question now is evidently the same as if we were required to find the share of A. and B. in a gain of $120, if A. had put in $2,400, and B. $3,600, and had continued these sums in trade 1 month.

Therefore, to perform an example in Compound Fellowship,

Consider the product of each partner's capital by the time it was continued in trade, as constituting his capital, and then proceed as in Simple Fellowship.

How are the capitals of the different partners employed in Compound Fellowship ?
How do we proceed to obtain the answer to example 1, lesson 155 ?
How then do we perform an example in Compound Fellowship ?

To prove Compound Fellowship,

Proceed as in Simple Fellowship.

2. Charles Jones, Henry Adams, and John Stevens formed a company, under the firm of Jones, Adams, and Co., and commenced trade the first of June, on $2,000 put in by Jones; the first of August, Adams put in $3,000, and the first of September, Stevens put in $4,000. At the end of the year their gains amounted to $1,500; what was each partner's share?

Ans. Jones's share $466.67, Adams's $500, Stevens's $533.33.

3. A. and B. formed a partnership the first of January, and put in $3,000 apiece. The first of April, A. put in $1,000 more, and the first of September, B. put in $500 more. At the end of the year they found the whole of their gain to be $2,000; what was each one's dividend?

Ans. A.'s dividend $1,084.34, B.'s $915.66.

4. A. and B. hired a pasture for $36; A. put in 6 cows for .5 of a year, and B. 4 cows for .25 of a year; what should each one pay? Ans. A. $27, B. $9.

5. A. B. and C. entered into partnership. A. kept his capital in 1 year. B. put in ¾ as much as A., and employed it 9 months. C. put in ⅘ as much as B., and employed it 6 months. They gained $3,000; what was each one's share of the profit?

Ans. A.'s share $1,610.74, B.'s $906.04, C.'s $483.22.

Explanation. Consider that A. put in $1.

6. A. and B. traded in company. A.'s capital was $5,000 at the commencement, but at the end of 4 months he took out $3,000, and kept the remainder in the company 6 months longer; B.'s capital was $3,000 at the commencement, but at the end of 5 months he put in $4,000 more, and continued the whole in the company 3 months longer. They found, on settlement, that a loss had been sustained of $1,800; what was each one's share of it?

Ans. A.'s share $847.06, B.'s $952.94.

To prove Compound Fellowship how do we proceed?

INSURANCE.

LESSON 156.

Insurance is an agreement to pay the damages which vessels, goods, houses, &c. may sustain from certain accidents, like the perils of the sea, fire, &c.

The written promise to pay such damages is called a *policy of insurance.*

The compensation paid to obtain insurance is called the *premium.* It is usually a certain per cent. of the sum insured.

The insurer is often called the *underwriter.*

When there is a loss on property, the whole of which is insured, the owner is reimbursed. When a part of the property only is insured, and the whole is lost, the owner is paid the sum insured; but if a certain part only is lost, the owner is paid the same part of the sum insured.

If a person desires to be insured so as to lose nothing in case of a total destruction of the property, not even the premium and other costs of insurance, he has the premium and other costs of insurance added to the value of the property, and insures the whole amount. Property so insured is said to be *covered.*

Policies are usually so made that the insurer is not liable for any losses under *five* per cent.

When property is greatly damaged, the insured can, if he pleases, abandon it as a total loss, in which case the underwriter must pay the sum insured, and take what is saved; for instance, if goods be damaged more than half their value by the perils insured against, or if a ship be damaged by such perils so as to require repairs exceeding half its original value, after deducting $\frac{1}{3}$ from the cost of

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What is insurance?

What is called a policy of insurance?

What is called the premium? What is it usually?

What is the insurer often called?

What if there be a loss on property, the whole of which is insured? What if a part of the property only be insured, and the whole be lost? What if a certain part only be lost?

What if a person desires to be insured so as to lose nothing, in case of a total destruction of the property, not even the premium and other costs of insurance? What is said of property so insured?

What is said of losses under 5 per cent.?

What if property be greatly damaged? Give some instances.

such repairs for the superiority of the new work over the old, according to custom. However, when property is abandoned in this way, the underwriter can make repairs, if he pleases, and deliver it to the insured, and if it is a vessel he can oblige him to pay ⅓ of the cost of the repairs, for the superiority of the new work over the old.

LESSON 157.

1. A man had his house and furniture insured against loss or damage from fire during one year, at ⅔ of 1 per cent., what did the premium amount to, if the property was valued at $3,800 ? Ans. $25.33⅓.

2. A merchant had $5,000 insured on his brig, valued at $8,000, during her voyage from New York to Havana. The premium was 2 per cent., for which he gave his note ; what must he receive from the underwriters if the vessel was damaged to the amount of $2,000, after deducting the amount of his note ? Ans. $1,150.

3. If I have $5,000 insured on my store for one year, and obtain the policy for $37.50, what per cent. premium do I pay ? Ans. .75 per cent.

4. What sum must be paid for insuring a ship worth $9,000 from Portland to Matanzas, at 3 per cent., from Matanzas to Canton, at 4 per cent., and from Canton to Portland, at 6 per cent. ? Ans. $1,170.

5. If I have a vessel valued at $2,112, bound to Lisbon from Baltimore, what sum must I get insured to cover it, and what will be the premium at 2½ per cent., the commission to agent for making insurance being ½ per cent., and the commission for recovering loss, if any be sustained, 1 per cent. ?

OPERATION.

Premium, ... 2½ per cent.
Commission for making insurance, ... ½ " "
" " recovering loss, ... 1 ".
—
4 " "

100
4
—
$2,112 then is 96 per cent. of the sum to be insured.

What can the underwriter do when property is abandoned in this way?

$
.96) 2112.00 (2200 sum to be insured to cover $2,112.
192 .02½ See Percentage, lesson 121.

192 44
192 11

$55.000 premium, 2½ per cent. of $2,200.

PROOF.

From sum to be insured;		$2,200
Deduct 2½ per cent., premium,	$55	
" ½ per cent., commis. for making ins.	11	
" 1 per cent., commis. for recov. loss,	22	
		88
Amount covered,		$2,112

Therefore, to find the sum to be insured to cover a certain amount,

Find what per cent. the costs of insurance are, subtract this per cent. from 100 *per cent., and divide the amount to be covered by the remainder.*

Then, to find the amount of the premium,

Multiply the sum to be insured by the premium per cent.

And to prove the work,

Deduct the costs of insurance from the sum to be insured, and the remainder will be the amount covered.

6. If you insure a ship worth $8,000, at 5 per cent., so as to cover her value during her passage from Manilla to Salem, what amount of premium must you pay?
Ans. $421.05.

7. What is the premium for insuring a quantity of goods worth $10,000, at 4 per cent.; and what is the sum covered by such an insurance?
Ans. the premium is $400, and the sum covered $9,600.

8. What amount of premium, at 3 per cent., must be paid to cover $8,000 worth of goods during their transportation from Boston to Tampico, the commission for making

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Explain how example 5, lesson 157, is performed.
How do we find the sum to be insured to cover a certain amount?
How then do we find the amount of the premium?
How do we prove the work?

insurance being ¼ per cent., and the commission for recovering loss ½ per cent. ? Ans. $249.35.

9. If $88.10 be the amount of the premium for insuring a quantity of goods during their transportation from New York to Bangor, at 1 per cent., what is the sum insured, and what is the sum covered ?

Ans. the sum insured is $8,810, and the sum covered $8,721.90.

GENERAL AVERAGE.

Lesson 158.

When a vessel is in danger, it is sometimes necessary to cut away the masts, spars, rigging, &c., or to throw overboard a part of the cargo, in order to preserve the remaining property. When such a sacrifice is made, each of the owners of the ship and cargo suffers a portion of the loss, according to his part of the property, whether sacrificed or not.

When a vessel is stranded, or meets with a like disaster through accident, and by extraordinary expense is got off, and enabled to pursue her voyage, the expense is apportioned on the owners of the ship and cargo.

When a vessel is obliged, from the occurrence of extraordinary accidents, to enter a port to repair, the provisions and wages of the seamen during her stay, and indeed all the expenses attending the delay, except the cost of repairs, are likewise borne by apportionment, or general average.

The damage a vessel or cargo may suffer from *accidents*, is borne by the owner of the property injured, or by the underwriter, if insured. Before making a general average, the amount of such damage must be deducted

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What is sometimes necessary when a vessel is in danger? When such a sacrifice is made, who suffers the loss?

What is said of a vessel's being stranded, and got off?

When a vessel is obliged, from the occurrence of extraordinary accidents, to enter a port to repair, what is said of the expenses attending the delay?

By whom is the damage a vessel or cargo may suffer from accidents *borne? What is* done before making a general average?

from the value of the injured property; the costs of insurance must also be deducted from the value of the property insured; likewise all charges that lessen the value of any property must be deducted from it. ⅔ of the money paid for freight must be added to the value of the ship; it being supposed that ⅓ of the amount of the freight is expended in wages and provisions for the seamen. In New York, only ½ of the freight is added to the value of the ship.

When the masts, spars, rigging, &c. of a vessel are destroyed for the general good, the damage is reckoned ⅔ as much as the cost of repairing; the new articles being supposed ⅓ better than the old ones.

The underwriter is liable for the expense contributed to a general average by insured property, even if the sum so contributed be less than five per cent. of the amount insured, whatever the policy may say.

1. A brig worth $9,016, covered by insurance at 2 per cent., and belonging to A. of Boston, was loaded with cloths and other goods for B., worth $15,000, insured at 3 per cent., and with hardware for C., worth $5,000, not insured. The amount of the freight was $1,482. On the voyage the brig encountered a heavy gale, and the master was obliged to throw overboard ⅘ of the goods belonging to C., and to cut away spars and rigging which it cost $158.25 to repair. What is each owner's portion of the loss; how much must C. receive, and how much must A. pay out?

OPERATION.

Ship,	$9,016
Deduct premium for insurance, 2 per cent.,	184
	$8,832
Add ⅔ of freight,	988
Ship, and freight,	$9,820

When the masts, spars, rigging, &c. of a vessel are destroyed for the general good, how much is the damage reckoned?
What is said of the liability of the underwriter in a general average?
Explain how example 1, lesson 158, is performed.

B.'s goods, ...	$15,000	9,820
Deduct premium for insurance, 3 per cent., ...	450	
	14,550	14,550
C.'s goods, ...		5,000
Value of property to bear the loss, ...		$29,370

LOSS.

$\frac{4}{5}$ of C.'s goods, ...	$4,000
Damage of ship $\frac{2}{3}$ of $158.25, ...	105.50
Freight lost on goods thrown overboard,	300
Whole loss, ...	4,405.50

```
                $
29370 ) 4405.50 ( .15 loss borne by each dollar of the
        2937 0              property.
        -------
        1468 50
        1468 50
        -------
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$9,820	multiplied by	$.15	gives	$1,473.00	A.'s loss.
14,550	multiplied by	.15	gives	2,182.50	B.'s loss.
5,000	multiplied by	.15	gives	750.00	C.'s loss.
				4,405.50	whole loss.

$4,000	value of C.'s goods sacrificed.
750	deduct C.'s share of the loss.
3,250	sum that C. must receive.
1,473	A.'s share of the loss.
405.50	deduct A.'s property lost and sacrificed.
1,067.50	sum that A. must pay out.

Note. The insurers must ultimately pay what the brig contributes to the general average of the loss, and also B.'s share of the loss. As for C., not being insured, he must suffer his share of the loss, or $750.

LESSON 159.

1. Ship Triton, belonging to L. Murdock, went ashore 9 miles from New York, during a storm, but by great exer-

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Who must ultimately pay what the brig contributes to the general average? B.'s share of the loss? C.'s share of the loss?

tions, after throwing over part of her cargo, she was got off, and towed up to the city by a steamboat. The following losses and expense are to be apportioned among the owners.

Loss of cable and anchor, with some spars and rigging ; ⅔ of the cost of new articles,	$425
Goods of John Williams, thrown overboard,	2420
Freight lost on the above goods,	121
Expense of steamboat 3 hours, at $70 an hour,	210
Protest,	16
Adjusting average,	45
Loss and expense to be apportioned,	3,237

Property to pay the preceding, the premium for insurance having been deducted ;

Ship,	$15,000	
½ of freight,	1,000	
	16,000	ship and freight.
Goods of John Williams, including those lost,	8,250	
Goods of Daniel Drake,	15,000	
Goods of T. Jones,	11,000	
Goods of S. Hyde,	10,750	
Total,	$61,000	

What will be each owner's share of the loss and expense ?

Ans. L. Murdock's $849.05, John Williams's $437.79, Daniel Drake's $795.98, T. Jones's $583.72, S. Hyde's $570.46.

2. Barque Berosus, from New Orleans to Philadelphia, met with much bad weather, sprung her foremast, had several sails carried away, and was otherwise so much damaged as to render it prudent to put into Norfolk to repair. All the expenses attending the delay, including pilotage, dockage, protest, commission, provision and wages of the seamen, were $345. The vessel was valued, in New Orleans, at $10,000, of which the sum of $7,000 was insured at 3 per cent., and had received damage to the amount of $1,140. The freight amounted to $1,050. All of the cargo was insured, at 2½ per cent., and consisted

of goods for A., valued at $9,000 in New Orleans, but which had been damaged to the amount of $300; goods for B., worth $5,000; goods for C., valued in New Orleans at $12,800, but which had been damaged to the amount of $587.50, and goods for D., worth $8,750. How much of the expense must the owner of the barque, and each owner of the cargo pay?

Ans. the owner of the barque must pay $74.80, A. $67.80, B. $39, C. $95.15, D. $68.25.

ALLIGATION,

OR MIXTURE.

LESSON 160.

Alligation is mostly used in mixing the precious metals.

Jewellers divide an ounce of gold into 24 parts, called *carats*. They express the quality, fineness, or purity of gold in carats. Thus a piece of gold 24 carats fine, is pure, or fine gold; a piece 22 carats fine, is of such a quality that an ounce of it contains 22 carats of pure or fine gold, and 2 carats of some base metal, like copper, called *alloy*. To express the quality, fineness, or purity of silver, jewellers name the quantity of pure or fine silver contained in 12 ounces of any kind. Thus, a piece of silver 12 ounces fine, is pure or fine, silver; a piece 11 oz. 3 pwts. 17 grs. fine, is of such a quality that 12 ounces of it contains 11 oz. 3 pwts. 17 grs. of fine silver, and the rest alloy. Alloy is considered of no value.

ALLIGATION MEDIAL.

A method of finding the average price or quality of several ingredients mixed together.

What is Alligation mostly used in?

How do jewellers divide an ounce of gold? What do they express in carats? Give some examples. How do jewellers express the quality, fineness, or purity of silver? Give some examples. How is alloy considered?

What is Alligation Medial?

1. A farmer mixed 4 bushels of wheat worth $.80 a bushel, with 6 bushels worth $1 a bushel, 8 bushels worth $1.30 a bushel, and 10 bushels worth $1.40 a bushel; what was the value of a bushel of the mixture? Ans. $1.20.

Explanation. What was the value of all the wheat mixed? How many bushels were there? Then what was 1 bushel of the mixture worth?

2. A grocer mixed 200 pounds of sugar worth 6 cents a pound, with 400 pounds worth 8 cents a pound, and 500 pounds worth 10 cents a pound; what were 100 pounds of the mixture worth? Ans. $8.55.

3. What was 1 pound of the mixture worth?
Ans. 8.55 cts. or $8\frac{1}{2}$ cts., about.

4. A jeweller melted together 9 ounces of gold 18 carats fine, 10 ounces 19 carats fine, and 7 ounces of pure gold, which is 24 carats fine; how many carats fine was the mixture? Ans. 20.

5. If 11 oz. of gold 18 carats fine, be melted with 2 lbs. 10 oz. 20 carats fine, 8 oz. of pure gold, and 3 oz. of copper alloy, how many carats fine will the mixture be?
Ans. $19\frac{3}{28}$.

6. A grocer bought 60 gallons of rum at $1 a gallon, mixed 15 gallons of water with it, and sold the mixture so as to gain 15 per cent.; at what price did he sell a gallon?
Ans. 92 cts.

7. A silversmith melted together 10 oz. of silver containing $\frac{1}{10}$ alloy, 9 oz. containing $\frac{1}{12}$ alloy, and 5 oz. containing $\frac{1}{20}$ alloy; what proportion of the mixture was alloy?
Ans. $\frac{1}{12}$.

8. How would a jeweller express the fineness of the mixture?

9. If 5 lbs. 3 oz. of silver 10 oz. 1 pwt. fine, be melted with 3 lbs. of pure silver, and 9 oz. of copper, of what fineness will the mixture be? Ans. 9 oz. 17 pwts. 6 grs., fine.

ALLIGATION ALTERNATE.

A method of finding how much of each ingredient must be taken to form a mixture of a certain price or quality.

What is Alligation Alternate?

Lesson 161.

1. A grocer has some sugar worth 7 cents a pound, and some worth 12 cents; in what proportion must the two kinds be mixed so that the compound may be worth 10 cents a pound?

OPERATION.

10	12
7	10
—	—
3	2

3 lbs. at 12 cts., and 2 lbs. at 7 cts. Ans.

Explanation. We find that the value of 1 lb. of the mixture exceeds the value of 1 lb. of the cheapest sugar by 3 cents, and that the value of 1 lb. of the dearest sugar exceeds the value of 1 lb. of the mixture by 2 cents. Now if we multiply the 3 cents by 2, and the 2 cents by 3, the products will be alike, and each will be 6. The value of 2 lbs. of the cheapest sugar, then, is 6 *cents less* than the value of 2 lbs. of the mixture, and the value of 3 lbs. of the dearest sugar is 6 *cents more* than the value of 3 lbs. of the mixture; so if we add 2 lbs. of the sugar worth 7 cents a lb., to 3 lbs. of that worth 12 cents, we get 5 lbs. of the same value a pound as the mixture required.

Therefore, to obtain the proportion in which two ingredients of given values are to be mixed to produce a compound of a certain price or quality,

Take the difference between the values of the cheapest ingredient and the compound to express the proportion of the dearest *ingredient, and the difference between the values of the dearest ingredient and compound to express the proportion of the* cheapest *ingredient.*

2. A grocer had some tea worth 45 cents a pound, and some worth 62 cents a pound; in what proportion must they be mixed so that the compound may be worth 50 cts. a pound? Ans. 12 at 45 cts., and 5 at 62 cts.

Note. The numbers expressing the proportion may be considered as meaning pounds, ounces, or any other denomination; also both numbers may be multiplied, or both divided by the same number, and the two products, or the two quotients, will evidently bear the same proportion to each other as the numbers do.

3. A watchmaker has some gold 20 carats fine, and some

Explain how example 1, lesson 161, is performed.

How do we obtain the proportion in which two ingredients of given values are to be mixed to produce a compound of a certain price or quality?

What is observed of the numbers expressing the proportion?

17 carats fine ; how many ounces of each kind must be melted so that the mixture may be 19 carats fine ?

Ans. 2 oz. 20 carats fine, and 1 oz. 17 carats fine.

4. A silversmith has some silver $\frac{1}{8}$ alloy, and some $\frac{1}{15}$ alloy ; in what proportion must he mix the two kinds to produce a compound $\frac{1}{12}$ alloy ?

Ans. in the proportion of $\frac{1}{60}$ to $\frac{1}{24}$, or of 2 to 5.

5. A grocer has some rum worth $2 a gallon ; in what proportion must this rum, and water worth $0 a gallon, be mixed, to produce a liquor worth $$1\frac{1}{2}$ a gallon ?

Ans. $1\frac{1}{2}$ of rum to $\frac{1}{2}$ of water, or 3 of rum to 1 of water.

6. If you have some gold 23 carats fine, in what proportion must it be melted with alloy so that the mixture may be 20 carats fine ? Ans. 20 of gold to 3 of alloy.

7. If you have some silver 5 per cent. copper, and some 23 per cent. copper, how many ounces of each must be melted together so that the mixture may be 10 per cent. copper ?

Ans. 13 oz. of 5 per cent. copper, and 5 oz. of 23 per cent. copper.

LESSON 162.

1. A miller has several kinds of corn worth 60, 70, 80, 110, and 120 cents a bushel ; in what proportions must these kinds be mixed so that the compound may be worth 100 cents a bushel ?

OPERATION.

$$\begin{array}{rr} 100 & 120 \\ 60 & 100 \\ \hline 40 & 20 \end{array}$$

40 at 120 cents, and 20 at 60 cents.

$$\begin{array}{rr} 100 & 120 \\ 70 & 100 \\ \hline 30 & 20 \end{array}$$

30 at 120 cents, and 20 at 70 cents. Answer.

$$\begin{array}{rr} 100 & 110 \\ 80 & 100 \\ \hline 20 & 10 \end{array}$$

20 *at 110 cents,* and 10 at 80 cents.

Explanation. We find how a compound worth 100 cents a bushel, can be formed by mixing the corn worth 60 cents with that worth 120 cents, the corn worth 70 cents with that worth 120 cents, and the corn worth 80 cents with that worth 110 cents. These three compounds, each worth 100 cents a bushel, when mixed together give another compound worth 100 cents a bushel.

Now if the miller chooses, he can mix 40 quarts at 120 cents with 20 quarts at 60 cents, 30 pecks at 120 cents with 20 pecks at 70 cents, and 20 bushels at 110 cents with 10 bushels at 80 cents ; in fact, he may consider the numbers expressing the proportions in which 2 of the kinds are to be mixed, as representing any measure that he pleases. It is usual, however, to consider all of the numbers expressing the proportions, as representing the same measures. If all the preceding numbers are considered as representing bushels, the 40 bushels at 120 cents, and the 30 bushels at 120 cents are added together, and the quantity of each kind to be mixed are 20 bushels at 60 cents, 20 bushels at 70 cents, 10 bushels at 80 cents, 20 bushels at 110 cents, and 70 bushels at 120 cents. We may multiply or divide the numbers expressing the proportions in which 2 or all of the kinds are to be mixed by any number, and we shall still have the true proportions. Moreover, we can combine the several kinds by 2 and 2 in many ways different from the preceding, and thereby obtain many different, but correct answers ; for instance, we can find in what proportions to mix the corn at 60 cents with that at 110 cents, the corn at 70 cents with that at 110 cents, and the corn at 80 cents with that at 120 cents. The only precaution to be observed in combining 2 kinds, is to take one kind *cheaper*, and the other *dearer* than the required compound.

Therefore, when several ingredients of different values are to be mixed, so as to form a compound of a certain value,

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Explain how example 1, lesson 162, is performed.

In what way can the miller mix the different kinds? What in fact may he do? What, however, is usual?

What if all the preceding numbers are considered as representing bushels? What if we multiply or divide these numbers? What is said of getting different answers? Give an example. What is the only *precaution to be observed?*

Take one ingredient whose value is less, and another whose value is more than the compound required, and find the proportion in which they are to be mixed to produce the compound; combine the other ingredients by 2 and 2 in the same manner and mix the whole in the proportions obtained.

2. A goldsmith has several kinds of gold, 16, 18, 19, and 23 carats fine; in what proportions must the different kinds be mixed so as to produce a compound 20 carats fine?

Ans. 3 of 16 carats and 4 of 23, 3 of 18 and 2 of 23, 3 of 19 and 1 of 23.

3. A silversmith has several kinds of silver, containing $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$ alloy; what proportions of the different kinds must he take so as to form a mixture containing $\frac{1}{9}$ alloy?

Ans. 4 of $\frac{1}{8}$ alloy and 5 of $\frac{1}{10}$, 10 of $\frac{1}{8}$ and 5 of $\frac{1}{12}$.

4. If you have pure gold, which is 24 carats fine, and some 17 and 20 carats fine, how must you mix the different kinds so as to make a compound 22 carats fine?

Ans. 5 of 24 carats with 2 of 17, 2 of 24 with 2 of 20.

5. A trader has some spirit worth $2.80 a gallon, some worth $2.10, some worth $1.60, and some worth $1.50; how many gallons must he take of each kind so as to make a compound worth $1.75 a gallon?

Ans. 15 at $2.80 and 105 at $1.60, 25 at $2.10 and 35 at $1.50.

LESSON 163.

1. A farmer has 30 bushels of rye worth 80 cents a bushel, and wishes to mix with it some worth 90 cents, and some worth 112 cents a bushel; what quantity must he take of that worth 90 cents, and of that worth 112 cents, to make with the 30 bushels a compound worth $1 a bushel?

Ans. 30 bushels at 90 cts., and 75 at 112 cts.

Explanation. How many bushels at 80 cents, at 90 cents, and at 112 cents, must be taken to make a compound worth $1 a bushel? How many bushels of the two last mentioned kinds, are to be mixed with 1 bushel of the first, to make such a compound? How many bushels of the two last mentioned kinds are to be mixed with 30 bushels of the first to make such a compound?

What course do we take, when several ingredients of different values are to be mixed, so as to form a compound of a certain value?

2. I have 8 ounces of gold 23½ carats fine, that I wish to melt with some 18, some 19, and some 21 carats fine, so that the mixture may be 20 carats fine ; how many ounces of each kind must I use ?
Ans. 14 oz. 18 carats, 4 oz. 19 carats, and 4 oz. 21 carats.

3. If you have some wine worth $3 a gallon, and some worth $2, what quantity of each sort must you mix with 10 gallons of water worth $0 a gallon, so that the compound may be worth $1½ a gallon ?
Ans. 7½ gallons of each sort.

4. A grocer has 15 pounds of sugar worth 10 cents a pound, and 5 pounds worth 6 cents a pound ; what quantity of some worth 11 and some worth 15 cents a pound, must he mix with the two quantities, so that the compound may be worth 10 cents a pound ?
Ans. 3⅓ lbs. at 11 cts., and 3⅓ lbs. at 15 cts.

Explanation. The two first named kinds being united together, what is the value of a pound of the mixture ?

5. A jeweller has 6 ounces of gold 16 carats fine, and 10 ounces 22 carats fine, which he wishes to melt with some 21 carats fine, so that the mixture may be 20 carats fine ; how many ounces of the gold 21 carats fine must he use ?
Ans. 4 oz.

6. A man has 8 ounces of silver 10 oz. fine, 10 ounces 9 oz. 14 pwts. fine, and 2 ounces 11 oz. 10 pwts. fine, which he wishes to melt with some 11 oz. 4 pwts. fine, and some pure silver, so that the mixture may be 11 oz. fine ; what quantity of the two last kinds must he take ?
Ans. 16⅔ oz. 11 oz. 4 pwts. fine, and 16⅔ oz. of pure silver.

7. A farmer has oats worth 40, 60, and 70 cents a bushel; what quantity of each of these kinds must he take to make 40 bushels worth 50 cents a bushel ?
Ans. 24 bu. at 40 cts., 8 at 60 cts., and 8 at 70 cts.

Explanation. How many bushels of each kind must be mixed in order to obtain a compound worth 50 cents a bushel ? How much of each kind must be taken to make 1 bushel of such a compound ? How much of each kind must be taken to make 40 bushels of such a compound ?

8. A jeweller wishes to make a piece of gold to contain 12 ounces 21 carats fine, by melting together 3 kinds, 18½, 20, and 23 carats fine ; how many ounces of each of these *kinds must be taken ?*
Ans. 3⅕ oz. 18½, 3⅕ oz. 20, and 5⅗ oz. 23 carats.

9. A grocer has different kinds of wine, worth \$3, \$2, and \$$1\frac{1}{2}$ a gallon ; how much of each of these kinds must be mixed with water so that the compound may fill a 20 gallon keg, and be worth \$$1\frac{3}{4}$ a gallon ?

Ans. 10 gals. at \$3, $1\frac{3}{7}$ gal. at \$2, $1\frac{3}{7}$ gal. at \$$1\frac{1}{2}$, and $7\frac{1}{7}$ gals. of water.

PROMISCUOUS QUESTIONS

IN

RULE OF THREE, FELLOWSHIP, INSURANCE, &c.

LESSON 164.

To be performed in the mind.

1. How much sugar at 8 cents a pound, can you obtain in exchange for 6 lbs. of wool, at 41 cents a pound ?

2. What must I pay to get my house and furniture insured at $1\frac{1}{2}$ per cent., their value being estimated at \$2,500 ?

3. A schooner worth \$4,000, was loaded with goods worth \$4,000 by a merchant ; in a storm it became necessary to throw the goods overboard to preserve the vessel ; what sum must the owner of the vessel pay to the merchant on account of his loss ?

4. A man paid \$20 for 25 casks of lime ; what sum would 31 casks have cost him ?

5. A. can finish a piece of work in 2 hours, and B. in 3 hours ; how long will it take both to finish it ?

6. If you mix 4 bushels of oats worth 50 cents a bushel with 6 bushels worth 60 cents a bushel, what will be the value of the compound ?

7. How many gallons of cider worth 12 cents a gallon should be mixed with some worth 7 cents, so that the compound may be worth 10 cents a gallon ?

8. A. owns $\frac{5}{8}$ of a brig, B. $\frac{2}{8}$, and C. $\frac{1}{8}$; if she earns \$1,000, what will be each one's share of the profit ?

9. If you commence trade the 1st of January on \$2,000, and the 1st of July are joined by a partner with \$1,000, what will be *your* share of \$1,200 profit, found to have *been gained at* the end of the year ?

10. What quantity of honey at 12 cents a pound must a man give for an axe worth $1.50, and 10 pounds of cheese worth 9 cents a pound.

11. If 6 men consume 1 bl. of flour in 20 days, how long will 2 bls. last 1 man ?

12. If 12 bushels of wheat be worth $25, what are 4 bushels worth ?

LESSON 165.

1. I wish to cover the value of my brig, worth at $5,000, during a voyage from New York to Smyrna, the premium for insurance being 4 per cent. ; what amount must I insure, and what premium must I pay ?
Ans. I must insure $5,208.33⅓, and pay $208.33⅓ premium.

2. A man paid $3.90 for the use of $80, 9 months ; how much should he pay for the use of $375, 11 months, at the same rate ? Ans. $22.34.

3. A lumber dealer sold a lot of 80 thousand pine shingles for $1350 ; what should he sell 33 thousand for, at the same rate ? Ans. $144.37½.

4. 7 men can remove 896 cubic yards of gravel in 8 days ; how many men will it take to do the same labor in 14 days ? Ans. 4.

5. If a merchant barters 80 lbs. of rice, worth 4 cts. a pound, at 5 cts., what should corn worth $1 a bushel be bartered at in exchange ? Ans. at $1.25

6. If 6⅜ yds. of cloth, 1¾ yd. wide, cost $25.50, what should 11 yds. of cloth of the same quality, ⅝ wide, cost ?
Ans. $15.71.

7. A. owns ⁶⁄₁₀ of a vessel, and B. the other ⁴⁄₁₀ ; what is each one's share of the earnings for one year, amounting to $1,859 ?
Ans. A.'s share is $1,115.40, and B.'s, $743.60.

8. A. can do a piece of work in 6 days, and B. in 9 days ; how long will it take them both to perform it ?
Ans. 3⅗ days.

9. If I lend you $2,000 for 1 year, how long should you allow me the use of $500 to reciprocate the favor ?
Ans. 4 years.

10. A man gave $1.50 for ⅓ of a cord of wood ; what should he pay for 3⅙ cords, at the same rate ?
Ans. $14.25.

LESSON 166.

1. If you give \$6,627 for 85 T. 6 cwt. 3 qrs. of American bar iron, what quantity can you get for \$453.33, at the same rate ? Ans. 5 T. 16$\frac{3}{4}$ cwt., about.

2. All the taxes to be paid in a certain town, in 1835, amounted to \$4,000, which the assessors increased 2 per cent. ; all the real and personal estate in the town was valued at \$200,000 ; there were 180 polls, the tax on each of which was fixed at \$2 ; what did A.'s tax amount to, his real estate being valued at \$3,000, personal estate at \$185, with 1 poll ? Ans. \$61.24.

3. A cistern containing 1,000 gallons was emptied by 2 cocks in 5 hours ; how long will it take 3 such cocks to empty it ? Ans. 3 h. 20 m.

4. If you own $\frac{1}{5}$ of a tract of land containing 840 acres, and sell $\frac{5}{8}$ of your share for \$656.25, how much should you ask for 30 acres of the remainder, at the same rate ?
Ans. \$187.50.

5. A jeweller has gold 18, 19, and 22 carats fine ; what quantity of each must be taken to make 1 lb. 20 carats fine ?
Ans. $\frac{2}{7}$ lb. 18 carats, $\frac{2}{7}$ lb. 19 carats, and $\frac{3}{7}$ lb. 22 carats.

6. If 2 lbs. 3 oz. of silver 11 oz. fine, be melted with 1 oz. of copper, and 1 lb. of silver 10 oz. 10 pwts. fine, what will be the fineness of the mixture ?
Ans. 10 oz. 11 pwts. 12 grs. fine.

7. A., B. and C., traded in company ; A. put in \$1,000, and employed it 1 year ; B. put in \$4,000, and employed it 8 months, and C. put in \$2,500, and employed it 10 months ; they gained \$2,000 ; what was each one's share ?
Ans. A.'s share \$347.82, B.'s \$927.54, C.'s \$724.64.

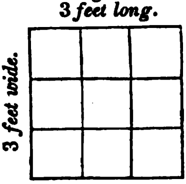
8. How much must I insure on my house to cover its value, it being worth \$600, and the premium for insurance being $2\frac{1}{2}$ per cent. ? Ans. \$615.38.

MENSURATION.

Lesson 167.

Definitions. A corner is called an *angle,* and a square corner a *right angle.* An angle greater or less than a right angle, is called an *oblique angle.*

Figure 3.

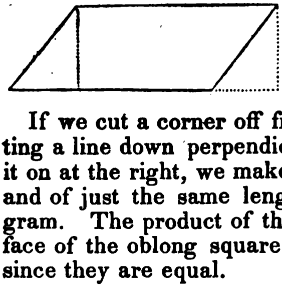


By examining figure 3, we find as we did in Compound Numbers, that an oblong square 1 ft. wide and 3 ft. long, contains 3 sq. ft.; that an oblong square 2 ft. wide and 3 ft. long, contains 2 times 3, or 6 sq. ft., and that a square 3 ft. long and 3 ft. wide, contains 3 times 3, or 9 sq. ft.

Therefore, to get the surface of an oblong square, or of a square,

Multiply the length by the breadth.

Figure 4.



A four sided figure, having its opposite sides equal and parallel, and its angles oblique, like figure 4, is called a *parallelogram.*

If we cut a corner off from the left of figure 4, by dotting a line down perpendicular to the lower side, and add it on at the right, we make an oblong square just as large, and of just the same length and breadth as the parallelogram. The product of this length and breadth is the surface of the oblong square, and also of the parallelogram, since they are equal.

Therefore, to get the surface of a parallelogram,

Multiply the length by the breadth.

What is a corner called? A square corner? What is called an oblique angle?

What do we find by examining figure 3, lesson 167?

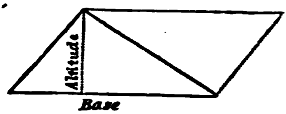
How do we get the surface of an oblong square, or of a square?

What is called a parallelogram?

How do we proceed to find a way to obtain the surface of a parallelogram?

How do we get the surface of a parallelogram?

Figure 5.



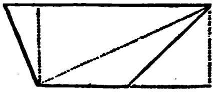
A three sided figure, like figure 5, is called a *triangle*. If we dot a line down from a corner, or angle of the triangle, perpendicular to the opposite side, the dotted line is called the *altitude* of the triangle, and this opposite side is called the *base*.

If we now dot lines parallel to the base and one of the sides, as in figure 5, we complete a parallelogram evidently twice as large as the triangle ; now the surface of the parallelogram is equal to the product of its length by its breadth, the length being the same as the base of the triangle, and the breadth the same as its altitude.

Therefore, to get the surface of a triangle,

Take half the product of the base by the altitude.

Figure 6.



A four sided figure, having two sides parallel but unequal, like figure 6, is called a *trapezoid*.

If we divide the trapezoid into 2 triangles, the surface of one will be equal to half the product of its base, or the lower side of the trapezoid, by its altitude, or the distance between the parallel sides ; the surface of the other will be equal to half the product of its base, or the upper side of the trapezoid, by its altitude, or the distance between the parallel sides.

Therefore, to get the surface of a trapezoid,

Take half the product of the two parallel sides by the distance between them.

What is called a triangle ? What is called the altitude of a triangle ? The base ?

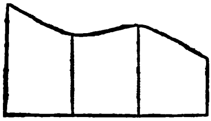
How do we proceed to find a way to obtain the surface of a triangle ? How do we get the surface of a triangle ?

What is called a trapezoid ?

How do we proceed to find a way to obtain the surface of a trapezoid ? How do we get the surface of a trapezoid ?

Lesson 168.

Figure 7.



If we have a long irregular piece, like figure 7, we can measure the breadth in various places, thereby dividing it into trapezoids.

Now to get the surface of the whole figure,

Find the surface of the trapezoids separately, and add them together.

We also can get the surface if we

Multiply the average breadth by the length.

To find the average breadth, measure the breadth at each end, and in several places, at equal distances apart, then take half the sum of the breadths at the two ends, add it to the sum of the intermediate breadths, and divide the result by the number of trapezoids. This way is correct; for to find the average breadth of each trapezoid, we take half the sum of its two parallel sides; so the sum of all these average breadths contains half the sum of the first and last parallel sides, and the sum of all the intermediate ones; the sum of the average breadths of all the trapezoids divided by the number of trapezoids evidently gives the average breadth of the whole piece.

Note. The usual way of finding the average breadth of a long irregular piece, is to take the breadth at each end, and in several other places, at equal distances apart, and divide the sum of the breadths by their number. This course is incorrect, and absurd, but there are many other similar mistakes made in averaging; thus, if the temperature is 30 degrees at 6 o'clock in the morning, 60 degrees at noon, and 48 at 6 o'clock in the evening, many persons add 30, 60, and 48 together, and divide by 3 to get the average temperature of the day; on the contrary, we must add half of 30 and 48 to 60, and divide by 2, the number of intervals. If 20 cubic ft. of water run over a dam in a second, at 6

How do we proceed to get the surface of an oblong irregular piece? How also can we get the surface?

How do we find the average breadth? Why is this way correct?

What is the usual way of finding the average breadth of a long irregular piece? What is said of this course? If the temperature be 30 degrees at 6 o'clock in the morning, 60 degrees at noon, and 48 degrees at 6 o'clock in the evening, what is the usual, and what is the correct way to get the average temperature of the day?

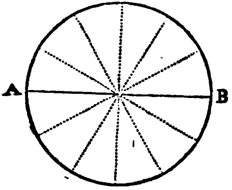
If 20 cubic ft. of water run over a dam in a second, at 6 o'clock in the morning, 16 cubic ft. at 10 o'clock, 14 at 2 o'clock, and 18 at 6 o'clock *in the evening,* what is the usual, and what is the correct way to get *the average* number of cubic ft. a second?

o'clock in the morning, 16 cubic ft. at 10 o'clock, 14 cubic ft. at 2 o'clock, and 18 cubic ft. at 6 o'clock in the evening, many persons add 20, 16, 14, and 18 together, and divide by 4 to get the average number of cubic ft. passing over the dam in a second; on the contrary, we must add half of 20 and 18, to 16 and 14, and divide by 3, the number of intervals. There are many other cases where the learner will see the propriety of averaging in a similar manner.

To find the surface of an irregular figure of 4 sides, or of a figure of 5, 6, 7, &c., sides,

Divide it into triangles, and the sum of their surfaces will be the surface of the figure.

Figure 8.



The curved line that bounds a circle is called the *circumference*. A line drawn through the centre of a circle, touching the circumference at both ends, like the line A B, is called the *diameter*. Half the diameter is called the *radius*.

The circumference of a circle has been found by calculation to be about 3.14159 times the diameter; for rough calculations it may be reckoned 3 times the diameter.

Let us imagine a great number of triangles, very small indeed, to be formed in a circle, with their tops in the centre, and having for bases portions of the circumference so small that they may be considered straight lines. The surface of each triangle is equal to half the product of its base by its altitude, the altitude being the radius; the surface of all these triangles, then, or

The surface of the whole circle, is equal to

Half the product of the circumference by the radius.

Lesson 169.

1. How many square feet are there in a house lot 51 ft.

How do we find the surface of an irregular figure of 4 sides, or of a figure of 5, 6, 7, &c. sides?

What is called the circumference of a circle? Diameter? Radius?

The circumference has been found by calculation to be how many times the diameter? For rough calculations how may it be reckoned? How do we proceed to find a method of obtaining the surface of a circle?

What is the surface of a circle equal to?

9 in. long, and 32 ft. 6 in. wide, and what will it cost at $.55 a sq. ft.?

Ans. 1,681.875 sq. ft., and it will cost $925.03.

Note. See note after example 6, lesson 107, in Reduction of Compound Numbers.

2. How many acres are there in a piece of ground 60 rods square? Ans. 22½.

3. How large is the surface of one side of a board 18.67 ft. long, and 1.25 ft. wide? Ans. 23⅓ sq. ft., about.

4. How many square yards are there in a piece of cloth 27¾ yards long, and ⅞ of a yard wide? Ans. 24.28, about.

5. How much will it cost to plaster the ceiling of a room 19 ft. 6 in. long, and 15 ft. wide, at 10 cts. a sq. yd.?

Ans. $3.25.

6. There is a house that has 12 windows 5 ft. 9 in. high, and 3 ft. 3 in. wide, and 7 windows 5 ft. 3 in. high, and 3 ft. 2 in. wide; what will the glazing of these windows cost at $.20 a sq. ft.? Ans. $68.12½.

7. How much must I give for paving a yard 60 ft. 4 in. long, and 57 ft. wide, at $.40 for every 100 sq. ft.?

Ans. $13.76.

8. What is the surface of a field in the form of a parallelogram 16 rods 9 ft. long, and 5 rods wide, measuring square across? Ans. 22,522½ sq. ft.

9. A man bought a house lot in a triangular shape, the altitude being 40½ ft., and the base 73⅔ ft.; how much did it cost at 30 cents a sq. ft.? Ans. $447.52.

10. What is the surface of a trapezoid, one of the parallel sides being 35.6 ft., the other 91.4 ft., and the distance between them 10 ft.? Ans. 635 sq. ft.

11. If a piece of land be 40 ft. long, and measuring the breadths at every 10 ft. you find them to be as follows; 8 ft., 5 ft., 2 ft., 11 ft., and 13 ft., what is the surface of the whole piece? Ans. 285 sq. ft.

12. What is the surface of a circular fish pond 50 ft. in diameter? Ans. 1,963.49 sq. ft., about.

Explanation. What is the circumference of the pond?

LESSON 170.

1. What is the surface of a semicircle, or half a circle, *like half of* figure 8, lesson 168, the diameter of the circle *being 12 ft.?* Ans. 56.549 sq. ft., nearly.

2. What is the surface of a quadrant, or quarter of a circle, the diameter of the circle being 5½ rods ?

Ans. 5.94 sq. rods, nearly.

3. What is the diameter of a circle in feet and decimals, the circumference being 18 ft. 9 in. ? Ans. 5.968 ft., about.

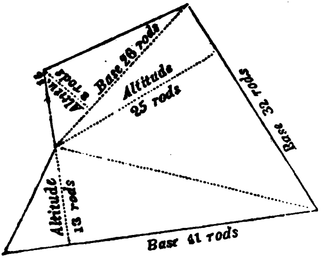
4. The surface of a field in the shape of an oblong square, is 1 acre, and its length is 16 rods ; what is its breadth ? Ans. 10 rods.

5. A parallelogram containing 225 sq. ft. is 12 ft. 6 in. wide ; how long is it ? Ans. 18 ft.

6. A triangle containing 2 acres, has a base 20 rods long ; what is its altitude ? Ans. 32 rods.

7. How many square yards of papering are there in a room 18 ft. long, 16 ft. wide, and 10 ft. high, if we deduct the space occupied by 2 doors, each 7 ft. high, and 3 ft. 10 in. wide, by 2 windows, each 5 ft. 4 in. high, and 3 ft. 6 in. wide, by a fireplace 5 ft. square, and by a mop-board 10 in. wide, extending entirely around the bottom of the room, except the spaces occupied by the doors and fireplace ? Ans. 57.543, about.

Figure 9.



8. Find the surface of figure 9, divided by the dotted lines into 3 triangles, whose bases and altitudes are marked.

Ans. 4 A. 3 qrs. 10½ sq. rods.

9. How many yards of carpeting, 1 yd. wide, will be necessary for a room 20 ft. long, and 18 ft. 9 in. wide, if we deduct the space occupied by the hearth, which is 5 ft. long, and 4 ft. 2 in. wide ? Ans. 39.352 nearly.

10. How much cloth ¾ of a yd. wide, is equal to 26¾ yds. 1⅛ yd. wide ? Ans. 40⅛ yds.

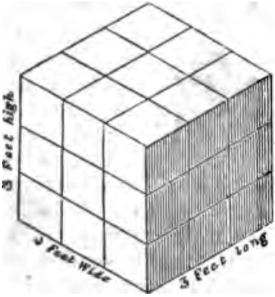
11. How many square miles, and how many acres are there in 6 miles square ?

Ans. 36 sq. miles, and 23,040 acres.

12. How much land is there in a piece 100 ft. long, the breadths measured every 20 ft. being as follows ; 7 ft., 11 ft., *5 ft., 6 ft., 16 ft.,* and 19 ft. ? Ans. 1,020 sq. ft.

LESSON 171.

Figure 10.



A square block like figure 10, is called a *cube*.

By examining the figure, we find as we did in Compound Numbers, that the surface of the top contains 3 times 3, or 9 sq. ft. If we take a piece off from the top 1 ft. thick, we find the quantity in it to be once 3 multiplied by 3, or 9 cubic ft.; if we take a piece off 2 ft. thick, we find the quantity to be 2 times 3 multiplied by 3, or 18 cubic ft.; and if we take the whole we find the quantity to be 3 times 3 multiplied by 3, or 27 cubic ft.

Therefore, to get the quantity in a cube,

Multiply the length, breadth, and thickness together.

Figure 11.

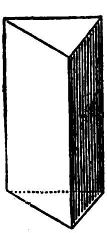
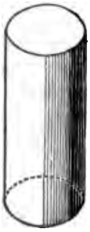


Figure 12.



A body of uniform shape and size, whose ends are cut off perpendicular to its length, is called a *right prism*, or simply a *prism*. Thus, a brick is a prism, a square stick of timber is a prism, a stick of timber hewed 3 square, that is, triangular, like figure 11, is a prism, also a stick hewed 5, 6, 7, 8, &c. square, is a prism, &c.

However, if the body be round, like figure 12, it is called a *right cylinder*, or simply a *cylinder*. Thus, a grindstone is a cylinder, a round stick of timber is a cylinder, &c.

What is called a cube?
What do we find by examining figure 10, lesson 171?
How do we get the quantity in a cube?
What is called a right prism, or simply a prism? What things are prisms?
What is called a right cylinder, or simply a cylinder? What things are cylinders?

Now if we take a piece out of figure 10 in the shape of a prism, say 2 ft. wide, 1 ft. thick, and 3 ft. long, it is plain that the surface of the end multiplied by 3, gives the number of cubic feet the piece contains. A prism of 3 sides, or of 5, 6, 7, &c. sides, or a cylinder, is evidently equal to a prism whose ends contain the same surface, but which are square.

Therefore, to get the quantity in a prism or cylinder,

Multiply the surface of one end by the length.

Figure 13.

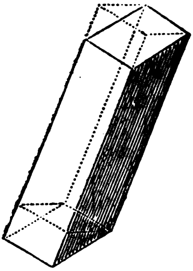
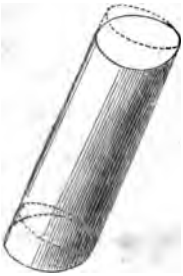


Figure 14.



When the ends of a prism or cylinder are not perpendicular to its length, but are parallel to each other, as in figures 13 and 14, the prism is called an *oblique prism*, and the cylinder an *oblique cylinder*.

If we cut a piece square off from the bottom of an oblique prism, or an oblique cylinder, and place it on the top, as the dotted lines in figures 13 and 14 represent, we form a right prism, or a right cylinder, of the same length and size as the oblique prism, or oblique cylinder. The same can be done in an oblique prism whose ends are triangular, or in any other shape.

Therefore, to get the quantity in an oblique prism, or cylinder,

Multiply the surface of one end of a right prism, or cylinder, of the same size, by the length.

How do we proceed to find a method of obtaining the quantity in a prism, or cylinder?

How do we get the quantity in a prism, or cylinder?

What is called an oblique prism, or an oblique cylinder?

How do we proceed to find a method of obtaining the quantity in an oblique prism, or cylinder?

How do we get the quantity in an oblique prism, or cylinder?

LESSON 172.

Figure 15.

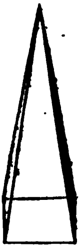


Figure 16.

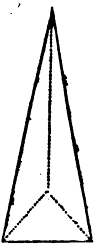


Figure 17.

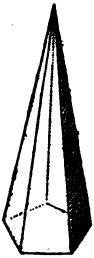
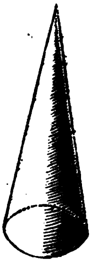


Figure 18.

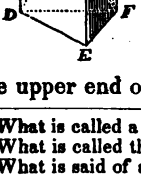


A body that tapers regularly from its bottom, or base, till it comes to a point, like figures 15, 16, and 17, is called a *pyramid*. If the base, however, be round, as in figure 18, the body is called a *cone*.

If the top of a pyramid or cone be cut off parallel to the base, what remains is called the *frustum of a pyramid or cone*.

It is plain that all pyramids and cones, which have the same height, and whose bases have equal surfaces, are equal in size, since they all taper regularly from their bases.

Figure 19.



To discover how to measure a pyramid, take off part of a 3-sided prism, like figure 19, by cutting from the corner B down to D and F; cut the figure that remains from B through to A and F. The first part we cut off, is a pyramid that has the same base as the lower end, or base of the prism, and is as high as the prism. That part of the remainder, cut off at the right, is also a pyramid, which, when placed upside down, has the same base as the upper end of the prism, and is as high as the prism.

What is called a pyramid? A cone?

What is called the frustum of a pyramid or cone?

What is said of all pyramids and cones which have the same height, and whose bases have equal surfaces?

How do we proceed to find a method of obtaining the quantity in a pyramid or cone?

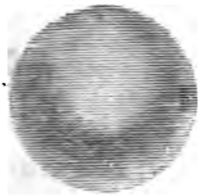
These two pyramids, then, are of the same size, since their bases and heights are equal. With regard to the third piece, cut off at the left, let us first suppose the prism turned down on the further side; the pyramid cut off at the right, now has a base composed of half of the further side, or face of the prism, and the piece, or pyramid, cut off at the left, has a base just as large, being composed of the other half; besides, it is just as high, since it tapers regularly up to the point B, the present top of the pyramid cut off at the right; it is therefore just as large. The one at the right being just as large as the first one cut off, it follows that a triangular prism is composed of three pyramids, equal to the pyramid whose base is one end of the prism, and whose height is the same as the length of the prism.

Therefore, as the quantity contained in a prism is equal to the product of the surface of one end by the length,

The quantity contained in a pyramid, or cone, is equal to

One third of the product of its base by its height.

Figure 20.



The surface of a sphere, globe, or ball, like figure 20, has been found by calculation to be equal to

The product of its circumference by its diameter.

This being the case, let us suppose a globe divided into a great number of pyramids, very small indeed, whose bases are at the surface of the globe, and tops in the centre, the height of each being the radius, or ½ of the diameter. The quantity in each pyramid is equal to ⅓ of the product of the radius by the base.

Therefore, the quantity in all of the pyramids, or in the whole globe is equal to

One third of the product of the radius by the surface.

What is the quantity contained in a pyramid or cone equal to?

What has the surface of a sphere, globe, or ball been found by calculation to be equal to?

How do we proceed to find a method of obtaining the quantity in a globe?

To what is the quantity in a globe equal?

We can measure a large body of almost any shape by dividing it into prisms, pyramids, and frustums, and the quantity in the several prisms, pyramids, and frustums, will be the quantity contained in the body.

If the body be very irregular, like a large rock, find the average length, breadth, and thickness, and then find the quantity in it as though it was a prism.

A small irregular body can be measured by immersing it in a vessel containing water enough to cover it, say in a cylindrical vessel, like a pail, tub, &c.; the space filled by the rising of the water will evidently be the quantity contained in the body.

Lesson 173.

1. How many cubic feet are there in a pile of bricks in the form of a cube, 6 ft. 6 in. long, 6 ft. 6 in. wide, and 6 ft. 6 in. high? Ans. $274\frac{5}{8}$.

2. How many cubic feet, and how many feet of wood are there in a cubic pile of wood, measuring 4 ft. on each side? Ans. 64 cubic ft. and 4 ft. of wood.

3. How many cords of wood are there in a load 8 ft. long, 4 ft. 5 in. wide, and 5 ft. 4 in. high; and what is it worth at $6 a cord? Ans. 1.4575 C. and it is worth 8.74\frac{1}{2}$.

Explanation. We cannot in practice get the length, width, or height of a load of wood nearer than $\frac{1}{10}$ of a foot, so in measuring wood when we change inches to decimals, we should omit all decimals less than .1 of a foot.

4. A pile of wood containing 5 C. 5 ft. of wood, is 20 ft. long, and 4 ft. wide; how high is it? Ans. 9 ft.

5. What quantity is there in a load of wood 7 ft. 2 in. long, 4 ft. 6 in. wide, and 4 ft. 7 in. high? Ans. 1 C. 1.315 ft.

6. How many cords of wood are there in a pile 30.4 ft. long, 4 ft. wide, and 4.3 ft. high? Ans. 4.085 C.

7. A stick of timber is 35 ft. 8 in. long, 2 ft. wide, and 1 ft. 3 in. deep; how much is it worth at $9 a ton? Ans. $16.05.

8. A stick of timber, hewed 3 square, is 28 ft. long, the

How can we measure a large body of almost any shape?
What if the body be very irregular, like a large rock?
How can a small irregular body be measured?

base of the triangle composing the end is 3 ft., and the altitude is 1 ft. 4 in.; how many cubic feet does the stick contain? Ans. 56.

9. How many cubic feet are there in a barn 40 ft. square at the bottom, and measuring 20 ft. from the ground to the eaves, and 35 ft. from the ground to the top of the roof? Ans. 44,000.

10. A laborer dug a cellar 30 ft. long, 22 ft. wide, and 8 ft. deep, and was paid 6 cents for every cubic yard excavated; what sum did he receive? Ans. $11.73.

11. How many cubic feet are there in a grindstone 5 in. thick, and 4 ft. in diameter, making no allowance for the eye? Ans. 5.236, nearly.

12. How many tons are there in a pine log 42 ft. long, and 4 ft. in diameter? Ans. 10.556, nearly.

Lesson 174.

1. There is a wedge 1 ft. 2 in. from the point to the centre of the head, the head is 2½ in. thick, and the breadth of the wedge is 3 in.; how many cubic inches are there in it? Ans. 52.5.

2. The ends of a stick of timber 20 ft. long, 1¼ ft. broad, and 1¼ ft. thick, are sawed off parallel to each other, but bevelling, like figure 13, lesson 171; how much is the stick worth at $8 a ton? Ans. $6.

3. How many cubic feet are there in a cylindrical stick of timber, 41 ft. long, and 2 ft. 3 in. in diameter, whose ends are cut off parallel to each other, but oblique to the direction of the stick, like figure 14, lesson 171? Ans. 163.019, about.

4. Supposing the largest Egyptian pyramid to be 700 ft. square at the base, and 500 ft. high, how many cubic yards does it contain, and how much would the cost of erecting it be at $2 a cubic yard?

Ans. it contains 3,024,691.358 cubic yds., about, and the cost of erecting would be $6,049,382.72.

5. The surface of the base of a triangular pyramid is 21.25 sq. ft., and the height is 18.33 ft.; how many cubic feet does it contain? Ans. 129.8375.

6. A certain church has a steeple shaped like a cone, 12 ft. in diameter at the base, and 44.5 ft. high; how many cubic feet are there in it? Ans. 1,677.609, about.

7. *A steeple has* a gilded ball on the top 2.25 ft. in di-

ameter ; how many square feet of gilding are there on the ball, and how many cubic feet are there in it ?

Ans. 15.904 sq. ft. of gilding, about, and 5.964 cubic ft. in it, about.

8. How many cubic feet are there in a rock, the average length of it being 18.5 ft., breadth 11 ft., and thickness 9.33 ft. ? Ans. 1,898.655.

9. There is a churn 1.5 ft. in diameter at the bottom, .8 of a ft. in diameter at the top, and 3.5 ft. high ; how many cubic feet of cream will it hold ? Ans. 3.748, nearly.

Explanation. How much is the diameter of the churn contracted in rising 3.5 ft. ? How many feet then must it rise to taper out to a point, and form a cone ? How many cubic feet will the whole cone contain ? How many cubic feet will the little cone formed on top of the churn contain ? Then how many cubic feet does the churn contain ?

10. How many wine gallons does a churn contain that is 16 in. in diameter at the bottom, 10 in. in diameter at the top, and 36 in. high ? Ans. 21.053, nearly.

LESSON 175.

1. There is a tub 3 ft. deep, 4 ft. in diameter at the top, and 3 ft. at the bottom ; how many wine gallons does it contain ? Ans. 217.38, about

Explanation. Imagine it to continue downward until it comes to a point and forms a cone.

2. The top of a pyramid was taken off so as to leave a portion 18 ft. high, 8 ft. square at the base, and 2 ft. square at the top ; how much did the portion left contain ?

Ans. 504 sq. ft.

3. How many square feet of surface has a cubic block of wood, each side of which is 2 ft. square ?

Ans. 24 sq. ft.

4. How many square feet of boards, 1 in. thick, will make a box measuring on the outside 4 ft. long, 1.5 ft. wide, and 2 ft. high ? Ans. $31\frac{5}{9}$.

Explanation. Observe that the boards at the top, bottom, and sides overlap those at the ends 1 inch, and that the boards at the top and bottom overlap those at the sides *1 inch.*

5. What is the surface of a 3-sided prism, not including the ends, which is 12 ft. long, one face or side being 2½ ft. broad, another 1½ ft., and the third 1 ft.? Ans. 60 sq. ft.

6. How many square feet of bark are there on a log 20 ft. long, and 3 ft. in diameter? Ans. 188.495, about.

Note. Suppose the convex surface of a cylinder to be divided into oblong squares of the same length as the cylinder, and very narrow indeed; if we now multiply the breadth of each by the length, we get its surface. Therefore,

If we multiply the sum of all the breadths, or the entire circumference of the cylinder, by the length, we get the whole convex surface.

7. There is a pyramid with a base 13 ft. square, and the altitude of each triangle composing the sides is 23 ft.; what is the surface of the pyramid, not including the base? Ans 598 sq. ft.

8. What is the surface of a cone, not including the base, the slant height being 18 ft., and the diameter of the base 6 ft.? Ans. 169.646 sq. ft., nearly.

Note. Suppose the convex surface of a cone to be divided into a great number of triangles, very small indeed, the tops of which shall be at the top of the cone, and the bases at the bottom; it is plain that the surface of each of the triangles will be equal to one half of the product of its base multiplied by the slant height of the cone. Therefore,

If we take half the product of the sum of all the bases, or the circumference of the base of the cone, multiplied by the slant height of the cone, we get the surface of all the triangles, or the whole convex surface of the cone.

9. A cone 4 ft. in diameter at the base is cut off where the diameter is 2 ft.; what is the convex surface of the remaining part, if the slant height of it is 7 ft.? Ans. 65.973 sq. ft., about.

Explanation. What was the convex surface of the whole cone? Of the part cut off at the top?

10. If the earth or globe is 7,920 miles in diameter, how many square miles are there in the surface of the northern half of it? Ans. 98,530,315.488 sq. miles.

LESSON 176.

Boards are usually sawed 1 inch thick; and when we say 5, 10, 20, &c., feet of boards, we mean 5, 10, 20, &c., square feet 1 inch thick.

How do we proceed to find a way to obtain the convex surface of a cylinder?

How do we get the convex surface of a cylinder?

How do we proceed to find a way to obtain the convex surface of a cone?

How do we get the convex surface of a cone?

How are boards usually sawed? What do we mean when we say 5, *10, 20, &c., feet of boards?*

Planks and *joists* are sawed thicker; when we say there are 5, 10, 20, &c., feet in a plank or joist, we mean there are 5, 10, 20, &c., feet of boards 1 inch thick in the plank or joist.

Round timber is often measured by the following arbitrary rule.

Multiply ¼ of the average girt of the log by itself, and this product by the length.

Note. This rule is incorrect, giving about ⅕ less than the true quantity.

1. How many feet are there in a pile of 40 boards, each being 19 ft. 4 in. long, 1 ft. 3 in. wide, and 1 in. thick? Ans. 966⅔.

2. How many feet are there in 2 planks 33 ft. long, 2 ft. 3 in. wide, and 3 in. thick? Ans. 445.5.

3. A man bought 20 pine joists, each of which was 18 ft. long, 5 in. wide, and 3 in. thick, at $20 a thousand feet; what did they cost him? Ans. $9.

4. What sum must I give for 50 pine planks 30 ft. long, 2.5 ft. wide, and 4 in. thick, at $16 a thousand feet? Ans. $240.

5. There is a pine log 30 ft. long, the average girt of which is 10 ft.; how many cubic feet does it contain by the preceding arbitrary rule? Ans. 187.5.

6. If you work by the preceding arbitrary rule, how many tons will there be in a stick of round timber 40 ft. long, with an average girt of 8 ft., 40 cubic ft. being reckoned to a ton? Ans. 4.

7. What sum must you receive for building a wall 87 ft. long, 7 ft. thick, and 12.5 ft. high, at $1.80 a perch, there being 24¾ cubic feet in a perch? Ans. $553.64.

8. There is a brick house 48 ft. long, and 26 ft. wide; the walls are 19 ft. high, and 1 ft. thick; the gable ends are 15 ft. high, and 8 in. thick; there are 2 doorways 8 ft. high, and 5 ft. wide, and 18 windows in the lower part, and 4 in the gable ends, each being 6 ft. high, 3½ ft. wide; now how many bricks are there in the house, making no allowance for the lime, a brick being 8 in. long, 4 in. wide, and 2 in. thick? Ans. 67,014.

9. How many cubic inches are there in a marble image,

How are planks and joists sawed? What do we mean when we say there are 5, 10, 20, &c., feet in a plank or joist?

Recite the arbitrary rule by which round timber is usually measured. *What is said of* the correctness of this rule?

which, being immersed in the water contained in a pail 10 in. in diameter, caused the water to rise 1 in. ?

Ans. 78.54, nearly.

10. A stone being immersed in some water contained in a box 12 in. wide and 20 in. long, caused the water to rise 2½ in. ; how many cubic inches were there in the stone ?

Ans. 600.

GAUGING.

LESSON 177.

To gauge a cask, that is, to find how much it holds,

Find on the inside in inches, the length, the bung diameter, and the head diameter ; then if the staves be much curved between the bung and head, add to the head diameter ⅔ of the difference between the bung and head diameters ; but if the staves be but little curved, add ⅗ of the difference ; the sum is found by experience, to be about the diameter of a cylinder of the same length and capacity as the cask ; the quantity contained in the cylinder is the quantity the cask will hold.

Note 1. We are generally obliged to take all the dimensions, except the bung diameter, on the outside ; when such is the case, we must make a proper allowance, say from 1 to 2 inches for the thickness of the two heads ; and the head diameter, measured within the chimes, must be diminished from .3 to .6 of an inch, on account of the greater thickness of the stave inside of the head. Observe that the average of the two head diameters must be taken when they are unequal.

Note 2. In gauging, we can consider the circumference of a circle 3.14 times the diameter ; this will be near enough.

1. How many wine gallons will a cask hold, whose staves are moderately curved, the head diameter being 16 in., bung diameter 20 in., and length 28 in. ?

Ans. 32.2, about.

Explanation. What is the diameter of a cylinder of the same length and capacity as the cask ? How many cubic inches does such a cylinder hold ? How many wine gallons are there in this number of cubic inches ?

What is the rule for gauging a cask ?

How are we generally obliged to take all the dimensions of a cask ?

When such is the case, what allowances must be made ? If the two head diameters are unequal, what is to be done ?

In gauging, how can we consider the circumference of a circle ?

2. How many bushels of salt will a cask hold, whose length is 42 in., head diameter 26 in., and bung diameter 33 in., the staves being much curved? Ans. 14.4, about.

3. How many imperial gallons of milk are there in a cask, whose length is 25 in., bung diameter 19 in., and head diameter 16 in., the staves curving but little?
Ans. 22.4, about.

4. What number of beer gallons will a cask hold, whose length is 33 in., bung diameter 22 in., and head diameter 16 in., the staves curving moderately? Ans. 35.3, nearly.

5. How many bushels of wheat are there in 500 casks, the length of each being 48 in., the bung diameter 36 in., the head diameter 28 in., and the staves much curved?
Ans. 9,734.6, about.

Lesson 178.

To find the quantity of liquor in a cask which is not full,

Set the cask upright; then if the height of the liquor be not more than ¼ of the height of the cask, consider the space filled as forming the frustum of a cone, and calculate the quantity accordingly. If the height of the liquor be more than ¼, but not more than ½ of the height of the cask, consider the space filled as ½ of a cask, and calculate the quantity accordingly.

Note 1. When the liquor fills more than ½ of the cask, measure the hollow space by the preceding rule, and then to find the amount of liquor, subtract this quantity from the whole contents of the cask.

Note 2. When casks lie on one side, guagers employ certain arbitrary rules to find the empty space in them. These rules, which are not very correct, may be learned by practice.

1. There is a barrel 34 in. long which contains a quantity of molasses; when placed on one head, the surface of the liquor is 8 in. high; the head diameter of the barrel is 16 in., and the diameter at the surface of the molasses is 18 in.; what quantity of molasses does the barrel contain?
Ans. 7.9 gals., nearly.

2. How much wine is there in a cask standing upright, the height of the cask being 30 in., the height of the wine 14 in., the head diameter 15 in., and the diameter at the

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How do we find the quantity of liquor in a cask which is not full?
What is done when the liquor fills more than ½ of the cask?
How is the empty space found when casks lie on one side?

surface of the liquor 18 in., the staves being much curved? Ans. 13.7 gals., about.

3. A cask of water 29 in. long, when placed on one head, lacks 6 in. of being full; the head diameter is 15 in., the bung diameter 18 in., and the diameter at the surface of the water 16 in.; how many imperial gallons of water are there in it, if the staves are much curved? Ans. 19.6, about.

4. How many bushels of wheat are there in a hogshead standing upright, with staves moderately curved, the height being 46 in., the height of the wheat 25 in., the head diameter 26 in., the bung diameter 36 in., and the diameter at the surface of the wheat 34 in.? Ans. 9.9, about.

TONNAGE OF VESSELS.

LESSON 179.

The following arbitrary rule is employed, at present, to find the number of tons a vessel will carry.

Subtract $\frac{3}{5}$ of the breadth of the vessel from the length; then multiply the remainder, the breadth, and the depth together, and divide the product by 95.

To find the length, breadth, and depth. *The length* is measured from the fore part of the main stem to the after part of the stern post, above the upper deck; *the breadth* is measured at the broadest part above the main wales; *the depth* in single decked vessels is measured from the under side of the deck plank to the ceiling of the hold; but in double decked vessels $\frac{1}{2}$ of the breadth is called the depth.

Note. In some places ship carpenters, to find the number of tons, multiply the length of the keel, the breadth, and depth together, and divide the product by 95.

1. A ship carpenter built a schooner, which is a single decked vessel, on contract; the length was 84.4 ft., the breadth 24 ft., and the depth 10 ft.; how many tons were to be paid for? Ans. 176$\frac{80}{95}$.

Recite the arbitrary rule employed, at present, to find the number of tons a vessel will carry.

How do we find the length, breadth, and depth?

In some places how do ship carpenters find the number of tons?

2. What is the tonnage of a double decked vessel, the length being 106.2 ft., and the breadth 27 ft. ?
Ans. $345\frac{30}{95}$ tons.

3. What is the tonnage of a double decked vessel, the length being 150 ft., and the breadth 36 ft. ?
Ans. $875\frac{78}{95}$ tons, about.

4. How many tons will a single decked vessel carry, whose length is 75 ft., breadth 20 ft., and depth 9 ft. ?
Ans. $119\frac{35}{95}$.

5. What number of tons are there in a sloop, which is a single decked vessel, 60 ft. long, 18.5 ft. wide, and 7.2 ft. deep ? Ans. $68\frac{53}{95}$, about.

6. How many tons are there in a double decked vessel, the length being 190 ft., and the breadth 44 ft. ?
Ans. 1,667, nearly.

SQUARE ROOT.

Lesson 180.

If a number be multiplied by itself, the product is called the *square* of that number ; thus, 4 is the square of 2, 25 is the square of 5, &c.

That number, which multiplied by itself will produce a certain other number, is called the *square root* of this other number ; thus, 2 is the square root of 4, 5 the square root of 25, &c.

The square of a number is easily found, being obtained by multiplying the number by itself, but it is more difficult to get the square root of a number ; however, when the square root is a whole number, not exceeding 10, it is readily found by trying a few times.

Examples to be performed in the mind.

What is the square root of 49 ? Of 36 ? Of 64 ? Of 25 ? Of 81 ? Of 16 ? Of 100 ? Of 9 ? Of 4 ?

What is called the square of a number ? What is the square of 2 ? Of 5 ?

What is called the square root of a number ? What is the square root of 4 ? Of 25 ?

What is said of the ease of finding the square and square root of a *number ?*

When the square root of a number exceeds 10, this manner of obtaining it is tedious; for instance, if the square root of 169, of 256, of 361, or of 625, is required, we are obliged to try many times before we find a number which, multiplied by itself, will produce either of them. In order to discover a method of getting the square root with facility when it exceeds 10, we first observe how we find the square of a number more than 10, say of 27. To make the operation more plain, we separate 27 into two parts, 20 and 7.

OPERATION.

```
     20    7
     20    7
     --------
    140   49
400 140
-------------
400 280   49
```

Adding the several products.

```
 400
 280
  49
 ---
 729 square of 27.
```

Explanation. We see that the square of 27 consists of 20 times 20, of 20 times 7, of 7 times 20, and of 7 times 7.

Therefore, the square of a number containing two figures, consists of

The square of the tens, twice the product of the tens by the units, and the square of the units.

Now the square of 10 is 100, the square of 100 is 10,000, the square of 1,000 is 1,000,000, &c. It appears then, that the square of units is found in the two right hand figures, because the square of 10, or 100, is the smallest number possible consisting of three figures; the square of tens is found in the two next figures, because the square of 100, or 10,000, is the smallest number possible consisting of five figures. It can be shown in the same way, that the square of hundreds is found in the two figures at the left of the square of the tens; that the square of thousands is found in the two figures at the left of the square of hundreds, &c.

What if the square root of a number exceeds 10?

How do we proceed to discover a method of getting the square root with facility when it exceeds 10?

What does the square of 27 consist of?

What then does the square of a number containing two figures consist of?

Where is the square of units found? Why? Where is the square of tens found? Why? What else can be shown?

Let us find the square root of 729.

OPERATION.

$\dot{7}29$(27 Ans.

4

47)329

329

Explanation. The square of the units of the root is in 29, and the square of the tens in 7. The greatest square in 7 is 4, the square root of which is 2; this root we place at the right like a quotient, subtract 4, the square of 2, from 7, and bring down the 29. The remaining number, 329, contains twice the product of the tens by the units, and the square of the units; now twice any number of tens multiplied by units, gives nothing less than *tens*, so if we divide the 32 tens in 329 by twice the tens already obtained, or 4, we shall get the number of units, or too large a number, since the square of the units usually increases the tens a little. In fact, we get 8, which we place at the right of the 2, and also at the right of the 4 we divided by; we now multiply 48 by 8, which if 8 be right gives the square of the units and the product of twice the tens by the units. The result being 384, greater than 329, we conclude that 8 is too large, we rub it out, substitute 7, and multiplying 47 by 7, obtain 329; 7 then is right, and 27 is the square root of 729.

LESSON 181.

What is the square root of 1,447,209?

OPERATION.

$\dot{1}4\dot{4}7\dot{2}09$(1,203 Ans.

1

22) 44

44

2403) 7209

7209

Explanation. The square of the units of the root is in the two right hand figures; the square of the tens is in the two next figures, and so on. We therefore begin at the right, and divide 1,447,209 by dots into parts, of two figures each. We find the root of the two left hand parts, or of 144, just as before. This is evidently correct, since the square of the two first figures in the root, is contained in 144. To get the remaining part of the root, consider the 12 already obtained as the tens, bring down 72, and divide 7 by twice 12, or 24,

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Explain how you find the square root of 729.
Explain how you find the square root of 1,447,209.

which is contained in 7, 0 times; we place 0 then at the right of 12, and considering 120 as the tens, finish as before.

From what precedes, we derive the following rule for obtaining the square root;

Begin at the right, and separate the number, by dots, into parts of two figures each. Find the greatest square in the left hand part, write its root as you do a quotient, subtract the square from the left hand part, and bring down the two next figures at the right of the remainder for a dividend. Double the root already found for a divisor, and omitting the right hand figure of the dividend, find how many times the divisor is contained in the rest of it, and place the result at the right of the root already found, and also at the right of the divisor. Multiply the divisor thus increased, by the last figure in the root, subtract the product from the whole dividend, and bring down the two next figures, and so on; but if the product be greater than the dividend, diminish the last figure in the root until the product shall be equal or less, and then subtract it from the dividend.

When a divisor is not contained in a dividend, the right hand figure of which is omitted, write 0 in the root, and at the right hand of the divisor, and bring down the next two figures, and divide as before.

To prove the Square Root,

Multiply the root by itself, and the original number will evidently be produced if the work be right.

LESSON 182.

1. What is the square root of 256? Ans. 16.
2. What is the square root of 324? Ans. 18.
3. What is the square root of 97,476,129? Ans. 9,873.
4. What is the square root of 1,002,001? Ans. 1,001.
5. What is the square root of 998,001? Ans. 999.
6. A man bought a square house lot, containing 2,025 sq. ft.; how long was each side of it? Ans. 45 ft.

Recite the rule for obtaining the square root.

What if a divisor is not contained in a dividend, the right hand figure of which is omitted?

To prove the square root how do we proceed?

7. A farmer determined to set out 784 apple trees in an exact square; how many rows will he have, and how many trees in a row? Ans. 28 rows, and 28 trees in a row.

8. I have 2 pieces of land, each in the shape of an oblong square; one is 80 ft. by 37, and the other 32 ft. by 20; how long must the side of a square be to contain as much as the two pieces? Ans. 60 ft.

9. A general formed 4,624 men into an exact square; how many were there on one side of the square? Ans. 68.

10. I wish to put 1,250 sq. ft. of land into an oblong square, the length of which shall be twice the breadth; what will the breadth be? Ans. 25 ft.

Lesson 183.

1. What is the square root of 0.36?

OPERATION.

.0360(.189 about. Ans.
1
28) 260
224
369) 3600
3321
279 remainder.

Explanation. In the first place we observe that a number must contain twice as many decimals as its root; for the square of the root, that is, the root multiplied by itself, contains twice as many decimals as the root. See Decimal Fractions, lesson 80. The number must therefore contain two, four, six, eight, or ten, &c., decimals, and .036 having three, we make it contain four, by annexing a 0, which does not alter its value. See Decimal Fractions, lesson 75. It is evident we must now proceed to find the root as in whole numbers. After obtaining two figures of the root, we have a remainder 36, and annexing two 0s for a dividend, which amounts to the same thing as bringing them down, had they been originally placed at the right of .0360, we obtain another figure. We can get as many more in this way as we think fit. There are evidently three decimals in the root, which we have obtained, as there are six in the number, including those annexed to the remainder, 36.

Explain how example 1, lesson 183, is performed.

Therefore, when a number contains decimals, if they do not consist of two, four, six, or eight, &c., figures,

Annex a 0, and observe that there will be half as many decimals in the root, as in the number including the 0s annexed to the remainders.

2. What is the square root of 3.3?

OPERATION.

```
      3.30(1.816, about. Ans.
      1
      ---
  28) 230
      224
      ----
  361) 600
       361
       -----
 3626) 23900
       21756
       -----
        2144 remainder.
```

Explanation. There being but one decimal, we make another, and then get the root as in whole numbers, annexing two 0s to each remainder, and continuing the operation, until the root is sufficiently accurate.

A separating dot comes between decimals and whole numbers, as there always will, since the decimals are made to consist of two, four, six, or eight, &c., figures. Moreover, it is plain that as soon as decimals are brought down, we get decimals in the root, for the square of whole numbers never produces decimals, any more than the square of decimals produces whole numbers.

Many whole numbers do not have an exact square root in whole numbers; as 2, 3, 5, 6, 7, 8, &c.

What is done when a number contains decimals, if they do not consist of two, four, six, or eight, &c., figures?

How many decimals will there be in the root?

Explain how example 2, lesson 183, is performed.

Where will a separating dot always come? Why? When do we get decimals in the root? Why?

What is said of whole numbers having an exact root?

3. What is the square root of 2 ?

OPERATION.

2(1.414, about. Ans.
1
——
24)100
96
——
281) 400
281
——
2824)11900
11296
——
604 remainder.

Explanation. After finding the greatest root in 2, there is 1 remainder ; we can evidently carry the operation as far as proper by annexing two 0s as decimals, to each remainder, and proceeding just as we should were they originally placed at the right of 2.

Although we can obtain a root as near as desirable by annexing two 0s as decimals, to each remainder, and continuing the operation, still if the *exact* root is not found without adopting this course, it cannot be found by it ; for the last figure in each dividend will always be 0, from which the units in the square of the next figure in the root are to be subtracted, and as the square of no figure can produce a number with 0 in the units' place, there will always be a remainder.

To multiply a common fraction by itself, that is, to get the square of it, we square the numerator and denominator. See Common Fractions, lesson 71.

Therefore, to find the square root of a common fraction,

Find the square root of the numerator and denominator ; thus, the square root of $\frac{9}{16}$ is $\frac{3}{4}$, of $\frac{4}{9}$ is $\frac{2}{3}$, &c.

However, in many common fractions, like $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c., we cannot get the root of both the numerator and denominator in whole numbers. In such cases, and in mixed numbers, it is best to change the fractions to decimals first, and then get the root.

Explain how example 3, lesson 183, is performed.

What is said of obtaining an exact root by annexing two 0s as decimals to each remainder, &c. ?

How do we get the square of a common fraction ?

How do we find the square root of a common fraction ? What is the square root of $\frac{9}{16}$? Of $\frac{4}{9}$?

What course do we take with mixed numbers ; and what with common fractions, when we cannot get the root of both numerator and denominator in whole numbers ?

LESSON 184.

1. What is the square root of .0081 ? Ans. .09.
2. What is the square root of 628.195 ? Ans. 25.0638, about.
3. What is the square root of 895,372 ? Ans. 946.241, nearly.
4. What is the square root of $\frac{144}{256}$? Ans. $\frac{12}{16}$, or $\frac{3}{4}$.
5. What is the square root of $\frac{3}{16}$? Ans. .433, about.
6. What is the square root of $98\frac{1}{4}$? Ans. 9.912, about.
7. A square garden contains 6.25 sq. rods ; what is the length of one side of it ? Ans. 2.5 rods.
8. A field in the shape of an oblong square, 5 times as long as it is wide, contains 2,672.05 sq. ft. ; how many feet wide is it, and what is its length ?

Ans. width 23.117 ft., about ; length 115.585 ft., about.

9. A field containing 61,322 sq. ft. is $\frac{3}{4}$ as wide as it is long ; how wide is it, and what is its length ?

Ans. width 214.456 ft., about ; length 285.941 ft., about.

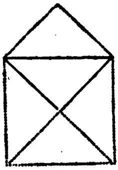
Explanation. It is $\frac{4}{3}$ as long as it is wide.

10. What is the square root of .00048 ? Ans. .0219, about.
11. There is a field 20.25 rods square, which I have agreed to exchange for another field equally large, but which shall be three times as long as it is wide ; what will be the length and breadth of the field ?

Ans. 35.073 ft. length, about ; 11.691 ft. breadth, about.

LESSON 185.

Figure 21.



If a triangle has a square corner, or right angle, the square of the side opposite the right angle will be equal to the sum of the squares of the two sides adjacent the right angle.

For, if the sides adjacent the right angle are equal, as in figure 21, we see that the square of one of them contains two triangles, and is half as large as the square of the side opposite the right angle, which contains four triangles of equal size. If the two sides ad-

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If a triangle has a square corner or right angle, how is the square of the side opposite the right angle to the sum of the squares of the two sides *adjacent the right* angle? Explain it.

jacent the right angle are not equal, it can also be shown that the sum of their squares is equal to the square of the side opposite the right angle.

1. A room is 18 ft. long, and 15 ft. wide; what is the distance between the opposite corners?
Ans. 23.431 ft., nearly.

2. The wall of a fort is 24 ft. high, and there is a ditch beside it 18 ft. wide; how long must a ladder be to reach from the outside of the ditch to the top of the wall?
Ans. 30 ft.

3. If the ladder be 20 ft. long, and the wall 16 ft. high, how wide is the ditch? Ans. 12 ft.

4. If the ladder be 25 ft. long, and the ditch 15 ft. wide, how high will the wall be? Ans. 20 ft.

5. A certain field, in the shape of an oblong square, is 40 rods long, and 36 rods wide; what is the distance between the opposite corners? Ans. 53.814 rods, about.

6. If you make a square with one side 6 ft. long, and the other 8 ft. long, what will be the length of a pole that will measure the distance from the end of one side to the end of the other? Ans. 10 ft.

Note. Carpenters usually make a square by fastening two pieces of wood together, one 6 ft. long, and the other 8 ft. long, and making the distance between the two ends 10 ft.

7. A carpenter building a house 24 ft. wide, wishes to have the gable end 12 feet high; how long must the rafters be? Ans. 16.97 ft., about.

Explanation. Make a figure of it on your slate.

8. What is the distance between the opposite corners of a square field, containing 2 A. 1 qr. 32 sq. rods?
Ans. 28 rods.

9. I wish to hew the largest square stick possible out of a log 16 in. in diameter; what size will the end of the square stick be? Ans. 11.3 in. square, about.

Explanation. The diameter of one end of the log is the distance between the opposite corners of the square.

10. If you have one pole 20 ft. long, and another 12 ft., how long must a third be, so that they may form a right angle when put together in the form of a triangle, the 20 ft. pole being opposite the right angle?
Ans. 16 ft.

CUBE ROOT.

LESSON 186.

If we multiply a number and its square together, the product is called the *cube* of that number ; thus, 8 is the cube of 2, 27 is the cube of 3, &c.

That number, which, multiplied by its square, will produce a certain other number, is called the *cube root* of this other number ; thus, 2 is the cube root of 8, 3 the cube root of 27, &c.

The cube of a number is easily found, being obtained by multiplying the number and its square together, but it is more difficult to get the cube root of a number ; however, where the cube root is a whole number, not exceeding 10, it is readily found by trying a few times.

Examples to be performed in the mind.

What is the cube root of 343 ? Of 216 ? Of 512 ? Of 125 ? Of 729 ? Of 64 ? Of 1,000 ? Of 27 ? Of 8 ?

When the cube root of a number exceeds 10, this manner of obtaining it is tedious ; for instance, if the cube root of 2,197, or of 4,096, is required, we are obliged to try many times before we find a number, which, multiplied by its square, will produce either of them. In order to discover a method of getting the cube root with facility when it exceeds 10, we first observe how we find the cube of a number more than 10, say of 27. To make the operation more plain, we multiply the square of 27 in three parts, as we found it in the Square Root, lesson 180, by 27 in two parts, 20 and 7.

What is called the cube of a number ? What is the cube of 2 ? Of 3 ?

What is called the cube root of a number ? What is the cube root of 8 ? Of 27 ?

What is said of the ease of finding the cube and cube root of a number ?

What if the cube root of a number exceeds 10 ? How do we proceed to discover a method of getting the cube root with facility, when it exceeds 10 ?

OPERATION.

	400	280	49
		20	7
	2800	1960	343
8000	5600	980	
8000	8400	2940	343

Explanation. 8,000 is the cube of 20, 8,400 is formed of the square of 20 multiplied by 7, and of the product of 2 times 20 by 7 multiplied by 20, making 3 times the square of 20 multiplied by 7 ; 2,940 is formed of the product of 2 times 20 by 7 multiplied by 7, and the square of 7 multiplied by 20, making 3 times 20 multiplied by the square of 7 ; 343 is the cube of 7.

Adding up the several products.

8000
8400
2940
343

19,683 cube of 27.

Therefore, the cube of a number containing two figures, consists of,

The cube of the tens, three times the square of the tens multiplied by the units, three times the tens multiplied by the square of the units, and the cube of the units.

Now the cube of 10 is 1,000, the cube of 100 is 1,000,000, the cube of 1,000 is 1,000,000,000, &c. It appears, then, that the cube of units is found in the three right hand figures, because the cube of 10, or 1,000, is the smallest number possible consisting of four figures ; the cube of tens is found in the three next figures, because the cube of 100, or 1,000,000, is the smallest number possible consisting of seven figures. It can be shown in the same way, that the cube of hundreds is found in the three figures at the left of the cube of tens ; that the cube of thousands is found in the three figures at the left of the cube of hundreds, &c.

Let us find the cube root of 19,683.

Explain what the cube of 27 consists of?

What then does the cube of a number, containing two figures, consist of?

Where is the cube of units found? Why? Where is the cube of tens found? Why? What else can be shown?

OPERATION.

19683 (27 Ans.	20	20 tens.	7
8	20	3	7
1200) 11683	400	60	49
11683	3	49 square of the units.	7
3 times the square of the tens	1200	540	343
	7	240	
	8400	2940	
	2940		
	343		
	11683		

Explanation. The cube of the units is in 683, and the cube of the tens in 19. The greatest cube in 19 is 8, the cube root of which is 2 ; this root we place at the right, like a quotient ; subtract 8, the cube of 2, from 19, and bring down the 683. The remaining number, 11,683, contains 3 times the square of the tens multiplied by the units, 3 times the tens multiplied by the square of the units, and the cube of the units ; now 3 times the square of any number of tens multiplied by units, gives nothing less than *hundreds ;* so if we divide the 116 hundreds in 11,683 by 3 times the square of the 2 tens, or 1,200, we shall get the number of units, or too large a number, since 3 times the tens multiplied by the square of the units, and the cube of the units, usually increase the hundreds considerably. In fact, we get 9, which, on the proper trial, is found too large, as well as 8. Let us try 7 ; placing it at the right of 2, to find whether it is right or not, we add together 3 times the square of the tens multiplied by 7, 3 times the tens multiplied by the square of 7, or 49, and the cube of 7, or 343 ; the sum making 11,683, the same as the remaining number, we conclude that 7 is right, and that 27 is the cube root of 19,683.

Explain how you find the cube root of 19,683.

LESSON 187

What is the cube root of 1,740,992,427 ?

OPERATION.

1740992427 (1,203 Ans.
1

300) 740
728

4320000) 12992427
12992427

Explanation. The cube of the units of the root, is in the three right hand figures ; the cube of the tens is in the three next figures, and so on. We therefore begin at the right, and divide 1,740,992,427, by dots, into parts of three figures each. We find the root of the two left hand parts, or of 1,740, just as before ; this is evidently correct, since the cube of the two first figures in the root is contained in 1,740. To get the remaining part of the root, consider the 12 already obtained as the tens, bring down 992, and divide 12,992 by 3 times the square of 12 tens, or 43,200, which is contained in 12,992, 0 times ; we place 0 then at the right of 12, and considering 120 as the tens, finish as before.

From what precedes, we derive the following rule for obtaining the cube root ;

Begin at the right, and separate the number, by dots, into parts of three figures each. Find the greatest cube in the left hand part, write its root as you do a quotient, subtract the cube from the left hand part, and bring down the next three figures at the right of the remainder, for a dividend. Each of the remaining operations to find the root, is as follows ; Multiply the square of 10 *times the root already found by* 3, *for a divisor, find how many times it is contained in the dividend, and place the result at the right of the root already found. Add together* 3 *times the square of the tens, hundreds, &c. in the root, multiplied by the units,* 3 *times the tens, hundreds, &c.,*

Explain how you find the cube root of 1,740,992,427.

multiplied by the square of the units, and the cube of the units; if the result be greater than the dividend, diminish the last figure in the root, until the result shall be equal, or less, and then subtract it from the dividend. Bring down the next three figures at the right of the remainder for a dividend, and proceed to get the next figure in the root as before, and so on.

When a divisor is not contained in a dividend, write 0 in the root, and bring down the next three figures, and divide as before.

To prove the Cube Root,

Multiply the root and the square of the root together, and the original number will evidently be produced if the work be right.

Lesson 188.

1. What is the cube root of 2,197? Ans. 13.
2. What is the cube root of 15,625? Ans. 25.
3. What is the cube root of 9,261? Ans. 21
4. What is the cube root of 2,924,207? Ans. 143
5. What is the cube root of 729,000? Ans. 90
6. What is the cube root of 164,359,469,195,483? Ans. 54,777.

7. What is the length of one side of a cubic block of wood containing 1,728 cubic inches? Ans. 12 in.

8. The quantity of water that passes over the dam across a certain stream, is 60 cubic ft. a second; what is the length of a cubic cistern that will contain what passes over in one hour? Ans. 60 ft.

9. If you have 13,824 cubic in. of lead, how large a cube will it make if melted together? Ans. a cube 24 in. square.

10. A lumber dealer has 110,592 cubic ft. of hewed timber; what will be the length of a cubic pile to contain the whole? Ans. 48 ft.

Recite the rule for obtaining the cube root.
What if a divisor is not contained in a dividend?
To prove the cube root how do we proceed?

Lesson 189.

1. **What is the cube root of .0365 ?**

OPERATION.

```
         .036500(.331 about.   Ans.
          27
          ----
  2700 )  9500
          8937
          ------
326700 )  563000
          327691
          ------
          235309 remainder.
```

Explanation. In the first place we observe that a number must contain three times as many decimals as its root, for the cube of the root contains as many decimals as the root and its square. The number must therefore contain three, six, nine, or twelve, &c. decimals, and .0365 having four, we make it contain six, by annexing two 0s which does not alter its value. See Decimal Fractions, lesson 75. It is evident we must now proceed to find the root as in whole numbers. After obtaining two figures of the root, we have a remainder 563, and annexing three 0s for a dividend, which amounts to the same thing as bringing them down had they been originally placed at the right of .036500, we obtain another figure. We can get as many more in this way as we think fit. There are evidently three decimals in the root which we have obtained, as there are nine in the number, including those annexed to the remainder, 563.

Therefore, when a number contains decimals, if they do not consist of three, six, nine, &c. figures,

Annex as many 0s as will make three, six, or nine, &c. decimals, and observe that there will be one third as many decimals in the root as in the number, including the 0s annexed to the remainders.

Explain how example 1, lesson 189, is performed.

What is done when a number contains decimals, if they do not consist of three, six, or nine, &c. figures?

How many decimals will there be in the root?

2. What is the cube root of 2.41 ?

OPERATION.

2.410 (1.34 about. Ans.
1
—
300)1410
1197
——
50700)213000
209104
———
3896 remainder.

Explanation. The number of decimals being two, we make it three, and then get the root as in whole numbers; annexing three 0s to each remainder, and continuing the operation until the root is sufficiently accurate. A separating dot comes between decimals and whole numbers, as there always will, since the decimals are made to consist of three, six, or nine, &c. figures. Moreover, it is plain that as soon as decimals are brought down, we get decimals in the root, for the cube of whole numbers never produces decimals, any more than the cube of decimals produces whole numbers.

Many whole numbers do not have an exact cube root in whole numbers, as 2, 3, 4, 5, 6, 7, 9, &c.

3. What is the cube root of 5 ?

OPERATION.

5 (1.71 nearly. Ans.
1
——
300)4000
3913
———
86700)87000
87211

Explanation. After finding the greatest root in 5, there is 4 remainder; we can evidently carry the operation as far as proper, by annexing three 0s, as decimals, to each remainder, and proceeding just as we should were they originally placed at the right of 5.

Although we can obtain a root as near as desirable by annexing three 0s, as decimals, to each remainder, and continuing the operation, still, if the *exact* root is not found without adopting this course, it cannot be found by it, for the last figure in each dividend will always be 0, from

---

Explain how example 2, lesson 189, is performed. Where will a separating dot always come? Why? When do we get decimals in the root? Why?

What is said of whole numbers having an exact cube root?

Explain how example 3, lesson 189, is performed.

What is *said of obtaining* an exact root by placing three 0s, as decimals, *after each remainder, &c.*?

which the units in the cube of the next figure in the root are to be subtracted, and as the cube of no figure can produce a number with 0 in the units' place, there will always be a remainder.

To get the cube of a common fraction, we cube the numerator and denominator. See Common Fractions, lesson 71.

Therefore, to find the cube root of a common fraction,

Find the cube root of the numerator and denominator; thus, the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, of $\frac{27}{64}$ is $\frac{3}{4}$, &c.

However, in many common fractions, like $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c., we cannot get the root of both numerator and denominator in whole numbers. In such cases, and in mixed numbers, it is best to change the fractions to decimals first, and then get the root.

LESSON 190.

1. What is the cube root of .000825 ? Ans. .0938, nearly.
2. What is the cube root of 27.98 ? Ans. 3.036, nearly.
3. What is the cube root of 1,601,618 ? Ans. 117, about.
4. What is the cube root of $\frac{512}{2197}$? Ans. $\frac{8}{13}$.
5. What is the cube root of $\frac{7}{20}$? Ans. .7048, nearly.
6. What is the cube root of $881\frac{1}{3}$? Ans. 9.588, nearly.
7. If the earth, or globe, be 7,920 miles in diameter, how large a cube will it make ?
   Ans. a cube $6,383\frac{1}{2}$ miles square, nearly.
8. A block of wood, the ends of which are square, and the length twice the breadth, or depth, contains 13.718 cubic ft. ; what is its breadth, or depth, and length ?
   Ans. breadth or depth, 1.9 ft., length 3.8 ft.
9. I have 81 cubic inches of lead ; if I melt it, how large a block will it make, the base of which shall be square, and the height $\frac{8}{9}$ of its length or breadth ?
   Ans. a block $4\frac{1}{2}$ in. square at the base, and 4 in. high.

How do we get the cube of a common fraction ?

How do we find the cube root of a common fraction ? What is the cube root of $\frac{8}{27}$? Of $\frac{27}{64}$?

What course do we take with mixed numbers; and what with *common fractions* when we cannot get the cube root of both *numerator and denominator* in whole numbers ?

Explanation. The block will contain $\frac{8}{9}$ as much as a cubic block of the same length and breadth.

10. What is the cube root of .00064? Ans. .04.

11. There is a pile of wood 4 ft. wide, 6 ft. high, and 21⅓ ft. long; how large a cubic pile will it make?
Ans. a cubic pile 8 ft. square.

SPECIFIC GRAVITY.

Lesson 191.

The specific gravity of any substance is the ratio of its weight to the weight of an equal bulk of fresh water; thus, if the substance weighs 2 oz., and an equal bulk of water 1 oz., its specific gravity is 2; if it weighs 1 oz., and an equal bulk of water 2 oz., its specific gravity is ½, or .5. See Promiscuous Questions after Fractions, lesson 85.

Therefore, to get the specific gravity of any substance,

Divide its weight by the weight of an equal bulk of fresh water.

A cubic foot of pure fresh water, properly distilled, weighs 1,000 ounces, avoirdupois, at the temperature of melting ice.

The table on the following page shows the specific gravity of several of the most common and important substances. It is formed by dividing the weight of a cubic foot of each substance, in ounces, by 1,000. If we omit the point, we get the weight of a cubic foot of each substance in ounces.

Note. It is not necessary to commit the table to memory.

What is the specific gravity of any substance? What if the substance weighs 2 oz., and an equal bulk of water 1 oz.? What if it weighs 1 oz., and an equal bulk of water 2 oz.?
How then do we get the specific gravity of any substance?
What is said of the weight of a cubic foot of water?
How is the table formed?

METALS.

Platina, pure,	19.500
Platina, hammered,	21.500
Gold, pure and cast,	19.260
Gold, hammered,	19.360
Mercury,	13.560
Lead, cast,	11.350
Silver, pure and cast,	10.470
Silver, hammered,	10.510
Copper, cast,	8.790
Copper, hammered,	8.890
Brass, cast,	8.400
Brass, hammered,	8.500
Iron, cast,	7.210
Iron, hammered,	7.790
Steel,	7.840
Tin, cast,	7.300
Zinc, cast,	7.200

STONES, EARTHS, &c.

Granite,	2.700
Marble,	2.700
Slate,	2.700
Glass,	2.600
Flint stone,	2.580
Paving stone,	2.580
Free stone,	2.500
Clay,	2.200
Sand,	1.500
Anthracite coal, from	1.400
to	2.000
Bituminous coal, from	1.100
to	1.300
Brick, from 1.800 to	2.000

WOODS, &c.

Lignum Vitæ,	1.300
Ebony,	1.200
Hempen rope, or cable,	1.100
Mahogany,	1.000
Boxwood,	1.000
Shell bark hickory,	1.000
White oak, heart,	.930
Ash,	.800
Rock maple,	.760
White pine,	.570
Charcoal,	.400
Cork,	.240

LIQUIDS, &c.

Sulphuric acid,	1.840
Nitric acid,	1.220
Sea water,	1.030
Cow's milk,	1.030
Pure fresh water,	1.000
Whale oil,	.920
Tallow,	.920
Olive oil,	.910
Proof spirit,	.920
Alcohol,	.840

GASES.

Oxygen gas,	.09134
Carbonic acid gas,	.00164
Common air,	.00122
Nitrogen gas,	.00098
Hydrogen gas,	.00008

Note. The specific gravity of any solid, liquid, or gas, increases with the cold, and diminishes with the heat. Moreover, there is always some difference in the specific gravity of several varieties of the same substance.

1. I have 16 round bars of hammered copper, 12 ft. long and $\frac{1}{10}$ of a foot in diameter; what do they all weigh if we reckon the circumference 3 times the diameter?

Ans. 800 lbs. 1.6 oz.

Explanation. Find the weight of a cubic foot in the table.

What does the specific gravity of any solid, liquid, or gas, increase *with? Diminish* with? Is the specific gravity of several varieties of *the same* substance always the same?

2. A man melted a number of pieces of lead, and cast them in a prism 1.5 ft. long, 1 ft. high, and .75 ft. wide; what did it weigh? Ans. 798 lbs. $\frac{3}{4}$ oz.

3. There is a dish of mercury, or quicksilver, 8 inches square on the top, 6 inches square on the bottom, and 4 inches deep; how much mercury, by weight, does it contain? Ans. 96 lbs. $12\frac{1}{2}$ oz., about.

4. How large a cube will 58,400 ounces of tin make?
Ans. a cube 2 ft. square.

5. There is a monument in a grave-yard composed of black marble, in the shape of a cone, 4 ft. in diameter at the base, and 7 ft. high; what is its weight?
Ans. 4,948 lbs., about.

6. What is the weight of a thousand heavy bricks, each being 8 inches long, 4 inches wide, and 2 inches thick?
Ans. 4,629 lbs. 10 oz., about.

7. What is the weight of a coil of 5 inch rope, that is, of rope 5 inches in circumference, containing 45 fathoms, if we reckon the circumference 3 times the diameter?
Ans. 268 lbs. 9 oz., nearly.

8. What is the weight of a ball of white oak 1 ft. in diameter? Ans. 30 lbs. $6\frac{8}{10}$ oz., about.

9. What is the weight of a barrel of sea water, the length being 36 inches, head diameter 18 inches, and bung diameter 21 inches, the staves being quite curving?
Ans. 421 lbs., about.

10. There is an iron pipe 5 inches in diameter on the outside, and 4 inches on the inside; how much does a piece 100 feet long weigh? Ans. 2,212 lbs., nearly.

Lesson 192.

1. There is a bottle which weighs 6 oz.; when filled with water it weighs 36 oz., and when filled with olive oil it weighs 33 oz. $7\frac{2}{10}$ drams; what is the specific gravity of the oil? Ans. .915.

2. A bottle weighs 4 oz. when empty, 20 oz. when filled with water, and 33 oz. 7 drams when filled with sulphuric acid; what is the specific gravity of the acid?
Ans. 1.840, nearly.

3. There is a piece of dry white pine in the shape of a wedge, weighing 35 oz., the length is 1 ft., breadth 6 inches, and the thickness of the head 3 inches; what is its specific gravity? Ans. .560.

4. A large glass bottle, which is found to contain 98 cubic inches of water, weighs 1 lb. when the air is taken out of it by an air pump; when the air is admitted it weighs 1 lb. 30 grains, and when filled with hydrogen gas it weighs 1 lb. 2 grains; what then does a cubic foot of air weigh, what does a cubic foot of hydrogen gas weigh, and what is the specific gravity of air, and of hydrogen gas, as found by the experiment?

Ans. a cubic ft. of air weighs 529 grs., nearly; a cubic ft. of hydrogen gas weighs 35.27 grs., nearly; the specific gravity of air is .00121, nearly, and of hydrogen gas, .00008, about.

5. There is an ingot composed of gold and copper, the specific gravity of which is 17.800; in what proportion are the gold and copper mixed?

Ans. 9,010 of gold, to 1,460 of copper.

Explanation. See Alligation Alternate. Consider the two metals as cast.

6. I wish to make a gallon of proof spirit from some alcohol, or rectified spirit of wine, and some pure water; what quantity of each must I take?

Ans. ½ of a gallon of each.

7. A man has 16 gallons of French brandy, consisting of 10 gallons of alcohol, and 6 gallons of pure water; what is its specific gravity? Ans. .900.

8. How much water must be mixed with 4 gallons of rum, the specific gravity of which is 890, to reduce it to proof spirit? Ans. 1½ gallon.

LESSON 193.

To find the specific gravity of a small irregular shaped body, heavier than water.

First find its weight, then find how much it weighs when suspended by a string fastened to one of the scales of an accurate balance, the body being immersed in pure cold water. In this position it will evidently lose just the weight of an equal bulk of water; because, if it was of the same specific gravity as the water, it would be just buoyed up by it, and lose all its weight.

Now to find its specific gravity,

Divide its weight by the weight lost.

How do we proceed to find the specific gravity of a small irregular body heavier than water?

To find the specific gravity of a small irregular body lighter than water.

First find its weight, then take a piece of lead, or other substance, sufficiently heavy to sink the body in water when attached to it; find how much the lead weighs when immersed in water; then attach the body to it, and immersing both, find the weight, and subtract it from the weight of the lead in the water. The difference added to the weight of the body evidently gives the weight of an equal bulk of water.

Now to find the specific gravity,

Divide the weight of the body by the weight of an equal bulk of water.

We can also find the specific gravity of a small irregular shaped body by dividing its weight by the weight of a quantity of water of the same bulk. The bulk can be found by immersing it in water, as directed in Mensuration, lesson 172. This method is not so accurate as the preceding ones, especially if the body be quite small.

1. What is the specific gravity of a piece of gold money that weighs 216 grains out of the water, and 204 grains when immersed; and if the piece was formed by alloying pure gold with pure silver, what quantity of each ingredient is there in it?

Ans. the specific gravity is 18, and it contains 183 grs. of gold, and 33 grs. of silver, about.

2. A piece of pine charcoal weighs 437.5 grs., and a piece of lead weighs, when immersed in water, 956.25 grs.; attaching the charcoal to the lead and immersing them, they weigh 300 grs.; what is the specific gravity of the charcoal? Ans. .400.

3. If a piece of marble weighs 180 pounds, and you find the bulk by immersing it in water, as directed in Mensuration, lesson 172, to be 1 cubic foot, what is its specific gravity? Ans. 2.880.

How do we proceed to find the specific gravity of a small irregular body lighter than water?

How, also, can we find the specific gravity of a small irregular body? What is said of the accuracy of this method?

SIMPLE MACHINES,

OFTEN CALLED

MECHANICAL POWERS.

LESSON 194.

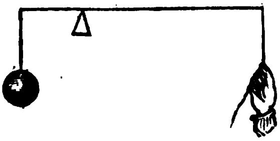
There are usually reckoned six simple machines; the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *screw*, and the *wedge*.

The force that raises a weight, or overcomes a resistance, is called the *power*. The power is usually the force of men, oxen, horses, moving water, wind, &c.

THE LEVER.

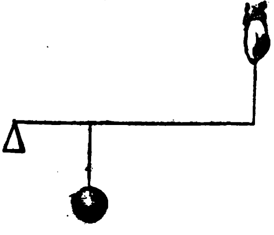
The support or prop round which the lever moves, is called the *fulcrum*.

Figure 22.



There are three kinds of levers. In the first kind the fulcrum is between the power and the weight, as in figure 22.

Figure 23.



In the second kind the weight is between the power and the fulcrum, as in figure 23.

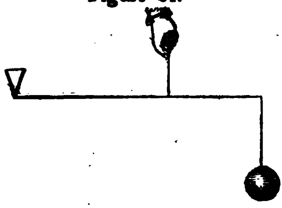
How many simple machines are there usually reckoned? Name them.

What is called the power? What is usually the power?

What is called the fulcrum?

How many kinds of levers are there? Explain them?

Figure 24.



In the third kind the power is between the weight and fulcrum, as in figure 24.

When these levers are extremely light, experiment shows that in order to sustain a weight with either of them,

The power must be to the weight in the same proportion as the distance from the fulcrum to the weight is to the distance from the fulcrum to the power.

To be performed in the mind.

1. In what proportion must the power be to the weight with the first kind of lever, in order to balance the weight, if the fulcrum is placed in the centre of the lever? If the weight be 10 lbs., how many pounds must the power be equal to?
2. A lever of the second kind is 6 ft. long, and the weight is placed 3 ft. from the fulcrum; in what proportion must the power be to the weight in order to sustain it? If the weight be 12 lbs., how much must the power lift?
3. A lever of the third kind is 8 ft. long, and the power is applied 4 ft. from the fulcrum; in what proportion must the power be to the weight to sustain it? If the weight be 13 lbs., how much must the power lift?
4. A lever of the first kind is 7 ft. long, and the fulcrum is 2 ft. from the weight; what proportion must the power bear to the weight to balance it?
5. A lever of the second kind is 10 ft. long, and the weight is placed 3 ft. from the fulcrum; in what proportion must the power be to the weight in order to sustain it?
6. A lever of the third kind is 9 ft. long, and the power is applied 4 ft. from the fulcrum; in what proportion must the power be to the weight in order to sustain it?

LESSON 195.

To be performed in the mind.

1. A lever of the first kind is 6 ft. long, and the fulcrum is 2 ft. from the weight; what proportion must the power

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When these levers are extremely light, what does experiment show?

bear to the weight in order to balance it? What part of the weight must the power be? If the weight be 42 lbs., how many pounds must the power be equal to? If the power be 9 pounds, what weight will it balance?

2. A lever of the second kind is 15 ft. long, and the weight is 5 ft. from the power; in what proportion must the power be to the weight to sustain it? What part of the weight must the power be? If the weight be 60 lbs., how many pounds must the power lift? If the power lift 12 lbs., how many pounds will it sustain?

3. A lever of the third kind is 12 ft. long, and the power is 9 ft. from the fulcrum; what proportion must the power bear to the weight in order to sustain it? What part of the weight must the power be? If the weight be 72 lbs., how many pounds must the power lift? If the power be 12 lbs., what weight will it sustain?

4. There is a lever of the first kind, 8 ft. long; what weight will 12 lbs. as a power, on the end of it, balance, if the fulcrum be placed 2 ft. from the weight?

5. A lever of the second kind is 9 ft. long, and a power can be applied that will lift 10 lbs.; what weight, placed 3 ft. from the fulcrum, will the power sustain?

6. A lever of the third kind is 7 ft. long, and a power can be applied 4 ft. from the fulcrum, that will lift 14 lbs.; what weight placed at the end of the lever will it sustain?

7. There is a lever of the first kind 6 ft. long, the power is equal to 10 lbs., and the weight is 30 lbs.; where must the fulcrum be placed, so that the power shall balance the weight?

8. A lever of the second kind is 12 ft. long, the power amounts to 7 lbs., and the weight is 28 lbs.; where must the weight be placed, so that the power shall sustain it?

9. A lever of the third kind is 8 ft. long, the power amounts to 16 lbs., and the weight is 14 lbs.; where must the power be placed, so as to sustain the weight?

10. If the weight is suspended 1 inch from the fulcrum in a steelyard, at what distance from the fulcrum must a poise weighing 1 lb. be placed, to balance a weight of 1 lb.? A weight of 2 lbs.? $2\frac{1}{2}$ lbs.? 3 lbs.? 11 lbs.?

If the first notch is 1 inch from the fulcrum, and the notches are $\frac{1}{16}$ of an inch apart, how many pounds and ounces will the poise balance, when placed in the 2d notch? 3d notch? 4th notch? 5th notch? 8th notch? 16th notch? 64th notch? 72d notch? 80th notch?

LESSON 196.

For the Slate.

1. A weight of 2,000 lbs. is on the end of a lever of the first kind, 1 ft. from the fulcrum; how many pounds as a power must be placed 10 ft. from the fulcrum to balance the weight? Ans. 200.

2. A lever of the second kind being 12 ft. long, where must a weight of 720 lbs. be placed, so that a power capable of lifting 50 lbs. shall sustain it?
Ans. 10 in. from the fulcrum.

3. A lever of the third kind is 15 ft. long, with a weight at the end of 90 lbs.; if a power be applied $1\frac{1}{2}$ ft. from the weight, how many pounds must it be capable of lifting in order to sustain the weight? Ans. 100.

4. A lever of the first kind, 11 ft. long, has a weight of 1,500 lbs. at one end, and a power equal to 150 lbs. can be applied at the other; where must the fulcrum be placed so that the power shall balance the weight?
Ans. 1 ft. from the end.

5. I wish to sustain $933\frac{1}{3}$ lbs. on a lever of the second kind, 14 ft. long, the fulcrum of which is 3 ft. from the weight; what power must I employ?
Ans. a power capable of raising 200 lbs.

6. If you have a lever of the third kind 10 ft. long, with a weight at the end of 145 lbs., where must you apply a power equal to 900 lbs. to sustain the weight?
Ans. 1 ft. $7\frac{1}{3}$ in. from the fulcrum.

7. If the place of suspending the weight in a steelyard be 2 inches from the fulcrum, what weight will a poise weighing 4 lbs. balance, if placed 3 ft. from the fulcrum?
Ans. 72 lbs.

8. The place of suspending the weight in a steelyard being $1\frac{1}{2}$ inches from the fulcrum, at what distance from the fulcrum must a poise weighing 4 lbs. be placed, so as to balance a weight of 56 lbs.? Ans. 1 ft. 9 in.

Observations on the Lever. We have regarded the lever as very light; the weight of it, however, in practice, will increase or diminish the power a little. By examining figures 22, 23, and 24, lesson 194, we see that the weight of the lever of the first kind, will generally increase the

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What *will the weight* of the lever do in practice? What do we see *by examining figures* 22, 23, and 24, lesson 194?

power, and that the weight of the levers of the second and third kinds, will generally diminish the power.

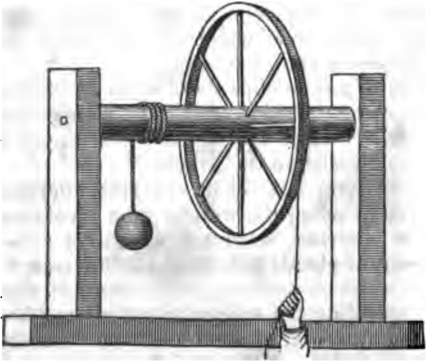
When the power and weight balance each other, the power must be increased a little in order to raise the weight, as there is some rubbing or friction on the fulcrum.

The lever is a very important instrument, and is used in a great many different forms; when you employ your weight on the end of an iron bar to overturn a log, the bar is a lever of the first kind; but if you place your shoulder beneath the bar to roll over the log, the bar is a lever of the second kind. A man who loads hay with a pitchfork, employs a lever of the third kind, one hand being the fulcrum, and the other the power. Common tongs are double levers of the third kind; blacksmiths' tongs are double levers of the first kind, the fulcrum being at the pivot.

LESSON 197.

THE WHEEL AND AXLE.

Figure 25.



The principle of the lever of the first kind, is employed in the wheel and axle; for the pivot, or axis, acts as a fulcrum, the radius of the axle is the distance of the weight from the fulcrum, and the radius of the wheel is the distance of the power from the fulcrum.

Therefore, to balance the weight,

The power must be to the weight as the radius of the axle is to the radius of the wheel.

When the power and weight balance each other, what must be done in order to move the weight? Why?

What is an example of a lever of the first kind? Second kind? Third kind? What are common tongs? Blacksmith's tongs; and where is the fulcrum?

What principle is employed in the wheel and axle? Explain how.

How must the power be to the weight in order to balance it?

1. What power must be applied to the circumference of a wheel 6 ft. in diameter, to balance 350 lbs., suspended from the axle, which is .5 of a foot in diameter ? Ans. 29 lbs. $2\frac{2}{3}$ oz.

2. The diameter of the wheel being 7 ft., and that of the axle being 9 in., what weight will a power equal to 12 lbs. balance ? Ans. 112 lbs

3. What must the diameter of a wheel be so that a power equal to 40 lbs. may balance a weight of 500 lbs., the axle being 1 ft. in diameter ? Ans. $12\frac{1}{2}$ ft.

4. What must the diameter of the axle be, so that a power equal to 30 lbs. may balance a weight of 270 lbs., the wheel being 9 ft. in diameter ? Ans. 1 ft.

5. There are 2 axles, side by side, 1 ft. in diameter, each of which has a wheel 10 ft. in diameter ; a belt passes round the first axle and the second wheel ; how many pounds on the second axle, will a power equal to 25 lbs. on the first wheel balance ? Ans. 2,500 lbs.

6. A wheel is 9 ft. in diameter, and the axle 10.8 in., the power is equal to 38 lbs., and the weight is 310 lbs.; which will overcome in this machine, the weight or the power ? Ans. The power.

Observations on the Wheel and Axle. When the power and weight balance each other, the power must be increased a little in order to overcome the friction on the axis, and raise the weight ; the power may also be diminished a little before the weight will overcome the friction and descend.

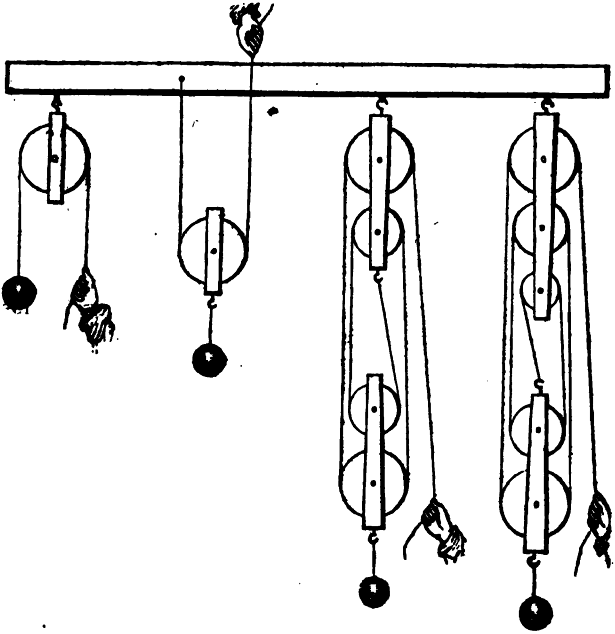
The wheel and axle are used in a great many different forms ; the crank of the windlass with which water is sometimes drawn, turns round in a circle, and acts as a wheel, while the body of the windlass acts as an axle ; the windlass of a vessel acts as a wheel and axle ; the capstan of a vessel is, in effect, a wheel and axle, where the wheel, represented by the levers, is horizontal, &c.

What is said of friction in the wheel and axle ?
Name some of the forms in which the wheel and axle are used ?

Lesson 198.

THE PULLEY.

Figure 26. Figure 27. Figure 28. Figure 29.



A pulley with an immovable block, like figure 26, is called a *fixed pulley.* A pulley with a movable block, like figure 27, is called a *movable pulley.* With one fixed pulley, like figure 26, the power must evidently be equal to the weight in order to balance it; this pulley merely serves to change the direction in which the power operates. With one movable pulley, like figure 27, the weight must be 2 times the power in order to be balanced, for the weight is

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What is called a fixed pulley? A movable pulley? With one fixed pulley, how is the power to the weight? What does the fixed pulley *serve for?* With one movable pulley, how is the power to the weight? *Why?*

supported by 2 ropes. In figure 28, we see that the weight is supported by 4 ropes, and is evidently 4 times the power required to balance it. In figure 29, we see that the weight is supported by 5 ropes, and is evidently 5 times the power required to balance it.

Therefore, to find what weight the power will balance,

Multiply it by as many ropes as support the weight.

1. What weight will a power equal to 80 lbs. balance by means of pulleys, the weight being supported by 4 ropes ? Ans. 320 lbs.

2. There are 3 movable pulleys, and one end of the rope is attached to the block which contains them ; what power will be necessary to balance a weight of 420 lbs. Ans. 60 lbs.

3. A system of pulleys support the weight by means of 8 ropes, and the power is applied through an axle, .8 of a foot in diameter, to which there is a wheel 8 feet in diameter ; what power at the wheel will balance a weight of 5,000 lbs. Ans. 62½ lbs.

Observations on the Pulley. When the power and weight balance each other, the power must be increased in order to raise the weight, ⅓ of the power being usually lost in overcoming friction ; the power may also be diminished about ⅓ before the weight will overcome the friction and descend.

4. What power operating on pulleys will be necessary to raise a weight of 360 lbs. supported by 6 ropes ? Ans. 90 lbs.

5. What weight will a power equal to 50 lbs. raise by means of 2 movable pulleys, one end of the rope being fastened to the block which contains them ? Ans. 166⅔ lbs.

6. There is a power equal to 30 lbs. and a weight of 200 lbs. ; if one end of the rope is fastened to the block of the movable pulleys, how many movable pulleys must there be so that the power shall raise the weight ? Ans. 5.

What do we see in figure 28, lesson 198? In figure 29, lesson 198?

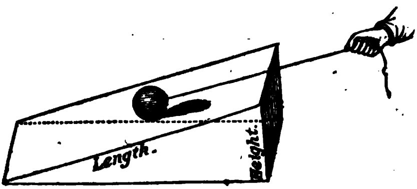
What is the rule to find what weight the power will balance?

What is said of friction in the pulley ?

LESSON 199.

THE INCLINED PLANE.

Figure 30.



If we place a very smooth body on an inclined plane, very smooth and slippery, we find that to hinder the body from moving down the plane,

A power must be employed, parallel to the plane, which shall be to the weight of the body as the height of the plane is to its length.

1. There is an inclined plane 8 ft. long, and 3 ft. high; what weight will be balanced on it by a stone weighing 12 lbs., acting by means of a line over a fixed pulley, at the top of the plane ? Ans. 32 lbs.

2. What power will be necessary to hinder an iron cylinder, weighing 600 lbs., from rolling down a plane, the length of which is 25 ft., and the height 5 ft. ? Ans. 120 lbs.

3. There is a platform 39 ft. long, one end of which is to be raised so that a power equal to 100 lbs. shall just sustain a wagon weighing 975 lbs, on it ; how high must the end be raised ? Ans. 4 ft.

4. There is an inclined plane, 90 ft. long, and 12 ft. high, on which is a wagon weighing, with its load, 6,000 lbs. ; what power applied through a system of pulleys supporting the weight by 8 ropes, will sustain the wagon on the plane ? Ans. 100 lbs.

Observations on the Inclined Plane. When the power and weight balance each other, the power must be increased a little, in order to overcome the friction, and cause

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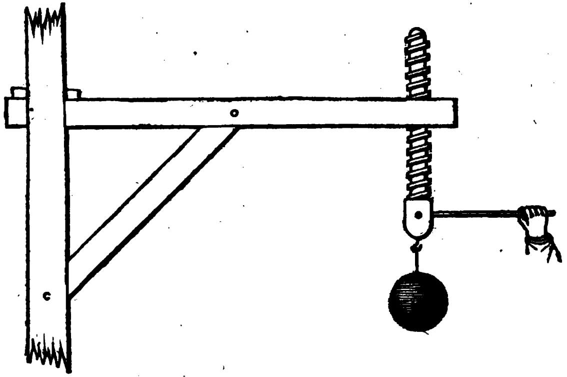
If we place a very smooth body on an inclined plane, very smooth and slippery, what do we find?
What is said of friction on an inclined plane?

the weight to move up the plane. The power may also be diminished a little, before the weight will overcome the friction and descend.

Lesson 200.

THE SCREW.

Figure 31.



The screw consists of two parts, the *external* screw and the *internal* screw. The external screw is a cylinder with a spiral projection, called the *thread*, coiled round it. This thread fits into a groove, cut in the internal screw. The thread is an inclined plane, and operates like one. It rises in passing round the screw the distance between the tops of the coils or turns. When the thread is triangular, this distance is evidently the same as that between the coils. Therefore, in order that the screw may balance the weight and sustain it without turning, we must apply to the thread a power which shall be to the weight as the distance between the tops of the coils is to the length of one turn or revolution of the thread. Now if the power is applied at the end of a lever, we may imagine the thread to extend to the end of the lever, and then it is obvious that

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What does the screw consist of? What is the external screw? Into what does this thread fit? What is the thread, and how does it operate? How much does it rise in passing round the screw? When the thread is triangular, what is this distance the same as? What is necessary *in order that a screw* may balance the weight and sustain it without *turning?*

To balance the weight,

The power must be to the weight as the distance between the tops of the coils is to the circumference described by the lever.

1. A certain screw has a triangular thread, the coils of which are 1 inch apart; what weight will be balanced on it by a power equal to 40 lbs., applied at the end of a lever 4 ft. from the centre of the screw?

Ans. 12,063.7 lbs., about.

Note. The circumference described by the lever, in example 1, is a mere trifle more than the circumference of a circle of which the lever is the radius, because the lever rises 1 inch, in turning round.

2. If a screw has a square thread with the tops of the coils $\frac{4}{5}$ of an inch apart, and a lever 36 in. long, what power applied to the lever will balance a weight of 1,500 lbs.? Ans. 5 lbs. $4\frac{9}{10}$ oz., about.

3. There is a weight of 7,540 lbs. to be sustained by a screw, to which a power equal to 25 lbs. can be applied by means of a lever 6 ft. long; the thread being square, what distance must the tops of the coils be apart?

Ans. $1\frac{1}{2}$ inch, nearly.

4. There is a downward pressure of 5,000 lbs. to be exerted by a power equal to 40 lbs., through the medium of a screw having a triangular thread the coils of which are $\frac{16}{100}$ of a ft. apart; how long must the lever be?

Ans. 3.18 ft., about.

Observations on the Screw. When the power balances the weight, the power must be increased in order to raise the weight; $\frac{1}{4}$ of the power being usually lost in overcoming friction. The power may also be diminished about $\frac{1}{3}$ before the weight will overcome the friction and descend.

5. There is a weight of 1,508 lbs. to be raised 5 ft. high by means of a screw which has a lever 5 ft. long, and a square thread with the tops of the coils $\frac{3}{4}$ of an inch apart; what power will be necessary? Ans. $4\frac{1}{2}$ lbs.

6. What weight will a power equal to 35 lbs. raise by means of a screw with a lever 4 ft. long, the thread being triangular, and the coils $1\frac{1}{3}$ inch apart?

Ans. 5,277.9 lbs., nearly.

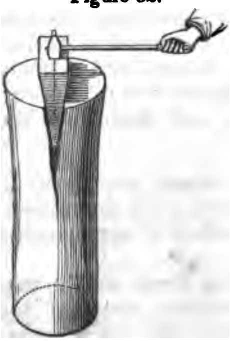
If the power is applied at the end of a lever, what is necessary to balance the weight?

What is said of the friction of the screw?

LESSON 201.

THE WEDGE.

Figure 32.



If you hold the weight in figure 30 firmly, and raise it by pushing along the inclined plane, the plane becomes a wedge; now to balance the weight, the power must evidently be to the weight as before, that is, *as the height of the plane is to its length,* or as the thickness of the head of the wedge is to the length of the slanting side. The only difference will be in the greater friction, and difficulty in moving the plane or wedge instead of the weight. A common wedge is two inclined planes placed together, as the dotted line in figure 32 represents.

Therefore, to balance the resistance,

The power must be to the resistance, as the thickness of the head is to the length of both the slanting sides.

1. There is a wedge with a head 2 in. thick, the length of each side being 10 in.; what amount of resistance will a power equal to 50 lbs. applied to the head balance? Ans. 500 lbs.

2. What power applied to a wedge 3 in. thick at the head, and 12 in. long on one side, will balance a resistance equal to 1,800 lbs.? Ans. 225 lbs.

3. A power equal to 80 lbs. is to be exerted on a wedge to balance a resistance amounting to 1,200 lbs.; what must the thickness of the head be, the length of a side measuring 15 in.? Ans. 2 in.

Observations on the Wedge. Friction greatly modifies the operation of the wedge; thus, if the power be a weight

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Explain how the inclined plane in figure 30 becomes a wedge. Now to balance the weight, how must the power evidently be to the weight? What will the only difference be? What is a common wedge?

To balance the resistance, how must the power be to the resistance?

placed on the head of a common wedge, it often will not overcome $\frac{1}{10}$ part as much resistance as we should expect from the preceding proportion, whereas if the wedge be struck, it many times will overcome 10 times this resistance.

The operation of an ordinary wedge, therefore, can only be estimated or guessed at by those accustomed to its use. The *principle* of the wedge, however, is employed in many machines, like presses used in printing and manufacturing, in which friction operates regularly, and diminishes the effect of the power about $\frac{1}{3}$.

By different combinations of these simple machines, all others, however complicated, are formed; and the preceding rules enable us to calculate the effect of any machine whatever.

There is no power gained by using these simple machines, or any machine; on the contrary, some of the power is always lost; still, they enable us to perform what would be difficult or impossible from a direct application of the power. For instance, a man cannot lift a millstone, but by working 20 minutes with a screw and lever, he can raise it several feet; the sum of his exertions, however, is about $\frac{1}{3}$ greater than the power employed by 20 men who lift it the same height in 1 minute, because $\frac{1}{3}$ of his labor is expended in overcoming friction.

What thing greatly modifies the operation of the wedge? Explain this modification.

How can the operation of an ordinary wedge, therefore, be obtained? In what is the *principle* of the wedge employed? How does friction operate in them?

What are formed by different combinations of these simple machines? What do the preceding rules enable us to calculate?

Is there any power gained by the employment of these simple machines, or of any machine? What on the contrary is true? What do they enable us to perform? Give an instance. What is said of the sum of his exertions?

PROMISCUOUS QUESTIONS

IN

MENSURATION, SQUARE ROOT, CUBE ROOT, &c.

LESSON 202.

To be performed in the mind.

1. 729 cubic ft. of clay were dug out of a cubic pit; what was the depth of the pit?

2. How many square yards of carpeting will cover a floor 20 ft. long and 18 ft. wide?

3. I bought 8,100 sq. ft. of land, to be laid out in an exact square; how long will one side be?

4. A lever of the first kind is 12 ft. long, and the fulcrum is 2 ft. from the weight; what power applied to the lever, will balance a weight of 60 lbs.?

5. A weight of 100 lbs. is to be raised by a wheel and axle, the axle being 6 in. in diameter; what must the diameter of the wheel be so that a power equal to 10 lbs. shall balance the weight?

6. What weight can I raise by means of pulleys with a power of 40 lbs., the weight being supported by 6 ropes?

7. What power will balance an iron cylinder weighing 70 lbs. on an inclined plane, the length of which is 12 ft., and the height 3 ft.?

8. There is a circle 12 ft. in diameter; if you reckon the circumference as 3 times the diameter, how many square feet will it contain?

9. A marble monument in the shape of a pyramid is 2 ft. square at the base, and 6 ft. high; how many cubic feet does it contain?

10. How many square rods are there in a triangular piece of land, the base of which is 10 rods, and the altitude 8 rods?

11. What is the square root of 400?

12. What is the cube root of 125,000?

LESSON 203.

1. There is a block of granite in the shape of a frustum of a square pyramid, the top is 2 ft. square, the base 5 ft. square, and the height is 4 ft.; what is its *weight?* Ans. 8,775 lbs.

2. What is the cube root of 74,088 ? Ans. 42.

3. There is a house 33 ft. long, 24 ft. wide, 19 ft. from the sill to the eaves, and 30 ft. from the sill to the top of the roof; how much will it cost to paint the outside at 25 cts. a sq. yd., no allowance being made for windows and doors ? Ans. $67½.

4. I own a piece of ground 250 ft. long, the widths of which, measured every 50 ft., are as follows; 35 ft., 18 ft., 16 ft., 24 ft., 10 ft., and 20 ft.; how many square feet of land are there in the piece ? Ans. 4,775 sq. ft.

5. The quantity of water flowing over a mill dam, July 1st, was found to be 28 cubic ft. a second at 6 o'clock in the morning, 23 cubic ft. at 9 o'clock, 19 cubic ft. at 12 o'clock, 18 cubic ft. at 3 o'clock, and 22 cubic ft. at 6 o'clock in the evening; what quantity, on an average, passed over the dam in a second ? Ans. 21¼ cubic ft.

6. There is a square garden containing 2,500 sq. ft.; how far is it between the opposite corners ?
Ans. 70.71 ft., or 70 ft. 8½ in., about.

7. How many feet, board measure, are there in a stick of timber 2 ft. square, and 22 ft. 9 in. long ? Ans. 1,092.

8. What is the surface of a circle 15 ft. in diameter ?
Ans. 176.714 sq. ft., about.

9. There is a pine log 4 ft. in diameter, and 27 ft. long, the ends of which are cut off obliquely, but parallel to each other; how many cubic ft. does it contain ?
Ans. 339.29, about.

10. How many square inches of leather will cover a foot ball 6 in. in diameter ? Ans. 113$\frac{1}{10}$ sq. in., nearly.

LESSON 204.

1. How large a cube will 90,800 ounces of lead make ?
Ans. A cube 2 ft. square.

2. How many gallons of milk are there in a cask of which the length is 26 in., the head diameter 17 in., and the bung diameter 22 in., the staves curving but little ?
Ans. 29 gals., nearly.

3. What is the carpenters' tonnage of a single decked vessel, the keel measuring 60 ft., the breadth of the beam 20 ft., and the depth of the hold 9½ ft. ? Ans. 120 tons.

4. There is a weight of 730 lbs. to be raised by a lever of the first kind 10 ft. long, to which a power can be applied equal to 146 lbs.; how far must the fulcrum be

placed from the weight so that the power may balance it ? Ans. 1 ft. 8 in.

5. A railroad has an inclined plane 2,000 ft. long, the height of which is 250 ft. ; what power must be applied by means of an engine at the top of the plane, to pull up a train of cars weighing 15,000 lbs., $\frac{1}{8}$ of the power being expended in overcoming friction ?

Ans. a power capable of raising 2,812$\frac{1}{2}$ lbs.

6. There is a steelyard the fulcrum of which is 1 in. from the place of suspending the weight ; how far from the fulcrum must a poise weighing 2 lbs. be placed to balance a weight of 3 lbs. ? Ans. 1$\frac{1}{2}$ in.

7. There is a wheel 7$\frac{1}{2}$ ft. in diameter, the axle of which is 9 in. in diameter ; how large a weight will a power equal to 100 lbs. applied to the wheel balance ?

Ans. 1,000 lbs.

8. There are 2 pulleys in a movable block, and one end of the rope is fastened to the block of the fixed pulleys ; how large a weight can be raised with this machine by a power equal to 96 lbs., $\frac{1}{4}$ of the power being lost by friction ? Ans. 288 lbs.

9. There is a screw with a lever 6 ft. long, measuring from the centre of the screw, the thread is square, and the tops of the coils are $\frac{1}{2}$ of an inch apart ; what power will be sufficient to balance a weight of 679 lbs. ?

Ans. $\frac{3}{4}$ of a lb., about.

10. In the preceding question, what power will be sufficient to *raise* the weight ? Ans. 1$\frac{1}{8}$ lb., about.

PROGRESSION BY DIFFERENCE,

OFTEN CALLED ARITHMETICAL PROGRESSION.

LESSON 205.

A row or series of numbers, constantly increasing or decreasing, so that the difference between two adjacent numbers is the same in all parts of the row or series, is called

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What is called a progression by difference ?

a *progression by difference*; thus, 2, 4, 6, 8, 10, 12, is an increasing progression by difference, and 16, 11, 6, 1, is a decreasing progression by difference.

The numbers which form a progression by difference, are called the *terms*.

The difference between any two terms, is called the *common difference*.

There are five things in progression by difference, any three of which being known, the other two may be found; these things are,

1st, The first term.
2d, The last term.
3d, The number of terms.
4th, The common difference.
5th, The sum of all the terms.

By examining these five things, we see that three different things can be selected in ten different ways. Now in each way three things are chosen, there are two to be found, so there are twenty different calculations to make in order to obtain the answer to every kind of question that may arise in progression by difference.

We shall now point out the method of obtaining the answers to the most important questions.

1. A bookseller has 100 books on his counter; the first is worth 5 cts., the second 8 cts., the third 11 cts., and so on; each being worth 3 cts. more than the one next preceding; what is the value of the last book?

OPERATION.

```
 99
  3
----
297
  5 cts. value of first book.
----
302 cts. value of the last
         book. Ans.
```

Explanation. The most obvious way to obtain the value of the last book, is to add 3 cts. to 5 cts., then 3 cts. to the result, and so on, adding 3 cts. to the value of every book except the last. It is plain, then, that the answer can be more readily found by adding to 5 cts. 99 times 3 cts.

Give an example of an increasing and of a decreasing progression by difference?

What are called the terms?

What is called the common difference?

How many things are there in progression by difference? How many must be known to find the others? Name these things?

What do we see by examining these five things?

Explain how example 1, lesson 205, is performed.

2. A bookseller has 80 books on his counter; the first is worth 162 cts., the second 160 cts., the third 158 cts., and so on, each being worth 2 cts. less than the preceding one; what is the value of the last book?

OPERATION.

79
2
——
158

162 cts. value of first book.
158
——
4 cts. value of the last book. Ans.

Explanation. The most obvious way to obtain the value of the last book, is to subtract 2 cts. from 162 cts., then 2 cts. from the remainder, and so on, taking 2 cts. from the value of every book except the last. It is plain, then, that the answer can be more readily found by subtracting from 162 cts. 79 times 2 cts.

Therefore, to find the last term, when the first term, number of terms, and common difference are known,

Multiply 1 *less than the number of terms by the common difference, and add the product to the first term, if it is an increasing progression, but subtract the product from the first term, if it is a decreasing progression.*

3. What is the first term of an increasing progression by difference, the last term being 266, the common difference 6, and the number of terms 40? Ans. 32.

Explanation. Call the first term the last, and the last term the first.

4. Suppose that an empty bottle, on being put 1 ft. beneath the surface of the ocean, is pressed on all sides by the water to the amount of 50 lbs., and that it sustains an additional pressure of 50 lbs. for every foot it sinks; what pressure will it bear 501 ft. beneath the surface?

Ans. 25,050 lbs.

5. A man bought 34 yds. of cloth, and agreed to give 12 cts. for the first yard, 12$\frac{1}{3}$ cts. for the second yard, 12$\frac{2}{3}$ cts. for the third yard, and so on; what did the last yard cost him? Ans. 23 cts.

Explain how example 2, lesson 205, is performed.

How do we find the last term, when the first term, number of terms, and common difference are known?

LESSON 206.

1. **A teacher gave the scholar that stood at the head of his class 32 chestnuts, the next 29, the next 26, and so on to the last, who received only 2 ; how many scholars were there in the class ?**

OPERATION.

32 number the first receives.
2 number the last receives.
—
Common difference. 3) 30
—
10
1
—
11 scholars. Ans.

Explanation. The most obvious way to obtain the answer, is to take 3 chestnuts from 32, then 3 from the remainder, and so on, taking 3 from the share of every one except the last, and then count the number of shares. But the answer can be more readily found ; for if we subtract 2 from 32, the remainder, 30, is as many times 3 as there are shares except the last ; so by dividing 30 by 3, and adding 1 to the quotient, we obtain the answer.

Therefore, to find the number of terms, when the first term, last term, and common difference are known,

Divide the difference between the first and last terms by the common difference, and add 1 to the quotient.

2. The first term of a progression by difference being 8, the last 100, and the common difference 4, what is the number of terms ? Ans. 24.

3. Experiment shows us that a heavy body falls through the air 16 ft. the first second, 48 ft. the next second, 80 ft. the third second, and so on ; now how many seconds has a stone been falling that descends 336 ft. in a second ? Ans. 11.

4. A young man having 50 cts., determined to add 6¼ cts. to it every day ; after some time, on counting his money, he found he had $13 ; how many days had he adhered to his resolution ? Ans. 200.

Explanation. 50 cts. is one term ; so there is 1 term more than the number of days.

Explain how example 1, lesson 206, is performed.

How do we find the number of terms when the first term, last term, *and common* difference are known?

5. There are 21 cannon balls of different weights lying in a heap, the smallest weighs 4 lbs., and the largest 44 lbs., and there is a regular increase of weight from the smallest to the largest ; what is this increase ?

OPERATION.

```
21        44
 1         4
—        ——
20   2|0 ) 4|0
           —
           2 lbs. Ans.
```

Explanation. The difference between 4 lbs. and 44 lbs., is the whole increase of weight ; now the regular increase is added to every ball except the last ; so we divide the whole increase, 40 lbs., by 20 to obtain what each ball increases.

Therefore, to find the common difference when the first term, last term, and number of terms are known,

Divide the difference between the first and last terms by 1 *less than the number of terms.*

6. The first term of a progression by difference being 122, the last term 7, and the number of terms 24, what is the common difference ? Ans. 5.

7. A man's expenses for the first year after marriage were $180, but they gradually increased for 29 years, when they were $2,000 a year ; what was the yearly increase ? Ans. $65.

8. The weight of 5 men forms a progression by difference, the first man weighing 135 lbs., and the last 239 lbs. ; what is the common difference of weight between them ?

Ans. 26 lbs.

LESSON 207.

1. A grocer bought 7 barrels of flour at different times ; the first barrel cost $4, and the price of each succeeding barrel increased regularly till the seventh cost $16 ; how much did the whole cost ?

Explain how example 5, lesson 206, is performed.

How do we find the common difference, when the first term, last term, and number of terms are known ?

OPERATION.

$$\begin{array}{r} 4 \\ 16 \\ \hline 2)\,20 \\ \hline 10 \\ 7 \\ \hline \$70 \text{ Ans.} \end{array}$$

Explanation. As the price regularly increases from the first barrel to the last, it is obvious that the average price of the first and last barrels, is the average price of each. We multiply the average of $4 and $16, then, by 7 for the answer.

Therefore, to find the sum of all the terms, when the first term, the last term, and the number of terms are known,

Multiply the average of the first and last terms by the number of terms.

2. There is a progression by difference, the first term of which is 5, the last 595, and the number of terms is 60; what is the sum of all the terms ? Ans. 18,000.

3. If a stone has been falling 14 seconds, and moves 464 ft. in a second, from what height has it fallen, the descent in the first second being 16 ft. ? Ans. 3,360 ft.

4. There are a number of rows of corn, the first of which contains 3 hills, the second 7, the third 11, and so on to the last, which has 43 hills ; how many hills are there in all the rows ? Ans. 253.

Explanation. What is the number of rows ?

Note. We have given no rules to find the answers to many of the twenty different questions in progression by difference; the answers, however, can usually be obtained by means of what precedes, sometimes indirectly, that is, after finding something else first, as in the preceding question.

5. A man paid $2.20 for a number of small books, the prices of which formed a progression by difference ; the price of the first was 5 cts., and the price of the last 35 cts. ; how many books did he buy ? Ans. 11.

Explanation. Examine the last rule.

Explain how example 1, lesson 207, is performed.

How do we find the sum of all the terms, when the first term, the last term, and the number of terms are known ?

What have we given no rules for ? How can the answers be usually *obtained ?*

6. In 9 seconds, an iron ball descended 1,296 ft., falling 272 ft. the last second; how far did it fall the first second? Ans. 16 ft.

7. Suppose a number of soldiers to be formed into a body resembling a wedge, having 7 men in the first rank, 9 in the second, 11 in the third, and so on, till there are 25 ranks; how many soldiers will there be in the whole body? Ans. 775.

8. If a body of soldiers drawn up like the preceding had 4 men in the first rank, 8 in the second, 12 in the third, and so on to the last, which contained 36 men, how many men were there in the whole body? Ans. 160.

9. The last term of a decreasing progression by difference is 99, the number of terms is 30, and the common difference is 3; what is the sum of the terms? Ans. 1,665.

PROGRESSION BY QUOTIENT,

OFTEN CALLED GEOMETRICAL PROGRESSION.

LESSON 208.

A row or series of numbers constantly increasing or decreasing, so that if any number in it be divided by the one that precedes, the quotient will be the same in all parts of the row or series, is called a *progression by quotient;* thus, $\frac{1}{2}$, 1, 2, 4, 8, 16, is an increasing progression by quotient, and 125, 25, 5, 1, is a decreasing progression by quotient.

The numbers which form a progression by quotient are called the *terms.*

The quotient formed by dividing any term by the preceding one is called the *ratio.* In a decreasing progression by quotient, the ratio is less than 1.

There are five things in progression by quotient, any

What is called a progression by quotient? Give an example of an increasing and of a decreasing progression by quotient?

What are called the terms? What is called the ratio? In a decreasing progression by quotient what is the ratio less than?

How many things are there in progression by quotient?

three of which being known, the other two may be found; these things are,

1st, The first term,
2d, The last term,
3d, The number of terms,
4th, The ratio,
5th, The sum of all the terms.

By examining these five things, we see that three different things may be selected in ten different ways. Now in each way three things are chosen, there are two to be found, so there are twenty different calculations to make, in order to obtain the answer to every kind of question that may arise in progression by quotient.

We shall now point out the method of obtaining the answers to the most important questions.

1. A man commenced trade with $645, and by skilful management, doubled his capital every year for 6 years; what sum did he have at the end of that time?

OPERATION.

```
 2      $645
 2        64
 —     ———————
 4      2580
 2     3870
 —    ————————
 8   $41,280  Ans.
 2
 —
16
 2
 —
32
 2
 —
64
```

Explanation. We observe first that there are 7 terms, which are the sum he began with, and the sum he had at the end of each year. Now the most obvious way to obtain the answer is, to multiply together $645 and 2, 6 times; but this way of proceeding being tedious, we find the answer by multiplying 2 by itself 5 times, and then multiplying $645 by the result. See Multiplication, lesson 38.

2. A man commenced trade with $2,025, but by unskilful management, possessed only $\frac{2}{3}$ as much capital at the

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How many must be known to find the others? Name these things.
What do we see by examining these five things?
Explain how example 1, lesson 208, is performed.

end of every year as he had at the beginning; what was he worth at the end of 4 years?

OPERATION.

2 numer. of $\frac{2}{3}$	3 denom. of $\frac{2}{3}$	$2025
2	3	16
4	9	12150
2	3	2025
		$
8	27	81)32400(400 Ans.
2	3	324
16	81	00

Explanation. We observe first, that there are 5 terms, which are the sum he began with, and the sum he had at the end of each year. Now the most obvious way to obtain the answer, is to multiply together $2,025 and $\frac{2}{3}$, 4 times; but this way of proceeding being tedious, we find the answer by multiplying $\frac{2}{3}$ by itself 3 times, and then multiplying $2,025 by the result. See Multiplication, lesson 38.

Therefore, to find the last term, when the first term, the number of terms, and the ratio are known,

Multiply the ratio by itself 2 times less than the number of terms, and multiply the result by the first term.

3. What is the first term of a progression by quotient, the last term being 4, the number of terms 6, and the ratio $\frac{1}{4}$? Ans. 4,096.

Explanation. Call the first term the last, and the last term the first; the ratio then will be $\frac{4}{1}$ or 4.

4. Suppose the number of inhabitants in the United States, in 1820, was 10,000,000, and that the number doubles every 30 years, how many will there be in the year 2000, that is, after the number has doubled 6 times? Ans. 640,000,000.

5. If a man had but $1 at compound interest in the year 1600, what would it have amounted to in 1828, allowing it to double every 12 years? Ans. $524,288.

Explain how example 2, lesson 208, is performed.
How do we *find the* last term, when the first term, the number of *terms, and the ratio* are known?

LESSON 209.

1. What is the sum of the following progression by quotient, 2, 8, 32, 128, the ratio of which we observe is 4 ?

Explanation. We can obviously find the sum by adding the numbers together ; but this way of proceeding will be very tedious when there are many numbers. To find an easier method, multiply the terms, 2, 8, 32, all except the last, by the ratio, 4.

The products are 8 32 128
From these subtracting, 2 8 32

we obtain 126, the difference between the first and last terms. This difference is plainly 3 times 2, 8, 32, since it is the difference between these numbers, and 4 times these numbers ; if we now divide 126 by 3, and add 128, the last term, to the quotient, we shall have the sum of 2, 8, 32, 128. The operation is as follows ;

128 last term.
subtract 2 first term.
3)126
42
Add last term 128
170 sum. Ans.

2. What is the sum of the following progression by quotient, 486, 162, 54, 18, 6, 2, the ratio of which we see is $\frac{1}{3}$?

Explanation. Direct addition being usually tedious, to find an easier method, multiply the terms 486, 162, 54, 18, 6, all except the last, by the ratio $\frac{1}{3}$,

and from 486 162 54 18 6
subtracting the products 162 54 18 6 2

we obtain 484, the difference between the first and last terms. This difference is plainly $\frac{2}{3}$ of 486, 162, 54, 18, 6, since it is the difference between these numbers and $\frac{1}{3}$ of them ; if we now divide 484 by $\frac{2}{3}$, and add 2, the last term,

---

Explain how example 1, lesson 209, is performed.

to the quotient, we shall have the sum of 486, 162, 54, 18, 6, 2. The operation is as follows;

486 last term.
Subtract 2 first term.
484
Divide by $\frac{2}{3}$ 3
2)1452
726
Add last term 2
728 sum. Ans.

Therefore, to find the sum of the terms, when the first term, the last term, and the ratio are known,

Divide the difference between the first and last terms by the difference between 1 and the ratio, and add the last term to the quotient.

3. A person has 2 parents, 4 grand-parents, 8 great-grand-parents, and so on; now allowing 25 years to a generation, what number of ancestors had a person, born in 1825, descended from since the year 1325; supposing that the person's ancestors had in no case married relatives? Ans. 4,194,302.

Explanation. Begin by finding the last term.

4. A gentleman offered to sell a beautiful cottage to a young fop, and receive in payment 1 cent for the first window in it, 10 cts. for the second, 100 cts. for the third, and so on; the young witling, thinking it a fine bargain, agreed accordingly; what did the house cost him, there being 11 windows in it? Ans. $111,111,111.11.

5. What is the sum of the following progression by quotient, 4, 6, 9, &c., the last term of which is $45\frac{5}{8}$? Ans. $128\frac{7}{8}$.

LESSON 210.

1. What is the sum of the progression by quotient, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c., continued on for ever? Ans. 4.

Explain how example 2, lesson 209, is performed.
How do we find the sum of the terms, when the first term, the last term, and the ratio are known?

Explanation. After the progression has been continued on for a very great number of terms, the last term will be so near 0 in value, that it may be considered nothing.

2. What is the sum of the progression by quotient, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c., continued on for ever? Ans. $1\frac{1}{2}$.

3. What is the sum of the progression by quotient, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c., continued on for ever? Ans. $\frac{1}{9}$.

4. What is the value of the repeating decimals, .3333, &c.? Ans. $\frac{1}{3}$.

Explanation. These decimals are equal to $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c.

5. What is the value of the repeating decimals, .139139139, &c. Ans. $\frac{139}{999}$.

Explanation. These decimals are equal to $\frac{139}{1000}$, $\frac{139}{1000000}$, &c.

6. If a farmer should sow 5 kernels of wheat, and its produce, every year for 9 years, how many bushels would there be in the last harvest, supposing that each harvest amounts to 10 times the quantity sowed, and that 8,000 kernels make 1 pint? Ans. 9,765 bu. 2½ pks.

7. What is the amount of the following sums; 384, 192, 96, 48, 24, 12, 6, 3? Ans. 765.

Note. We have given no rules to find the answers to many questions in progression by quotient, but hardly one of those not noticed will occur in the course of a man's life. However, when we find such a question, the answer can be found by an obvious though lengthy method.

LESSON 211.

PROGRESSION BY QUOTIENT APPLIED TO COMPOUND INTEREST.

To obtain the amount of a sum at simple interest for 1 year, say at 6 per cent., we multiply the principal by .06, and add the principal to the product, or multiply it by 1.06, which is obviously the same thing.

To obtain the amount of a sum at compound interest for

What have we given no rules for? Do such questions often occur? What if we find such a question?

In what two ways do we obtain the amount of a sum at simple interest *for 1 year*, say at 6 per cent.?

a number of years, say at 6 per cent., we can multiply it by 1.06, the product by 1.06, and so on, multiplying by 1.06 as many times as there are years. The original sum, then, and the amounts at the end of each year, form a progression by quotient. The first term is the original sum, the number of terms is 1 more than the number of years, and the ratio is 1.06, 1.07, 1.08, &c., according to the per cent.

1. What will $80 amount to in 12 years at 6 per cent., compound interest ? Ans. $160.98.

Explanation. In order to perform this and the following examples with ease, examine the table in Compound Interest.

2. Find the amount of $6,525.12½ for 8 years, at 7 per cent., compound interest. Ans. $11,211.38.

3. What is the compound interest on $1,000 for 10 years, at 6 per cent. ? Ans. $790.85.

4. What will $100 amount to in 9 years at 5 per cent., compound interest ? Ans. $155.13.

5. How long will it take $50 to amount to $84.47 at 6 per cent., compound interest ? Ans. 9 years.

PROGRESSION BY QUOTIENT APPLIED TO DISCOUNT AT COMPOUND INTEREST.

6. What is the present worth of $3,581.70 due in 10 years, without interest, discounting at the rate of 6 per cent., compound interest ? Ans. $2,000.

Explanation. The number of terms is 11, and the ratio evidently $\frac{1}{1.06}$, to find the last term.

7. What principal at 6 per cent., compound interest, will amount to $143.45, in 8 years ? Ans. $90.

8. What sum ought to be paid now to discharge a debt of $500, due in 6 years, without interest, supposing money to be worth 7 per cent., compound interest ? Ans. $333.17.

9. What discount should be made for the immediate payment of $1,000, due in 5 years, without interest, money being worth 10 per cent., compound interest ? Ans. $379.08.

Explanation. What is the present worth ?

How can we obtain the amount of a sum at compound interest for any number of years, say at 6 per cent.? What do the original sum, *and the amounts* at the end of each year form? What is the first term? *Number of terms?* **Ratio?**

ANNUITIES.

LESSON 212.

A sum of money payable at regular intervals during a certain time or for ever, is called an *annuity* ; thus, rents, pensions, salaries, &c., are really annuities.

If an annuity is not paid when due, it is said to be in *arrears*.

When an annuity is not to commence until after a certain time, it is said to be in *reversion*.

The sum of the instalments of an annuity not paid, together with the interest on each instalment after it becomes due, is called the *amount*.

ANNUITIES AT SIMPLE INTEREST.

1. The rent of a house was $100 a year, and the occupant paid nothing until 4 years after he entered it ; what was then due, 6 per cent. being the legal interest ?

OPERATION.

The rent of the fourth year being paid when due, is	$100
The amount of $100 for 1 year is	106
The amount of $100 for 2 years is................	112
The amount of $100 for 3 years is................	118
	Ans. $436

AN EASIER WAY.

$118 amount of $100 for 3 years.
100 rent of the fourth year.
2)218
109
4 years.
$436 Ans.

Explanation. The operation by the first method is obvious ; by examining it, we see that the amounts of the rents form a progression by difference; so we can get their sum in an easier way, by the rule for finding the sum of a progression by difference.

What is called an annuity ? What are really annuities ?
When is an annuity said to be in arrears ? In reversion ?
What is called the amount ?
Explain how example 1, lesson 212, is performed.

Therefore, to find the amount of an annuity in arrears, at simple interest,

Observe that the amounts of the several instalments, remaining unpaid, form a progression by difference, and find their sum, which is the whole amount, accordingly.

2. A man had an annuity of $240 a year, during life, but the company it was purchased from becoming embarrassed, the annuity remained unpaid, that is, in arrears, 30 years; what was then due, 6 per cent. being the legal interest? Ans. $13,464.

3. A man hired a house at a rent of $425 a year, but paid nothing until 10 years after the rent of the first year was due; what did the rent then amount to, 7 per cent. being the legal interest? Ans. $5,588.75.

4. An annuity of $20 a month remained unpaid for 3 years; what was then due, 6 per cent. being the legal interest? Ans. $783.

5. What is the present worth of an annuity of $100 a year, to commence 1 year from this time, and to continue 3 years, money being worth 6 per cent.?

OPERATION. *See Discount.*

$94.34 present worth of first instalment.
89.286 present worth of second instalment.
84.746 present worth of third instalment.

$268.37 Ans.

Therefore, to find the present worth of an annuity at simple interest,

Find the present worth of each instalment of the annuity, and the sum of these present worths will be the answer.

6. What is the present worth of a salary of $800, to be paid at the end of every year from this time, and to continue 4 years, 6 per cent. being the legal interest? Ans. $2,792.13.

7. What sum of ready money is equal to an annuity of

How do we find the amount of an annuity in arrears, at simple interest?

Explain how example 5, lesson 212, is performed.

How do we find the present worth of an annuity at simple interest?

$200, to commence 1 year from this time, and to continue 5 years, money being worth 10 per cent.? Ans. $778.52.

Note. If an annuity is to continue a long time, the present worth, found in this way, will often be larger than a sum, the yearly interest of which is equal to the annuity; thus, reckoning interest at 5 per cent., the present worth of an annuity of $100 a year to continue 50 years, will be more than $2,000, the yearly interest of which is $100. The rule then is absurd, though it may be used without much error to find the present worth of an annuity to continue for a short time.

LESSON 213.

ANNUITIES AT COMPOUND INTEREST.

1. An annuity of $100 a year has not been paid for 4 years; what is the amount due on it, money being worth 6 per cent., compound interest?

OPERATION.

The amount of the fourth year, now due, is....$100
The amount of $100 for 1 year is 106.00
The amount of $100 for 2 years is 112.36
The amount of $100 for 3 years is 119.1016

Ans. $437.46.

AN EASIER WAY.

From $119.1016 last term,
Subtract 100 first term.

Difference between 1 and the ratio. .06) 19.1016

318.36
Add last term 119.10

$437.46 Ans.

Explanation. The operation by the first method is obvious; by examining it, we see that the amounts of the instalments form a progression by quotient; so we can get their sum an easier way, by the rule for finding the sum of a progression by quotient.

Therefore, to find the amount of an annuity in arrears, at compound interest,

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If an annuity is to continue for a long time, what is said of its present worth? Give an example. Is the rule correct then? When may *it be used?*

Explain how example 1, lesson 213, is performed.

Observe that the amounts of the several instalments, remaining unpaid, form a progression by quotient, and find their sum, which is the whole amount, accordingly.

2. A man paid a certain sum to a company for an annuity of $250 a year during life, but the company becoming embarrassed, did not pay the annuity for 8 years; what was the amount then in arrears, allowing 6 per cent., compound interest? Ans. $2,474.37.

Explanation. The table in Compound Interest shows the products of 1.05, 1.06, and 1.07 multiplied by themselves any number of times up to 39.

3. A man had a salary of $400 a year, payable quarterly, but which was not paid for 2 years; what was then due, allowing quarterly compound interest, at 7 per cent a year? Ans. $850.75

4. What is the amount of an annuity of $333 a year, which has been in arrears 5 years, allowing 5 per cent., compound interest? Ans. $1,768.27.

5. What is the present worth of a pension of $100 a year, the first instalment of which is to be paid in 1 year, and which is to continue 3 years, 6 per cent., compound interest, being allowed?

OPERATION.

The present worth of first instalment is ……	$94.3396
The present worth of the last instalment is …	83.962
Divide by $\frac{.06}{1.06}$ the difference between 1 and the ratio $\frac{1}{1.06}$,	10.3776
	1.06
	622656
	103776
	.06)11.000256
	183.337
Add last term	83.962
Ans.	$267.30

How *do we find* the amount of an annuity in arrears, at compound *interest?*

Explanation. To get the present worth of the first instalment, we multiply $100, by $\frac{1}{1.06}$; see Progression by Quotient, lesson 211; to get the present worth of the second instalment, we multiply $100 by the product of $\frac{1}{1.06}$ multiplied by itself, and so on. We see then, that the present worths of the instalments form a progression by quotient, the ratio of which is $\frac{1}{1.06}$; so we find the first and last terms, and then get the sum of the terms by the rule in Progression by Quotient.

Therefore, to find the present worth of an annuity, discounting by compound interest,

Find the present worth of the first and last instalments, and then find the sum of all the present worths, by the rule for finding the sum of all the terms in Progression by Quotient.

6. What sum of ready money will purchase an annuity of $240 a year, to commence 1 year from this time, and to continue 9 years, money being worth 6 per cent., compound interest? Ans. $1,632.41.

7. What is the present worth of an annuity of $180 a year, to commence in 1 year, and to continue for ever, money being worth 7 per cent., compound interest? Ans. $2,571.43.

8. What is the present worth of an annuity of $500 a year, to commence in 6 years, and to continue 6 years, money being worth 7 per cent., compound interest? Ans. $1,699.24.

9. What annuity will $3,000 buy, to commence in 1 year, and to continue 3 years, money being worth 6 per cent., compound interest? Ans. $1,122.33.

Explanation. What sum will purchase an annuity of $1 a year, to commence in 1 year, and to continue 3 years? How many times is this sum contained in $3,000?

Explain how example 5, lesson 213, is performed.

How do we find the present worth of an annuity, discounting by compound interest?

PROMISCUOUS QUESTIONS

IN

PROGRESSION BY DIFFERENCE, &c.

Lesson 214.

1. An annuity of $200 a year remained unpaid 5 years; how much did it then amount to, 7 per cent. being the legal interest? Ans. $1,150.15.

2. The poise of a steelyard, when placed in the first notch, balances 4 oz., and at every notch it is moved, it balances 2 oz. more than before; in what notch is it when it balances 12 lbs.? Ans. in the 95th.

3. A countryman, in going to market, found the roads so bad that he went only 4 miles the first day; however, as the roads grew better, he increased his speed, and completed his journey in 6 days, going 14 miles the last day; how much did he increase his speed each day, on an average? Ans. 2 miles.

4. By the preceding supposition, how far did the countryman travel? Ans. 54 miles.

5. What sum of ready money will purchase an annuity of $300 a year, to begin in 1 year, and to continue for ever, money being worth 5 per cent., compound interest? Ans. $6,000.

6. There is a progression by quotient, the first term of which is 3, the number of terms 9, and ratio 4; what is the sum of all the terms? Ans. 262,143.

7. The first term of an increasing progression by difference is 7, the common difference is 3, and the number of terms 12; what is the last term? Ans. 40.

8. What sum of ready money is equal to 2 annual payments of $500, the first in 1 year, money being worth 6 per cent., simple interest? Ans. 918.13.

9. A man had a salary of $700 a year, which was not paid for 3 years; what was then due, allowing 6 per cent., compound interest? Ans. $2,228.52.

10. A merchant began trade with $3,000, and his gains were such that at the end of every year, he had $\frac{3}{2}$ as much as he had at the beginning; what was he worth at the end of 6 years? Ans. $34,171.87.

MONEY.

Lesson 215.

Money is composed of gold and silver, metals which are extremely precious, because they are of great use in the arts. Copper, a valuable but much inferior metal, is also used for money. The value of gold and silver never arose from whim or fancy, but always has existed from the nature of things, because these metals are indispensable for the manufacture, preservation, or embellishment of many articles of necessity or convenience. You cannot eat or drink gold and silver, it is true, neither can you eat or drink an axe, a plough, or a steamboat, which are, nevertheless, articles of great utility.

Gold and silver, in the form of money, do not represent property, but are property, and *serve as a medium of exchange*. They are admirably fitted for the purposes of exchange, because they are scarce and very precious, light in proportion to their value, clean, not liable to decay or be destroyed, &c. If a man in Massachusetts has 500 bushels of potatoes, and wishes to buy a farm in Michigan, he does not transport his potatoes to Michigan, and exchange them for the farm, because potatoes are of small value there, would spoil in carrying, and the transportation would cost more labor than would be necessary to raise five times the quantity. On the contrary, he gets 2 or 300 dollars for them, and carrying the money to Michigan, exchanges it for land.

Coins of gold, silver, or copper, are called *true or real money*. Money that has no coin to represent it, is called *imaginary money*, as a New England shilling, a mill, &c. Accounts are kept in imaginary money as well as in real money.

What is money composed of? What kind of metals are they? Why are they precious? What other metal is used for money? Did the value of gold and silver arise from whim or fancy? Why has it always existed? What is said of eating and drinking gold and silver?

Do gold and silver, in the form of money, represent property? What are they, and for what do they serve? Why are they admirably fitted for the purposes of exchange? What is a strong instance of the utility of money for the purposes of exchange?

What are called true or real money? What is called imaginary *money?* Give some examples? In what money are accounts kept?

The following is a description of the money *in which accounts are kept* in the principal places where we trade.

Continual alterations are taking place in the money of foreign countries, so that an accurate description will soon be erroneous.

Note. The scholar should read the whole, and commit to memory the answers to the questions.

GREAT BRITAIN.

Accounts in Great Britain are kept in pounds, shillings, pence, and farthings.

4 farthings	sign *qr.*	make	1 penny,	sign *d.*
12 pence		make	1 shilling,	sign *s.*
20 shillings		make	1 pound,	sign £ or *l.*

Note. Accounts in this country were formerly kept in pounds, shillings, pence, and farthings; 4 farthings made 1 penny, 12 pence 1 shilling, and 20 shillings 1 pound, just as in the money of Great Britain; the value of this money, however, was not the same as the English money, and varied in different parts of the country. In the origin this money had the same value as the English, but the governments of the colonies fabricated paper money, which, having no real value, depreciated, so that a silver dollar would buy 6 shillings of Massachusetts money, 8 shillings of New York money, &c.

A dollar is estimated,

In the money of Great Britain and Newfoundland, called English or *sterling* money,	at 4*s.* 6*d.*, or $\frac{9}{40}$ of a pound.
In the money of Canada and Nova Scotia, called *Canada currency,*	at 5*s.*, or $\frac{1}{4}$ of a pound.
In the old money of New England, Virginia, Kentucky, and Tennessee, called *New England currency,*	at 6*s.*, or $\frac{3}{10}$ of a pound.
In the old money of New York and North Carolina, called *New York currency,*	at 8*s.*, or $\frac{4}{10}$ of a pound.
In the old money of Pennsylvania, Delaware, Maryland, and New Jersey, called *Pennsylvania currency,*	at 7*s.* 6*d.*, or $\frac{3}{8}$ of a pound.

What is said of alterations of money in foreign countries?

In what are accounts kept in Great Britain?

Recite the table of the money of Great Britain.

In what were accounts formerly kept in this country? How many farthings made a penny, pence a shilling, and shillings a pound? Was it of the same value as the English money? Was the value the same in all parts of the country? What was the value in the origin? Why did it alter?

What is called sterling money? Canada currency? New England currency? New York currency? Pennsylvania currency?

What is a dollar estimated at in sterling money? In Canada currency? In New England currency? In New York currency? In *Pennsylvania currency?*

In the old money of Georgia and South Carolina, called *Georgia currency*, } at 4*s.* 8*d.*, or $\frac{7}{30}$ of a pound.

£1 sterling is worth	\$4.44$\frac{4}{9}$
£1 Canada currency is worth	4.00
£1 New England currency is worth	3.33$\frac{1}{3}$
£1 New York currency is worth	2.50
£1 Pennsylvania currency is worth	2.66$\frac{2}{3}$
£1 Georgia currency is worth	4.28$\frac{4}{7}$

It is often convenient to know something of these old currencies, though they are no longer used in the United States. Federal money was established by Congress in 1786, and is now universally employed.

LESSON 216.

FRANCE.

10 centimes sign c. make 1 decime.
10 decimes make 1 franc, sign fr. value \$.18$\frac{6}{10}$.

Accounts have been kept in this money since 1795. The French do not mention their decime any more than we do our dime; for instance, instead of 6 fr. 4 decimes, they say 6 fr. 40 c.

Accounts were formerly kept in different money, thus;

12 deniers made 1 sou.
20 sous made 1 livre, or livre Tournois, value \$.18$\frac{37}{100}$.

SPAIN, SPANISH AMERICA, AND SPANISH COLONIES.

Accounts are usually kept in reals,

34 maravedies making 1 real.

There are nine different kinds of reals, four of which are in general use; these are,

The *real vellon*, worth about 5 cents.
The *real of old plate*, worth usually about 9$\frac{4}{10}$ cents.
The *real of new plate*, worth 10 cents.
The *real of Mexican plate*, worth 12$\frac{1}{2}$ cents.

What is called Georgia currency?

What is a dollar estimated at in Georgia currency?

What is £1 sterling worth? £1 Canada currency? £1 New England currency? £1 New York currency? £1 Pennsylvania currency? £1 Georgia currency?

What is said of these old currencies, and of Federal money?

Recite the table of the money of France.

How long have accounts been kept in this money? Do the French *mention* their decime? What do they say for 6 francs 4 decimes?

What is the value of the real of Mexican plate?

In Spanish America, and the Spanish colonies, accounts are kept in reals of Mexican plate, and hard dollars, worth $1. In Spain, when real is mentioned without explanation, real of old plate is understood; 8 of these reals make what is usually called a *dollar of exchange*, an imaginary money, worth usually about 75 cents. This dollar is called *libra* in Alicant and Valencia. In Gibraltar, accounts are kept in *cob dollars*, worth $1. The real of this place is worth 8⅓ cents.

PORTUGAL, BRAZIL, AND PORTUGUESE COLONIES.

1,000 rees make 1 milree; value in Portugal, $1.24; in Brazil, $1.05, and in Portuguese Colonies, $1.00.

RUSSIA.

10 kopecs	make 1 grievener.
10 grieveners	make 1 ruble, value........$.75

Business is done with paper money. The paper rouble is usually 60 or 80 per cent. below par.

PRUSSIA.

12 pfennings	make 1 good-groschen.
24 good-groschen	make 1 rix dollar, value......$.69

This is the most usual way of keeping accounts.

SWEDEN AND NORWAY.

12 runstycken	make 1 skilling.
48 skillings	make 1 rix dollar, value $1.07

DENMARK.

12 pfennings	make 1 skilling.
16 skillings	make 1 mark.
6 marks	make 1 rixbank dollar, value ...$.53

HAMBURG.

12 pfennings	make 1 skilling.
16 skillings	make 1 mark, value, about.....$.30

The word lubs, a contraction of Lubecks, is often put after the money of Hamburg and Lubeck, to distinguish it from that of Denmark and other places; thus we say, 5 skillings lubs, 2 marks lubs.

In what are accounts kept in Spanish America, and the Spanish *colonies?*

Certificates or evidences of money in the bank are transferred and used as money. This bank money, called *banco*, is above par, and usually brings from 18 to 25 per cent. premium. A mark of bank money, or a *mark banco*, is usually worth about 35 cents.

BELGIUM AND HOLLAND.

100 cents make 1 florin or guilder, sign fl. value $.40

GENOA AND LEGHORN.

12 denari di pezza make 1 soldo di pezza.
20 soldi di pezza make 1 pezza, value, about $.90

also,

12 denari di lira make 1 soldo di lira.
20 soldi di lira make 1 lira, value, about..... $.16

ROME.

10 baiocchi make 1 paolo.
10 paoli make 1 scudo or Roman crown, value $1.00

NAPLES.

10 grani make 1 carlino.
10 carlini make 1 ducato, value.......... $.80

SICILY.

20 grani make 1 taro.
12 tari make 1 scudo or Sicilian crown, value $.95
30 tari make 1 oncia, value................ 2.40

MALTA.

20 grani make 1 taro.
12 tari make 1 scudo, value............. $.40
30 tari make 1 pezza, value............. 1.00

VENICE.

10 millesimi make 1 centesimo.
10 centesimi make 1 lira di Austria, value... $.16

Accounts have formerly been kept here in many different ways.

VIENNA AND TRIESTE.

4 pfennings make 1 kreuzer.
60 kreuzers make 1 florin or guilder, value... $.48
1½ florin make 1 rix dollar of account.

TURKEY, GREECE, AND THE NORTHERN STATES OF AFRICA.

The only invariable currency in these countries is composed of Spanish hard dollars and parts. In Turkey and Egypt money is usually reckoned by the *asper* and *para*,

3 aspers making 1 para, and
40 paras making 1 piastre, or Turkish dollar, value from 5 to 50 cents.

EAST INDIES.

MAURITIUS, OR THE ISLE OF FRANCE.

20 sous make 1 livre.
10 livres make 1 dollar, value......... $1.00
Also, 100 cents make 1 dollar.

HINDOSTAN.

Before European colonies were established in Hindostan, the principal currency was the *sicca rupee*, a silver coin, worth about 58 cents. This coin was also used as a weight. The British possessions in Hindostan are now divided into three presidencies, Bengal, Bombay, and Madras, the money of which differ.

CALCUTTA, in Bengal.

12 pice make 1 anna.
16 annas make 1 rupee, value, about........ $.46

A lac of rupees is 100,000 rupees, and a crore of rupees is 100 lacs, or 10,000,000 rupees.

BOMBAY.

100 rees make 1 quarter.
4 quarters make 1 rupee, value, about.. $.45

MADRAS.

Money is reckoned in rupees, worth about $.45; the rupee is divided into halves, quarters, eighths and sixteenths. The sixteenth is called the anna.

This is the new method, adopted in 1818, and is the only intelligible way of reckoning money in the place.

BENCOOLEN, in Sumatra.

8 satellers make 1 soocoo.
4 soocoos make 1 dollar, value,.......... $1.15

BATAVIA, in Java.

5 doits make 1 stiver.
2 stivers make 1 dubbel.
3 dubbels make 1 skilling.
4 skillings make 1 florin or guilder, value,....$.40

CANTON, in China.

10 cash make 1 candarine.
10 candarines make 1 mace.
10 mace make 1 tale, value,............ $1.48

Bargains can usually be made in Spanish dollars in any part of the East Indies.

JAPAN.

10 candarines make 1 mace.
10 mace make 1 tale, value,.........$1.43

WEST INDIES.

BRITISH ISLANDS.

Accounts are kept in pounds, shillings, pence and farthings, in the British West India islands, in the Swedish islands of St. Bartholomew, and by the British settlers on the French islands.

A pound in Jamaica and Bermuda currency is worth $3.00
A pound in Barbadoes currency is worth.......... 3.20
A pound in Bahama currency is worth............ 4.28 4/7
A pound in the currency of the Leeward and Windward Islands, employed also on St. Bartholomew, and by the British settlers on the French islands, is worth.................................. 2.22 2/9

FRENCH ISLANDS.

The French, in these islands, reckon money as follows ;

12 deniers make 1 sou.
20 sous make 1 livre, value........... $.11 1/9

ST. DOMINGO, OR HAYTI.

Accounts are kept in dollars and cents, as in the United States, but the dollar is worth only .... $.66

DANISH ISLANDS. St. Thomas, St. John, and Santa Cruz.

It is customary to keep accounts in rix dollars and *cents,* but the rix dollar, of colonial currency, is worth *only*.. $.64

Dutch Settlements. St. Eustatia, St. Martin, and Curazoa.

6 stivers	make 1 real or skilling.	
8 reals	make 1 piastre, value about........	$.73

SURINAM, BERBICE, DEMARARA, AND ESSEQUIBO.

8 doits	make 1 stiver.	
20 stivers	make 1 guilder, value about.....	$.33⅛

SPANISH ISLANDS. See Spain.

COINS.

LESSON 217.

Civilized nations coin the precious metals in pieces of certain values, so that they can be employed in making exchanges without weighing them, or examining their purity. Uncivilized or half civilized people, like many in the East Indies, often use these metals in bullion as money, and every time each ingot or piece is exchanged, are obliged to weigh it, and estimate its purity.

Gold and silver, when pure, are exceedingly soft and ductile ; so if they were coined in this state they would soon be defaced and worn out. Therefore, they are mixed or alloyed with a very small quantity of copper, which makes them quite hard, and but little impairs their ductility, incorruptibility, and other useful properties. There are two things then on which the value of each coin depends ; its weight, and the quantity of pure gold or silver which it contains.

We have mentioned the coins of the United States, and their values, in lesson 90, Federal Money ; it is proper to

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Why do civilized nations coin the precious metals ? How do uncivilized or half civilized people use these metals as money ? What are they obliged to do every time each ingot or piece is exchanged ?

What is the nature of gold and silver when pure ? What if they were coined in this state ? What, therefore, is done to them ? What effect does this have ? On what two things does the value of each coin depend ?

Observe further, that the gold pieces, coined since 1834, contain $\frac{26}{258}$ of alloy ; the eagle containing 232 grains of pure gold, and 26 grains of alloy, or 258 grains of standard gold. The alloy consists of copper and silver, the copper never being less than half of it.

The silver pieces contain $\frac{179}{1664}$ of alloy ; the dollar having 371¼ grains of pure silver, and 44¾ grains of pure copper, or 416 grains of standard silver. The copper coins consist of pure copper.

The gold pieces coined before 1834, contained about $\frac{1}{12}$ of alloy ; the eagle consisting of 247¼ grains of pure gold, and 22¾ grains of alloy, or of 270 grains of the standard gold of that time. These eagles are now worth $10.66½, and the half and quarter eagles in proportion.

The reason for altering the size of the eagle was this ; when the first eagles were coined, $\frac{1}{15}$ of an ounce of gold was worth as much as 1 ounce of silver, so they put $\frac{1}{15}$ as many grains of gold in $\frac{1}{10}$ of an eagle, as they put grains of silver in a dollar. In 1834, $\frac{1}{16}$ of an ounce of gold had become worth as much as 1 ounce of silver, so they put about $\frac{1}{16}$ as many grains of gold in $\frac{1}{10}$ of an eagle as there are grains of silver in a dollar.

The gold coins of Great Britain, France, Spain, Mexico, Columbia, Portugal, and Brazil, were made current in 1834, by law of Congress, and their values fixed as follows ;

Gold coins of Great Britain, Portugal and Brazil, at least 22 carats fine, at	$.948 a pwt.
Gold coins of France, $\frac{9}{10}$ fine, at	.931 a pwt.
Gold coins of Spain, Mexico, and Colombia, 20 carats $3\frac{7}{16}$ grains fine, at	.899 a pwt.

Foreign gold coins usually sell at a premium. The following are some of the principal foreign gold coins, with what may be considered their par value in this country ;

		Value.
Great Britain.	Guinea, 21 shillings, ½ guinea in proportion, ...	$5.07
	Sovereign, 20 shillings, ...	4.85
	Seven-shilling piece, ...	1.70

What is the value of the eagles coined before 1834 ?
Why was the size of the eagle altered ?
What gold coins were made current by law of Congress ?

France.	Napoleon, 20 francs, Double Napoleon in proportion,	$3.85
	New Louis,	3.85
Spain, Spanish Colonies, Mexico, and Colombia.	Doubloon of 8 escudos, 320 reals vellon,	16.00
	Doubloon of 4 escudos, 160 reals vellon,	8.00
	Doubloon or pistole, 80 reals vellon,	4.00
	Escudo, 40 reals vellon,	2.00
	Coronilla, 20 reals vellon,	1.00
	Patriot doubloon, parts in proportion.	15.53
Portugal and Brazil.	Dobra, 12,800 rees,	17.30
	Joanese, 6,400 rees,	8.65
	Half joanese, 3,200 rees,	4.32
	Escudo, 1,600 rees,	2.16
	Half escudo, 800 rees,	1.08
Russia,	Half Imperial, or 5 ruble piece,	3.92
Prussia,	Ducat,	2.27
Sweden and Norway,	Ducat,	2.23
Denmark,	Ducat-current, coined since 1757,	1.80
Hamburg,	Ducat,	2.28
Holland and Belgium,	10 guilder piece,	4.00
Genoa and Leghorn,	Sequin,	2.31
Rome,	Sequin,	2.25
Naples and Sicily,	Oncia,	2.50
Malta,	Spanish gold coins.	
Venice,	Sequin,	2.31
Vienna and Trieste,	Ducat,	2.29
Turkey, &c.,	Sequin, coined since 1818,	1.83
East Indies,	Star pagoda,	1.80
	Gold rupee,	7.10
	Japanese new copang,	4.92
West Indies,	Spanish gold coins.	

SILVER COINS.

The dollars of Mexico, Peru, Chili, and Central America, weighing not less than 415 grains each, and at least 10 oz. 15 pwts. fine, with those re-stamped in Brazil, of like weight and fineness, were made current in 1834, by law of

What foreign silver coins were made current by law of Congress?

Congress; the five franc piece of France, weighing not less than 384 grains, and at least 10 oz. 16 pwts. fine, was made current by the same law, and the value fixed at 93 cents.

The following are some of the principal foreign silver coins, with what may be considered their par value in this country;

		Value.
Great Britain, coined since 1816,	Crown, 5 shillings,	$1.10
	½ Crown, 2s. 6d.,	.55
	Shilling,	.22
	Sixpence,	.11
France.	5 franc piece,	.93
	2 franc piece,	.37$\frac{2}{10}$
	Franc,	.18$\frac{6}{10}$
	Piece of 50 centimes,	.09$\frac{3}{10}$
	Piece of 25 centimes,	.04$\frac{13}{20}$
Spain, Spanish Colonies, and Spanish America.	Dollar,	1.00
	½ dollar	.50
	¼ of a dollar,	.25
	⅛ of a dollar,	.12½
	$\frac{1}{16}$ of a dollar,	.06¼
	These are the same pieces that circulate in our country. Spanish dollars are to be found in all parts of the world, and are more extensively used than any other money.	
Portugal and Brazil,	New crusado, 480 rees,	$.60
	Teston, 100 rees,	.12½
Russia,	Ruble,	.75
Prussia,	Rix dollar,	.69
Sweden and Norway,	Rix dollar,	1.07
Denmark,	Rixbank dollar,	.53
Hamburg,	Crown dollar,	1.09
Holland and Belgium,	Florin or guilder,	.40
Leghorn,	Francescone,	1.05
Rome,	Scudo or Roman Crown,	1.00
Naples and Sicily,	Ducat,	.80
Malta,	Spanish dollar,	1.00
Venice,	Ducat-effective,	.77
Vienna and Trieste,	Rixdollar, 2 guilders,	.96
Turkey, Greece, and the Northern States of Africa,	Spanish dollars.	

East Indies,	Calcutta, sicca rupee,............	$.48
	Bombay and Madras, silver rupee,	.45
	Japanese schuit,.................	5.86

Rest of East Indies with West Indies, Spanish dollars.

EXCHANGE.

LESSON 218.

1. What sum in Canada currency, must you receive for $87.25 ? Ans. £21 16s. 3d.

Explanation. How many pounds and decimals of a pound are there in this sum ? Then how many pounds, shillings, &c., are there ?

2. What is the value of £25 12s. 8d. 2 qrs. sterling in Federal money ? Ans. $113.94.

Explanation. Change to pounds and decimals of a pound first ?

3. What is £1,000 in Newfoundland currency worth ? Ans. $4,444.44.

4. What is the value of $817.15 in sterling or English money ? Ans. £183 17s. $2\frac{1}{10}$d.

5. Change the following sums in sterling to Federal Money; £101 4s. 6d., £98 15s. 6d., £250 8s. 3d., and £49 11s. 9d. Ans. $2,222.22.

6. A man bought furs in Montreal to the amount of £78 11s. 8d. ; how many dollars and cents must he give in payment ? Ans. $\$314.33\frac{1}{3}$.

7. The salary of the governor of Massachusetts is £1,100 New England currency ; what is the amount in Federal money ? Ans. $3,666.67.

8. Change $637.50 to New England currency ? Ans. £191 5s

LESSON 219.

1. How many dollars and cents are there in £750 18s. *Georgia currency* ? Ans. $3,218.14.

2. Change $667.37 to Georgia currency.
Ans. £155 14s. 4d. 3qrs.

3. How many dollars and cents are there in £156 8s. 9d. New York currency? Ans. $391.09.

4. Change $800 to New York currency. Ans. £320.

5. Suppose a man in Delaware should recover £9,000 on a suit for money due his ancestors in 1780; what sum in Federal money should he receive? Ans. $24,000.

6. Change $1,700 to Pennsylvania currency.
Ans. £637 10s.

7. How many pounds sterling are there in £200 New England currency? Ans. £150.

8. A man in London owes me £100 5s. 4d., and I am indebted to him $500; what sum in English money must I pay to balance the account? Ans. £12 4s. 8d.

9. Multiply £15 5s. 4d. sterling by 5, and change the product to Federal money. Ans. $339.26.

LESSON 220.

1. If you owe a merchant in St. Petersburg, Russia, 12,000 paper rubles, worth 25 cents a piece, and he requests payment in Spanish dollars, allowing you 7 per cent. on them, what number of dollars must you send?
Ans. \$2,803$\frac{74}{100}$.

2. How many half joaneses, called half joes, must be shipped from Rio Janeiro, Brazil, to New York, to pay a debt of $6,897.75, Portuguese gold being 5 per cent. above par in New York? Ans. 1,520$\frac{668}{1000}$.

3. If a merchant in Tampico, Mexico, owes you 30,000 reals, what number of patriot doubloons should you receive in payment, these doubloons being 3 per cent. above par, and worth $16 apiece? Ans. 234$\frac{375}{1000}$.

4. How many dollars in specie must be sent to Havre, France, to pay a debt of 7,000 francs, specie in France being 2 per cent. above par? Ans. \$1,276$\frac{47}{100}$.

5. A merchant in Liverpool owes a mercantile house in Boston $10,000; how many sovereigns must be shipped to Boston to pay the debt, English gold being 1 per cent. above par? Ans. 2,041$\frac{44}{100}$.

6. A merchant sold a quantity of flour in Paramaribo, Surinam, for 15,000 guilders; how many Spanish dollars *should he* receive in payment? Ans. 5,000.

7. If you sell a cargo of rice in Amsterdam, Holland, for 24,000 florins, take your pay in 10 guilder pieces at par, and sell them at the United States mint for $4.03 a piece, what sum in Federal money do you obtain for the cargo? Ans. $9,672.

8. A merchant bought some tea in Canton for 800 Chinese tales, and paid for it in American coin at par; what sum did he pay? Ans. $1,184.

9. A young American in Palermo, Sicily, bought some articles for 85 scudi 9 tari; what sum in Federal money will pay for the articles? Ans. $81.46.

10. If you sell 90 quintals of cod fish in Guadaloupe, one of the French West India islands, for 3,600 livres, how many Spanish dollars should you receive in payment? Ans. 400.

LESSON 221.

1. If you buy a quantity of saltpetre in Calcutta, for 2,300 rupees, what number of Portuguese half joes, at 4 per cent. above par, and what odd change in dollars and cents, will pay for it? Ans. 235 half joes and $2.19.

2. A merchant bought silks in Genoa to the amount of 43,264 lire; how many Spanish doubloons, at 4 per cent. above par, will pay for the silks? Ans. 416.

3. If a merchant in Kingston, Jamaica, owes you £645 6s. 8d. currency, what number of Spanish dollars must he send you to cancel the debt? Ans. 1,936.

4. A trader sent a cargo of lumber to Barbadoes, and sold it for £600 currency; the whole cost of the lumber delivered being $925.37½, what profit did he make upon it? Ans. $994.62½.

5. What sum in Federal money is equal to £964 8s. 4d. of the currency of the Leeward Islands? Ans. $2,143.15.

6. What sum in Federal money must be given for a quantity of salt valued at £1,000 currency in Turks Island, one of the Bahamas? Ans. $4,285.71.

7. What sum in Federal money do 108 florins 36 kreuzers of the money of Trieste make? Ans. $52.13.

8. Change 8,736.45 dollars, in the currency of the Danish West India islands, to Federal money. Ans. $5,591.33.

9. Change 1,988 marks banco 15 skillings of Hamburg,

to Federal money, estimating the mark banco at 34 cents? Ans. $676.24.

LESSON 222.

It is not customary for merchants engaged in foreign trade, to pay in specie for the goods they import, or to receive specie for the goods they send abroad. The business of exchange is done by an easier and less risky method. For instance, John Smith, of New York, owes Peter Brinkerhoff, of Amsterdam, Holland, 6,000 florins; now, if Smith has a creditor in Amsterdam who owes him enough to cancel the debt, he sends him an order to pay Brinkerhoff; if he has no such creditor, he goes to Charles Brown, who has, say 10,000 florins due him from Thomas Van Horne, of Amsterdam, and gets an order from Brown directing Van Horne to pay Brinkerhoff the 6,000 florins. There is evidently no risk in transmitting this order to Amsterdam.

Such an order is called a *bill of exchange*, and is also styled *bill or exchange on Amsterdam*, or simply *New York on Amsterdam*. Exchanges between distant cities, as Boston and New Orleans, are made in the same way, but the order is called a *draft*.

In the preceding instance, if Smith obtains the order or bill on Amsterdam for 6,000 florins by paying the amount of that sum to Brown, exchange on Amsterdam is said to be at par; if he pays more than the amount of 6,000 florins for the accommodation, exchange on Amsterdam is said to be above par, in which case exchange is unfavorable to this country; if he gets the bill for less than the amount of 6,000 florins, exchange on Amsterdam is said to be below par, in which case exchange is favorable to this country.

Is it customary for merchants engaged in foreign trade, to make their exchanges by means of specie? Give an example of an easier and less risky method. Is there any risk in transmitting this order to Amsterdam?

What is such an order called? What is it also styled? How are exchanges between distant cities, as Boston and New Orleans, made, and what is the order called?

In the preceding instance, when is exchange on Amsterdam at par? Above par? Below par?

In which case is the exchange favorable to this country? Unfavorable?

There is a continual variation in the rates of exchange between different countries, so that they may be favorable to a place one time, and presently unfavorable to it.

1. Exchange on Cadiz, Spain, being worth 68 cents a dollar of exchange, what must I give for a bill of exchange on Cadiz, for 1,967 dollars of exchange, 7 reals and 15 maravedies? Ans. $1,338.19.
2. A merchant in Havre, France, gave 10,400 francs for a bill of exchange on New York for $1,876.37; was exchange in that place on New York above or below par, and what per cent.? Ans. above par about 3 per cent.
3. What sum in Federal money will a bill of exchange on Liverpool for £4,000 cost, exchange on that place being 8 per cent. above par? Ans. $19,200.
4. How many pounds, shillings, &c., must a merchant in Jamaica give for a bill of exchange on Baltimore for $1,600, exchange on Baltimore being 3 per cent. above par? Ans. £549 6s. 8d.
5. What must I give for a bill of exchange on Calcutta for 1,835 rupees, 12 annas, 3 pice, at the rate of 50 cents a rupee? Ans. $917.89.
6. How many rubles, grieveners, &c., must a merchant of St. Petersburg, Russia, give for a bill of exchange on New York for $8,232, exchange on New York being 4 per cent. above par? Ans. 11,415 rubles 4 kopecs.
7. What must a merchant of Philadelphia give for a draft or bill of exchange on Norfolk for $2,000, at 2 per cent. above par? Ans. $2,040.

LESSON 223.

1. A merchant in Boston owes 8,000 paper rubles in St. Petersburg, Russia; being unable to purchase a bill of exchange on that place, he obtains one on London, and sending it to his correspondent there, directs him to purchase a bill of exchange for 8,000 rubles on St. Petersburg, and remit it to the creditor; what sum in Federal money will pay the debt, if 1 dollar will purchase 4s. 2d.

What is said of variation in the rates of exchange?

exchange on London, and £1 in London will purchase 12 rubles exchange on St. Petersburg ? Ans. $3,200.

Explanation. See Chain Rule.

2. A merchant in New York has a creditor in Hamburg, to whom he owes 9,000 marks banco ; this merchant, having correspondents in London, Havre in France, and Amsterdam in Holland, draws a bill of exchange on London, for a proper amount, and directs London to draw on Havre, Havre on Amsterdam, and Amsterdam on Hamburg ; what sum in Federal money will discharge the debt, if exchange of New York on London is 1 dollar for 4s. 2d., exchange of London on Havre, 1£ for 24 francs, exchange of Havre on Amsterdam 2 francs for 1 florin, and exchange of Amsterdam on Hamburg 3 florins for 4 marks banco ? Ans. $2,700.

3. What is the gain or loss by remitting this way, instead of sending directly to Hamburg, exchange of New York on Hamburg being 4 per cent. above par ?
Ans. the gain is $576.

4. A merchant in Palermo, Sicily, owes 7,500 rix dollars in Stockholm, Sweden ; he is unable to purchase a bill of exchange on Stockholm, but can purchase one on London or on Havre, in either of which places his correspondents can purchase one on Stockholm, and send it to his creditor. Now, had he better remit the money by way of London, exchange of Palermo on London being 5 scudi for £1, and exchange of London on Stockholm 4s. 2d. for 1 rix dollar, or by way of Havre, exchange of Palermo on Havre being 1 scudo for 5 francs, and exchange of Havre on Stockholm 5 francs 25 centimes for 1 rix dollar ?
Ans. he will save 62½ scudi in remitting by way of London.

5. Suppose a merchant in Boston owes $5,000 in Charleston, South Carolina, between which places exchange is at par, and remits a proper amount by means of drafts and correspondents from Boston to Philadelphia, from Philadelphia to Baltimore, and from Baltimore to Charleston ; how much will he save by this circuitous remittance, if exchange between Boston and Philadelphia is 2 per cent. in favor of Boston, between Philadelphia and Baltimore 1 per cent. in favor of Philadelphia, and between Baltimore and Charleston at par ? Ans. $149.

Note. When money is remitted in a circuitous way, we must pay commission to the various correspondents or agents who transact part of the business; there is usually more interest lost than in a direct remittance; and besides, there is considerable risk attending several operations transacted at a distance from us.

WEIGHTS AND MEASURES.

LESSON 224.

The following is an account of the most important weights and measures in the principal places where we trade. Their value is expressed in the weights and measures of the United States; the gallons in wine measure, and the pounds in avoirdupois weight.

Note. The scholar should read the whole, and commit to memory the answers to the questions.

GREAT BRITAIN.

The weights and measures of Great Britain are the same as those of the Uniteh States, described in Compound Numbers, except the imperial gallon. See lesson 95, Compound Numbers.

FRANCE.

The old weights and measures of France are as follows;

Foot,	12.7893	inches.
Aune, or ell,	46.85	inches.
Toise,	6	feet.
Mile, ½ of a lieue or league,	1.212	mile.
Boisseau,	.369	bushel.
Quart, $\frac{1}{144}$ of a muid,	1.967	quart.
Commercial pound, or livre,	1.08	pound.

New Weights and Measures.

The French, during their revolution, ascertained the length of a quarter of a meridian of the earth, by means of

---

What is said of remitting money in a circuitous way?
What are the weights and measures of Great Britain the same as?
What did the French ascertain during their revolution?

some very exact observations and measures. A quarter of a meridian is a line running north and south, beginning at the equator, and ending at the pole. $\frac{1}{10000000}$ part of a quarter of the meridian was called a *metre*, a French word for measure. All measures, of whatever kind, were derived from the metre, and arranged in decimal proportions, like our dollars, dimes, cents and mills. They placed before any measure the Latin terms

deci to express $\frac{1}{10}$ of it,
centi to express $\frac{1}{100}$ of it,
milli to express $\frac{1}{1000}$ of it,

And the Greek terms

deca to express	10 times the number,
hecto to express	100 times the number,
kilo to express	1,000 times the number,
myria to express	10,000 times the number.

Thus, in Long Measure,

			English feet
10 milli-metres	make	1 centi-metre, value	.032809167
10 centi-metres	make	1 deci-metre,	.32809167
10 deci-metres	make	1 metre,	3.2809167
10 metres	make	1 deca-metre,	32.809167
10 deca-metres	make	1 hecto-metre,	328.09167
10 hecto-metres	make	1 kilo-metre,	3,280.9167
10 kilo-metres	make	1 myria-metre,	32,809.167

Land or Square Measure.

The unit of square measure is the square of the decametre, called the *are*, value about 1.076.4414 square feet.

Dry and Liquid Measure.

The unit of these measures is the cube of the decimetre, called the *litre*, value about .0353171 cubic feet.

What is a quarter of a meridian? What was called a metre? How were all measures derived and arranged? What did they place before any measure?

Recite the table for long measure, and state the length of the metre to hundredths.

What is the unit of land or square measure? What is it called?

What is the unit of dry and liquid measures? What is it called?

Cubic or Solid Measure.

The unit of cubic measure is the cube of the metre, called the *stere,* value about 35.3171 cubic feet.

Weight.

The unit of weight is the weight of a cubic centi-metre of distilled water, called the *gramme,* value about 15.434 grains.

The new system of weights and measures is not yet employed universally, except among the learned.

Spain, Spanish America, and Spanish Colonies.

Foot,	11.128	inches.
Vara, or yard,	33.384	inches.
Common league,	4.291	miles.
Fanega, $\frac{1}{12}$ of a cahiz,	1.599	bushel.
Arroba of wine,	4.245	gallons.
Pound, $\frac{1}{25}$ of an arroba of weight,	1.0144	pound.

Portugal, Brazil, and Portuguese Colonies.

Foot,	12.944	inches.
Vara, or yard,	43.2	inches.
Mile,	1.25	mile.
Fanga, $\frac{1}{15}$ of a moyo,	1.535	bushel.
Almuide of Lisbon,	4.37	gallons.
Pound, $\frac{1}{32}$ of an arroba,	1.0119	pound.

Russia.

Foot,	13.75	inches.
Arsheen, or yard,	28.	inches.
Werst, or Russian mile,	3500.	feet.
Chetwert,	5.952	bushels.
Wedro,	3.246	gallons.
Pound, $\frac{1}{40}$ of a pood,	.9026	pound.

What is the unit of cubic measure? What is it called?

What is the unit of weight? What is it called?

What should you call $\frac{1}{10}$ of an are? 100 ares? 1,000 ares? $\frac{1}{100}$ of a litre? 10 litres? 10,000 litres? $\frac{1}{10}$ of a stere? 1,000 steres? 10,000 steres? $\frac{1}{1000}$ of a gramme? 10 grammes? 100 grammes?

Is the new system of weights and measures universally employed?

PRUSSIA.

Foot, or Rhineland foot,	12.356 inches.
Ell, or yard,	26.256 inches.
Mile,	4.684 miles.
Sheffel,	1.5594 bushel.
Eimer,	18.14 gallons.
Commercial pound,	1.0311 pound.

SWEDEN AND NORWAY.

Foot,	11.684 inches.
Ell, or yard,	23.368 inches.
Mile,	6.648 miles.
Kann,	.297 peck.
Kann,	.69 gallon.
Pound, victualie weight,	.9376 pound.

DENMARK.

Foot, or Rhineland foot,	12.356 inches.
Ell, or yard,	24.712 inches.
Mile,	4.684 miles.
Barrel, or toende,	3.9472 bushels.
Viertel, $\frac{1}{30}$ of a hogshead,	2.041 gallons.
Commercial pound, $\frac{1}{100}$ of a centner,	1.103 pound.

HAMBURG.

Foot,	11.289 inches.
Hamburg ell, or yard,	22.578 inches.
Mile,	4.684 miles.
Fass,	1.494 bushel.
Viertel, $\frac{1}{20}$ of an ahm,	1.912 gallon.
Commercial pound,	1.068 pound.

BELGIUM AND HOLLAND.

Amsterdam foot,	11.147 inches.
Amsterdam ell, or yard,	27.0797 inches.
Schepel,	.7892 bushel.
Stoop,	5.125 pints.
Aam,	41. gallons.
Commercial pound,	1.0893 pound.

GENOA.

Palmo,	9.725	inches.
Braccio, or yard,	22.692	inches.
Mina,	3.426	bushels.
Barile of wine, ½ of a mezzorala,	19.61	gallons.
Commercial pound,	.76875	pound.

LEGHORN.

Braccio, or yard, ¼ of a canna,	23.25	inches.
Sacco,	2.06	bushels.
Barile of wine,	12.	gallons.
Pound,	.75	pound.

ROME.

Foot,	11.72	inches.
Mile,	.925	mile.
Rubbio,	8.356	bushels.
Barile of wine,	15.409	gallons.
Pound,	.7477	pound.

NAPLES.

Palmo,	10.375	inches.
Canna,	83.	inches.
Tomolo, $\frac{1}{36}$ of a carro,	1.451	bushel.
Barile of wine,	11.	gallons.
Rottolo, $\frac{1}{100}$ of a gross cantaro,	1.965	pound.
Pound, $\frac{1}{150}$ of a neat cantaro,	.7067	pound.

SICILY.

Palmo,	9.5	inches.
Canna,	76.	inches.
Common salma,	7.85	bushels.
Gross salma,	9.77	bushels.
Barile, ⅛ of a liquid salma,	2.8825	gallons.
Gross rottolo, $\frac{1}{100}$ of a gross cantaro,	1.925	pound.
Neat rottolo, $\frac{1}{100}$ of a neat cantaro,	1.75	pound.
Pound,	.7	pound.

MALTA.

Palmo,	10.2375	inches.
Canna,	81.9	inches.
Salma of grain,	8.221	bushels.
Rottolo, $\frac{1}{100}$ of a cantaro,	1.75	pound.

VENICE.

Braccio for silks,	24.8	inches.
Braccio for woollens,	26.61	inches.
Staio, $\frac{1}{4}$ of a moggio,	2.27	bushels.
Bigoncia, $\frac{1}{4}$ of an anfora,	34.24	gallons.
Gross pound,	1.0518	pound.
Neat pound,	.644	pound.

TRIESTE.

Braccio for silks,	25.2	inches.
Braccio for woollens,	26.6	inches.
Staio,	2.344	bushels.
Orna, or eimer,	15.	gallons.
Commercial pound,	1.236	pound.

SMYRNA.

Pic,	27.	inches.
Killow,	1.456	bushel.
Oke,	2.833	pounds.

EAST INDIES.

MURITIUS, OR THE ISLE OF FRANCE.

The weights and measures are those of England, and the old ones of France.

CALCUTTA.

Haut, or cubit, [illegible] a guz or yard,	18.	inches.
Coss, or mile,	1.136	mile.
Factory maund,	$74\frac{2}{3}$	pounds.
Bazar maund,	$82\frac{2}{15}$	pounds.

BOMBAY.

Haut, or cubit,	18	inches.
Candy,	25	bushels.
Seer, $\frac{1}{40}$ of a maund, $\frac{1}{800}$ of a candy,	$11\frac{2}{10}$	ounce.
Bag of rice, 6 maunds,	168	pounds.

MADRAS.

Haut, or cubit,	18.	inches.
Parah, $\frac{1}{80}$ of a garce,	1.75	bushel.
Maund, $\frac{1}{20}$ of a candy,	25.	pounds.

BENCOOLEN, in Sumatra.

Bamboo, $\frac{1}{800}$ of a coyang,	1	gallon.
Bahar,	560	pounds.

BATAVIA, in Java.

Foot,	12.36	inches.
Ell,	27.	inches.
Kann,	1.58	quart.
Pecul,	135.625	pounds.

The other weights and measures are like those of Holland.

CANTON.

Covid, or cobre,	14.625	inches.
Catty,	1⅓	pound.
Pecul,	133⅓	pounds.

JAPAN.

Inc, or tattamy,	6.25	feet.
Catty,	1.3	pound.
Pecul,	130.	pounds.

WEST INDIES.

BRITISH ISLANDS.

Weights and measures of Great Britain.

FRENCH ISLANDS.

The old and new systems of French weights and measures are used, and also the English wine gallon.

ST. DOMINGO, OR HAYTI.

Old French weights and measures, also the English wine gallon.

DANISH ISLANDS. St. John, St. Thomas, and Santa Cruz.

Danish weights and measures, also the English foot and yard.

DUTCH SETTLEMENTS. St. Eustatia, St. Martin, Curazoa, Surinam, Berbice, Demarara, and Essequibo.

Weights and measures of Holland. In Curazoa, however, they also employ the Spanish vara of 33.375 inches, and a pound weighing 1.17 pound.

SPANISH ISLANDS. See Spain.

LESSON 225.

1. If 7,000 pounds of sugar, old French weight, imported from Guadaloupe, cost 6 cents a pound, delivered in New York, at what price must it be sold by the American pound to gain $184.80 on the whole amount ?
Ans. at 8 cts.

2. How many acres are there in a French myriare ?
Ans. 247 A. 18 sq. rods 193½ sq. ft.

3. A merchant sold 650 bushels of corn in Cumana, Colombia, at 10 reals a fanega ; what price in Federal money did he get a bushel ? Ans. $.78174, nearly.

4. If you purchase 500 almuides of wine in Lisbon, for 1,311 Spanish dollars, how much a gallon does it cost you ?
Ans. $.60.

5. How many yards are there in 575 Hamburg ells ?
Ans. 360.62, about.

6. If 105 barili of wine are imported into St. John, New Brunswick, from Leghorn, and sold at 4s. 6d. Canada currency, an imperial gallon, how much does the whole bring in dollars and cents ? Ans. $944.75.

7. How many gallons are there in 25 Venetian anfora of wine ? Ans. 3,424, about.

8. If you buy 9,000 pounds of hides in Port au Prince, St. Domingo, what number of American pounds will you obtain ? Ans. 9,720.

9. A merchant imported 5,000 sheffels of wheat from Prussia, in 1837 ; how many bushels were there in that quantity ? Ans. 7,797.

10. If I buy 100 bags of rice in Bombay, for 840 rupees, what price a pound do I give in Federal money ?
Ans. .0225.

BOOK-KEEPING.

FARMERS' AND MECHANICS' METHOD.

LESSON 226.

The Farmers' and Mechanics' method of Book-Keeping should be employed by farmers and mechanics, and by all persons in their household expenses, or any small business in which they may be engaged.

Accounts are kept by this method in a single book; on page 1 are written the owner's name, his place of residence, the date, and a description of the book. If the book is to contain his general accounts, he writes Account Book for a description; if the book is to contain accounts concerning his household expenses only, he writes Household Expenses; if the book is to contain accounts concerning his farm in Medford only, he writes Accounts concerning Farm in Medford, &c.

On pages 2 and 3, an account is opened with a certain person; the things for which he is debtor are placed on the left hand page, and the things for which he is creditor are placed on the right hand page. On pages 4 and 5 an account is opened with another person in the same manner, and so on, always taking two pages for each person's account. But if you are likely to have extensive dealings with any person, more than two pages should be left for his account.

Example of Book-keeping by the Farmers' and Mechanics' method.

[Page 1.]

JOSEPH SMITH.

Cambridge, April 1, 1834.

ACCOUNT BOOK.

By whom should the Farmers' and Mechanics' method of Book-Keeping be employed?

In what are accounts kept by this method? What is written on page 1? On pages 2 and 3? 4 and 5?

How many pages do we always take for each person's account? *What if you are likely to have extensive dealings with any person?*

Page 2.

Dr. Alexander Dolland.

1834.		$	c.
April 5.	To one day's Work of myself and oxen,	2	25
" 19.	" 9 cwt. of Hay, at 95 cts. cwt., ...	8	55
July 21.	" 6 bu. of Potatoes, at 45 cts. a bu., .	2	70
Nov. 19.	" 3 cords of Wood, at $5.50 a cord, .	16	50
1835.			
Jan. 6.	" 4 bu. of Corn, at $1.12½ a bu.,....	4	50

Page 4.

Dr. William Clark, of Watertown, Innkeeper.

1834.		$	c.
April 12.	To 16 bu. of Potatoes, at 40 cts. a bu.,	6	40
July 3.	" 6 lbs. of Butter, at 22 cts. a lb., ...	1	32
" "	" 9 doz. of Eggs, at 16⅔ cts. a doz., .	1	50
" "	" 8 lbs. of Cheese, at 10 cts. a lb., ..		80
Aug. 1.	" Horse to Boston, ...		40
		10	42

Page 6.

Dr. Jeremiah Dustwich.

1834.		$	c.
May 2.	To labor of my oxen half a day,......		60
June 14.	" 7 lbs. of Lard, at 16 cts. a lb., ...	1	12

Page 3.

Alexander Dolland. *Cr.*

1834.		$	c.
April 19.	By 18 lbs. of Flour, at 4 cts. a lb.,....		72
May 7.	" 1 Plough, ...	7	50
" "	" 2 Shovels, at $1.25 apiece,........	2	50
Aug. 11.	" 1 bl. Flour,......................	8	00
Sept. 20.	" 7 lbs. of Black Tea, at 35 cts. a lb.,	2	45
" "	" Cash, ...	4	00

Page 5.

William Clark, of Watertown, Innkeeper. *Cr.*

1834.		$	c.
April 19.	By Cash,...........................	5	00
July 30.	" 10 lbs. of Sugar, at 9 cts. a lb., obtained by an order from Wm. Clark on Joel Davis, ...		90
Sept. 1.	" Cash to balance, ...	4	52
		$10	42

Page 7.

Jeremiah Dustwich. *Cr.*

1834.		$	c.
April 18.	By shoeing my horse, ...	1	50

Great care should be taken to keep all the accounts in a uniform manner; to write a plain and exact description of every transaction on the first opportunity, before any part of it is forgotten. By taking this course, many losses, misunderstandings, disputes, and lawsuits may be avoided.

When an account is settled, the amounts of the Dr. and Cr. columns are added up, and two heavy black lines drawn beneath.

It is always well to state a person's residence, if he lives out of town, and his occupation, if any question can be raised as to his identity.

In the latter part of the book there should be an index.

The following is an index to the preceding book.

INDEX.

		Page
A.		
B.		
C.	Clark, William,	4
D.	Dolland, Alexander,	2
	Dustwich, Jeremiah,	6
E.		
F.		
&c.		

What should we take great care to do? What shall we avoid by this course?

What is done when an account is settled?

By the preceding account book, whose account appear to be settled? Whose accounts appear unsettled?

What if a person lives out of town, or if a question can be raised as to his identity?

What should there be in the latter part of the book?

LESSONS 227 AND 228.

How should Jonathan Belcher of Hartford prepare his book and keep his accounts, beginning January 1, 1830, if he buys of George Hodges, Jan. 6, 4 lbs. of tea at 33 cts. a lb., 6 lbs. of sugar at 10 cts. a lb., and an iron kettle for $2 ; sells Abraham Adams, Jan. 16, 2 tons of hay at $17 a ton ; sells David Bartlett, Feb. 3, 11 bushels of corn at $1.10 a bushel, and 14 lbs. of butter at 16⅔ cts. a lb. ; buys a cart of John Farrell, Feb. 27, for $47, a plough for $10, and a horse for $67 ; pays John Farrell, March 9, $17 in cash, sells him two cows, one for $19 and the other for $20 ; sells David Bartlett, March 9, 25 bushels of potatoes at 30 cts. a bushel, and a quarter of veal, weighing 11 lbs., at 8 cts. a lb. ; delivers to John Farrell, March 12, an order on David Bartlett for $20, and charges Farrell with the order, giving Bartlett credit for it ; receives of Abraham Adams, March 15, $25 in cash, and sells him the same day, 16 bushels of oats at 60 cts. a bushel, 5 bushels of corn at $1 a bushel, and 45 lbs. of butter at 20 cts. a lb. ; buys of George Hodges, April 2, 1 bl. of flour for $7.50, 14 lbs. of rice at 5 cts. a lb., 3 lbs. of tea at 30 cts. a lb., and 2 gals. of molasses at 35 cts. a gal. ; works, together with his oxen, 4 days, April 2, 3, 5, and 6, for Lemuel Snow, at $2.50 a day ; receives, April 9, of Lemuel Snow, $5 in cash, and a barrel of cider worth $2 ; employs Pliny Shaw, April 10, to shoe his horse for $1.33⅓, and to perform other blacksmith's work to the amount of 97 cts. ; works 2 days, April 12 and 13, together with his oxen, in moving Robert Sampson, of East Hartford, at $2.50 a day ; pays George Hodges, April 14, $10 in cash ; lets Lemuel Snow have his oxen 3 days, April 15, 16, and 17, at $1.50 a day ; buys of George Hodges, April 27, 7 lbs. of coffee at 16 cts. a lb., 12 lbs. of salt fish at 4 cts. a lb., and 2 doz. of crackers at 10 cts. a doz. ; receives of Robert Sampson, April 30, $5 in cash, and sells him the same day 45 lbs. of cheese at 10 cts. a lb., and 8 doz. of eggs at 14 cts. a doz.; May 1, employs Pliny Shaw to repair his plough for $1, and to new steel his axe for 75 cts. ; hires Davis Weed, May 6, 7, and 8, to paint his house, at $1.75 cts. a day, and buys the paint of him for $18 ; hires John Smith, carpenter, May 7 and 8, to work on his house, at $1.62½ a day; pays David Weed, May 12, $5 in cash, and John Smith, carpenter, $3.35; sells Abraham Adams, May 20, 4 pigs at $2 apiece, and receives of him that amount in cash; pays John Farrell, May 25, $10 in cash; hires John Smith, carpenter, to work on his shed 5 days, May 27, 28, 29, 31, and June 1, at $1.50 a day; sells Lemuel Snow, June 1, a calf for $5.75, and receives of Robert Sampson $4.75 in cash; settles with John Farrell, June 10, paying him his due in cash; buys of George Hodges, June 15, 11 lbs. of sugar at 10 cts. a pound., 5 gals. of molasses at 33⅓ cts. a gal., and 12 yds. of cotton cloth at 22 cts. a yd.; sells John Farrell, June 16, 7 bushels of corn at $1 a bushel, and 5 bushels of rye at $1.30 a bushel.

Note. To record the preceding transactions, double 2½ sheets of letter paper, thereby making a short but wide book of 20 pages.

BOOK-KEEPING BY SINGLE ENTRY.

LESSON 229.

Retail merchants and traders with a moderate business, keep their accounts in a way somewhat like that which we have described. The original charges, however, are made in what is called a *day book*, where they are written one after another, in the order in which the transactions occur. During the hours of leisure, these charges are copied into another book, arranged just like that in lesson 226, the account of each man being placed under his name. This book is called the *leger*. The act of copying from the day book into the leger is called *posting*. Merchants use the day book, because it is inconvenient, during the hours of business, to turn over the leger and search out each customer's name in order to record the trade they make with him.

How do retail merchants and traders with a moderate business keep their accounts? In what are the original charges made? How are they written? Into what are these charges copied, and how is the account of each man placed? What is this book called? What is the act of copying the day book into the leger called? Why do merchants use *the day book?*

In case of a dispute or suit, the day book, or the book in which the original charges are made, is the only one to be examined or produced. The charges in the leger ought to be exact copies of those in the day book, unless the business be extensive, when it is customary to abridge them in posting, to save labor and room.

As soon as an account is posted, we mark it, by writing in the first column of the day book, opposite the account, the page of the leger to which the account has been transferred; and in the first column of the leger, opposite the account the page of the day book where the account is to be found.

When the day book is filled, we can commence another and continue on, but when the leger is filled, we must add up the Dr. and Cr. columns of all the unsettled accounts, and transfer the amounts to the top of the Dr. and Cr. columns of the same persons in the new ledger. The first day book is called day book A, the second, day book B, and so on ; the first leger is called leger A, and the second, leger B, and so on.

This method of keeping accounts with a day book and leger, is called Book-Keeping by *single entry.*

Example of Book-Keeping by Single Entry.

DAY BOOK.

Page 1.

JOHN REED,

Portsmouth, June 3, 1830.

DAY BOOK.

What book is the only one to be examined or produced in case of a dispute or suit? Of what ought the charges in the leger to be exact copies? What, however, is customary when the business is extensive?

What is done as soon as an account is posted?

What is done when the day book is filled? When the leger is filled? What are the first, second, &c., day books called? Legers?

What is this method of keeping books with a day book and leger called?

Page 2.

Portsmouth, June , 1830.

			$	c.
	Reuben Farnham, ...	Dr.		
2	To 1 bl. Pork, ...	$25.00		
	" 6 gals. Molasses, at 35 cts. a gal.,	2.10		
	" 6 yds. Calico, at 28 cts. a yd., ...	1.68		
			28	78
	David Brown, Machinist, ...	Dr.		
4	To 1 piece Sheeting, 27¾ yds., at 18 cts. a yd.,		5	00
	June 5.			
	Oliver Dana, of Kittery, ...	Cr.		
7	By 12 lbs. Butter, at 16⅔ cts. a lb., ...	$2.00		
	" 4 doz. Eggs, at 12½ cts. a doz., ...	50		
			2	50
		Dr.		
6	To 14 lbs. Flour, at 6½ cts. a lb., ...			91

Page 3.

Portsmouth, June 7, 1830.

			$	c.
	Thomas Avery, ...	Dr.		
8	To 56 lbs. Loaf Sugar, at 18 cts. a lb., ...		10	08
	William Dyer, ...	Cr.		
11	By 2 hhds. Molasses, at $28 a hhd., ...	$56.00		
	" 5 chests Black Tea, at $16 a chest,	80.00		
			136	00
		Dr.		
10	To Cash, ...		45	00
	June 8.			
	Samuel Everett, ...	Dr.		
12	To 16 lbs. Nails, at 9 cts. a lb., ...		1	44
		Cr.		
13	By 5 bu. Rye, at $1.25 a bu., ...	$6.25		
	" ½ cord of Wood, at $5 a cord, ...	2.50		
			8	75

Page 4.

Portsmouth, June 9, 1830.

			$	c.
	David Brown, Machinist, ...	Cr.		
5	By Labor, mending safe, ...	$1.00		
	" 1 Lock, ...	1.00		
	" Cash, ...	3.00		
			5	00

Account settled.

			$	c.
	June 10.			
	Thomas Avery, ... Dr.			
8	To 20 quintals Cod-fish, at $3.80 a quintal, ...	$76.00		
	" 3 pieces Brown Sheeting, 81 yds., at 14 cts. a yd., ...	11.34		
			87	34
	Cr.			
9	By Cash, ...		50	00
	Mr. Avery agrees to pay the balance of his account in 90 days, in cash,			
	June 11.			
	Reuben Farnham, ... Cr.			
3	By Cash, ...		15	00

Page 5.

Portsmouth, June 12, 1830.

			$	c.
	Reuben Farnham, ...			
2	To 16 lbs. Cheese, at 10 cts. a lb., ...	$1.60		
	" 8 lbs. Black Tea, at 35 cts. a lb., ..	2.80		
			4	40
	Oliver Dana, of Kittery, ... Dr.			
6	To 5 yds. Broadcloth, at $4.50 a yd., ..	$22.50		
	" 1 Hat, ...	4.75		
			27	25
	June 14.			
	Oliver Dana, of Kittery, ... Cr.			
7	By 15 bu. Corn, at 95 cts. a bu., ...		14	25
	June 15.			
	Reuben Farnham, ... Dr.			
2	To 1 tub Butter, 42 lbs., at 20 cts. a lb.		8	40
	Butter and tub weigh 46 lbs., tub is estimated to weigh 4 lbs.			
	Mr. Farnham agrees to pay what is due on his account in 10 days, in Corn at $1 a bu.			

Lesson 230.

EXAMPLE OF A LEGER.

Page 1.

JOHN REED,

Portsmouth, June 3, 1830.

LEGER.

Page 2.

Dr.

Reuben Farnham.

	1830.			$	c.
2	June 4.	To 1 bl. Pork,	$25.00		
		“ 6 gals. Molasses, at 35 cts. a gal.,	2.10		
		“ 6 yds. Calico, at 28 cts. a yd.,	1.68		
				28	78
5	“ 12.	“ 16 lbs. Cheese, at 10 cts. a lb.,	1.60		
		“ 8 lbs. Black Tea, at 35 cts. a lb.,	2.80		
				4	40
5	“ 15.	“ 1 tub Butter, 42 lbs. at 20 cts. a lb.,		8	40

Page 4.

Dr.

David Brown, Machinist.

	1830.			$	c.
2	June 4.	To 1 piece Sheeting, 27¾ yds., at 18 cts. a yd.,		5	00

Page 6.

Dr.

Oliver Dana, of Kittery.

	1830.			$	c.
2	June 5.	To 14 lbs. Flour, at 6½ cts. a lb.,			91
5	“ 12.	“ 5 yds. Broadcloth, $4.50 a yd.	$22.50		
		“ 1 Hat,	4.75		
				27	25

Page 8.

Dr.

Thomas Avery.

	1830.			$	c.
3	June 7.	To 56 lbs. Loaf Sugar, at 18 cts. a lb.,		10	08
4	“ 10.	“ 20 quintals Cod-fish, at $3.80 a quintal,	$76.00		
		“ 3 pieces Brown Sheeting, 81 yds., at 14 cts. a yd.,	11.34		
				87	34

Page 10.

Dr.

William Dyer.

	1830.			$	c.
3	June 7.	To Cash,		45	00

Page 12.

Dr.

Samuel Everett.

	1830.			$	c.
3	June 8.	To 16 lbs. Nails, at 9 cts. a lb.,		1	44

Page 3.

REUBEN FARNHAM. *Cr.*

	1830.			$	c.
4	June 11.	By Cash,		15	00

Page 5.

DAVID BROWN, Machinist. *Cr.*

	1830.			$	c.
4	June 9.	By Labor, mending safe,	$1.00		
		" 1 Lock,	1.00		
		" Cash,	3.00		
				5	00

Page 7.

OLIVER DANA, of Kittery. *Cr.*

	1830.			$	c.
2	June 5.	By 12 lbs. Butter, at 16⅔ cts. a lb.,	$2.00		
		" 4 doz. Eggs, at 12½ cts a doz.,	50		
				2	50
5	14.	" 15 bu. of Corn, at 95 cts. a bu.,		14	25

Page 9.

THOMAS AVERY. *Cr.*

	1830.			$	c.
4	June 10.	By Cash,		50	00

Page 11.

WILLIAM DYER. *Cr.*

	1830.			$	c.
3	June 7.	By 2 hhds. Molasses, at $28 a hhd.,	$56.00		
		" 5 chests Black Tea, at $16 a chest,	80.00		
				136	00

Page 13.

SAMUEL EVERETT. *Cr.*

	1830.			$	c.
3	June 8.	By 5 bu. Rye, at $1.25 a bu.,	$6.25		
		" ½ cord Wood, at $5 a cord.,	2.50		
				8	75

There should be an index to the leger precisely like one to an account book.

The following is an index to the preceding Leger.

		Page.
A.	Avery, Thomas,	8
B.	Brown, David,	4
C.		
D.	Dana, Oliver,	6
	Dyer, William,	10
E.	Everett, Samuel,	12
F.	Farnham, Reuben,	2
G.		
&c.		

Those who deal much in money, have a small book in which the cash received and paid, is recorded by itself. This book is called a *cash book.*

Example of a Cash Book formed from the preceding day book.

CASH BOOK.

Cash.

Received.	$	*Paid.*	$
1830		1830	
June 9, Of David Brown,	3.00	June 7. To Wm. Dyer,	45.00
" 10, " Thomas Avery,	50.00		
" 11, " Reuben Farnham,	15.00		

Note. We have stated that many persons abridge the day book in posting. Several things bought or sold the same day, they call *sundries* in the leger; when only one article is bought, or sold the same day, it is named, and cash is always named; thus David Brown's account, in the preceding leger, would stand as follows;

Dr. David Brown. || David Brown. *Cr.*

	1830		$	c.		1830		$	c.
2	June 4.	To Sheeting,	5	00	4	June 9.	By Sundries, $2.00		
							" Cash, 3.00	5	00

This method of abridging saves some labor in posting; besides much more can be written in the leger than by the other course. It should be adopted when the business is extensive.

Whose account in the preceding leger is marked as being settled? What is said of an index? What do those who deal much in money have? What is it called? *How do some persons abridge the day book in posting? What does this method of abridging save? When should it be adopted?*

The advantages of copying every charge in full into the leger, are as follows; any man's account can be drawn off with ease, without turning over the day book, and copying out the items of the account from perhaps ten or twenty different places; a new clerk, or a person but little skilled in Book-Keeping, can understand a leger so kept at a glance; a man can be shown the state of his account, and be satisfied with regard to each item of it without opening the day book, and exposing other people's accounts on the same pages as his own, showing him, perhaps, that some person has obtained an article for a smaller price than himself; also, if the day book is carried out to be used in a settlement, or a suit, the accounts can be kept in the leger during the interval, and afterwards, be copied into the day book.

LESSONS 231, 232, and 233.

Prepare a day book, leger and cash book, for Moses Harris of Lowell, and record in them the following transactions.

January 2, 1833, Mr. Harris bought of Cyrus Neal of Boston, 2 pieces of broadcloth, containing 56 yds., at $3.50 a yd., 6 pieces of sheeting, containing 159 yds. at 15 cts. a yd., 3 pieces of shirting containing 82 yds., at 14 cts. a yd., and 50 yds. of calico, at 20 cts. a yard. ; bought of David Pierce of Boston, 27 hats, at $3.15 a piece, and 16 pr. of boots at $3 a pr. ; bought of Henry Gray of Boston, 3 hhds. of Molasses at $26 a hhd., 8 bls. of flour at $7.50 a bl., and 4 chests of green tea at $24 a chest, and paid Gray $100 in cash ; Jan. 3, sold Tobias Cary of Tewksbury, 18 lbs. of feathers at 52 cts. a lb., and 26¾ yds. of shirting at 18 cts. a yd., and received of him 15 bushels of corn at 98 cts. a bushel ; bought of Joshua Higgins 250 lbs. of cheese at 8 cts. a lb., paid Higgins $10 in cash, and agreed that he should take the rest of his pay in goods ; Jan. 5, sold Henry Gray of Boston, 800 lbs. of butter, at 18 cts. a lb., 100 lbs. of cheese at 10 cts. a lb., and 20 doz. of eggs, at 20 cts. a doz. ; bought of Silas Strong, 4 cords of oak wood at $5 a cord, and sold him 5 yards of broadcloth at $4.50 a yd. ; Jan. 7, sold Joshua Higgins 1 hat for $4 50, 4 gals. of molasses, at 37½ cts. a gallon, and 2 lbs. of coffee at 18 cts. a lb., and bought of him, 16 doz. of eggs at 14 cts. a doz. ; paid Cyrus Neal of Boston, $80 in cash ; bought of Zebediah Burney of Boston, 400 lbs. of iron at 7 cts. a lb., 2 casks of nails, containing 225 lbs. a

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What are the advantages of copying each charge in full, into the leger?

piece, at 7½ cts. a lb., 25 brooms, at 14 cts. a piece, and 12 axes, at $1.25 a piece ; bought of Henry Gray of Boston, 60 bushels of salt at 40 cents a bushel, 1 hhd. of brown sugar, weighing 7 cwt. 3 qrs. 13 lbs., at 7 cts. a lb., 6 bls. of flour at $7 a bl., and 45 lbs. of raisins at 10 cts. a lb. ; Jan. 8, sold John Williams, carpenter, 42 lbs. of nails at 9 cts. a lb., 1 broad-axe for $2, and 16 lbs. of cheese at 12 cts. a lb. ; sold Thomas Gurney 1 piece of shirting containing 27⅓ yds., at 19 cts. a yd., and a pair of boots for $4, and received of him, $5 in cash ; Jan. 9, sent Henry Gray of Boston, $75 by Edward Stickney ; sold Tobias Cary of Tewksbury, 8 lbs. of brown sugar at 10 cts. a lb., 4 lbs. of green tea at 38 cts. a lb., 2 gals. of molasses at 36 cts. a gal., and 6 lbs. of raisins at 16 cts. a lb. ; sold Thomas Gurney 1 iron bar, weighing 15 lbs., at 10 cts. a lb., 1 stone hammer, weighing 12 lbs., at 16⅔ cts. a lb. ; bought of Jonas Howe of Dracut, 8 bls. of apples at $2.50 a bl., 25 bushels of rye, at $1 a bushel, and 14 bushels of corn at 95 cts. a bushel ; Jan. 10, received of Tobias Cary of Tewksbury, $2 in cash ; bought of Jonas Howe of Drucut, 11 bushels of beans at $1.50 a bushel, and sold him 1 axe for $1.75, 2 pr. of woollen mittens, at 37½ cts. a pr. 6 pr. of woollen stockings at 41⅔ cts. a pr., and 1 pr. of pantaloons for $4.75 ; Jan. 11, paid David Pierce of Boston, $100 in cash ; bought of Cyrus Neal of Boston, 1 bale of sheeting, containing 400 yds., at 14 cts. a yd., and 2 pieces of blue cassimere, containing 21 yds. a piece, at $1.16⅔ a yd. ; Jan. 12, bought of Joshua Higgins 6 hogs, the total weight of which was 1,584 lbs., at 9 cts. a lb., and sold him 3 bls. of flour at $8 a bl., 112 lbs. of rice at 5 cts. a lb., 1 bl. of molasses, containing 30 gals., at 33⅓ cts. a gal., and 1 quintal of cod fish for $4 ; Jan. 14, sold Henry Gray of Boston, 9 bls. of pork at $22 a bl. ; bought of Tobias Cary of Tewksbury, ½ of a pig, weight 118 lbs. at 10 cts. a lb., and 16 bushels of potatoes at 38 cts. a bushel ; bought of Silas Strong, 25 lbs. of butter at 16 cts. a lb., and sold him 18 lbs. of salt fish at 4 cts. a lb. ; Jan. 15, bought of Silas Strong a horse for $75, and paid the cash for him ; sold Thomas Gurney 75 lbs. of cast steel at 12½ cts. a lb., and 100 lbs. of iron at 5 cts. a lb. ; Jan. 16, sold Jonas Howe of Dracut, 8½ yds. of cassimere at $1.25 a yd., and 2 yds. of shirting at 18 cts. a *yd. ; sold* Tobias Cary of Tewksbury, 2 bls. of flour at

$8.50 a bl., and settled ; Jan. 17, sold John Williams, carpenter, a firkin of butter at 23 cts. a lb., the butter and tub weigh 58 lbs., and the tub is estimated to weigh 5 lbs., the tub is to be returned and the weight rectified ; Jan. 18, bought of Cyrus Neal of Boston, 1 piece of broadcloth, containing 29 yds., at $4 a yd., and 2 bales of shirting, containing 400 yds., at 14 cts. a yd.

Note. To record the preceding transactions, double 1 sheet of letter paper, thereby making a short but wide day book of 8 pages. Also double 3 sheets of letter paper, thereby making a short but wide leger of 24 pages. In the leger on the last page but two, write the index, and on the last page but one, write the cash book. The charges should also be posted into another leger of the same size, by the abridged method.

OBSERVATIONS.

If a merchant takes an exact inventory or account of his unsold goods and of all the rest of his property, including the debts owed to him, and deducts from this sum the amount of the debts he owes, he will obtain the present value of his property. This, compared with the value of his property at a certain time before, shows what he has gained or lost since that time.

Book-Keeping by single entry is unfit for merchants engaged in extensive business ; they usually keep their accounts by a system called *double entry*. A description of this system requires a treatise ; a short account of it, like those appended to some arithmetics, is quite useless and insignificant. All persons should understand Book-Keeping by the methods which we have explained. Book-Keeping by double entry is necessary for a few only.

How can a merchant find the present value of his property, and his gain or loss since a certain time ?

What is Book-Keeping by single entry unfit for ? How do they usually keep their accounts ? What is said of Book-Keeping by double entry ? What is said of the necessity of understanding Book-Keeping by the different methods ?

BUSINESS FORMS.

Lesson 234.

BILLS.

No. 1.

Form of a Common Bill.

Mr. Joseph Snow
to James Thompson, ... Dr.

Feb. 7, 1837.	To 11 bu. of Potatoes, at 38 cts. a bu.,	$4.18
" 15, "	" 7 lbs. of Butter, at 20 cts. a lb., . . .	1.40
Mar. 4, "	" 2 days' Labor, at $1.25 a day, ...	2.50
		$8.08

April 3, 1837. Received payment.
James Thompson.

No. 2.

Form of a Bill, taken from a Merchant's Books.

See Reuben Farnham's Account in the preceding leger.

Mr. Reuben Farnham
to John Reed, ... Dr.

1830.				
June 4.	To 1 bl. Pork, ...	$25.00		
" "	" 6 gals Molasses, at 35 cts. a gal., . .	2.10		
" "	" 6 yds. Calico, at 28 cts. a yd., ...	1.68		
			28	78
" 12,	" 16 lbs. Cheese, at 10 cts. a lb., ...	1.60		
" "	" 8 lbs. Black Tea, at 35 cts. a lb., .	2.80		
			4	40
" 15,	" 1 tub Butter, 42 lbs., at 20 cts. a lb.,		8	40
			41	58
	Cr.			
" 11,	By Cash, ...		15	00
" 21,	" Cash to balance, ...		26	58

John Reed.

Portsmouth, June 21, 1830.

Observations. If Paul Dow, agent for James Thompson, receives the pay of bill No. 1, he acknowledges the receipt of the money thus;

Received payment,
For James Thompson,
Paul Dow.

If Samuel Short, clerk or agent of John Reed, settles bill No. 2, he signs it thus;

John Reed,
By Samuel Short.

Note. A bill should not be signed until payment is received, or a settlement effected.

1. If John Carter works 3 days, January 3, 5, and 6, 1835, for Ephraim Wheeler, at $1.16⅔ a day; sells Mr. Wheeler 18 cwt. of hay, Feb. 2, at 95 cts. a cwt.; sells him a pig for $11, Feb. 17, and mends a plough for him, March 3, for which he expects 75 cts., how should he make out his bill, and how should his son, Walter Carter, acknowledge the receipt of the money, March 7, 1835?

2. How should Thomas Cole, clerk of John Reed, make out Oliver Dana's bill from the preceding leger, and settle it, June 23, 1830?

3. How should John Reed make out the bill of William Dyer, from the preceding leger, before settlement, June 25, 1830? How after settlement?

4. David Wilkins, on examining his books, finds Simpson & Clark charged with 11 bushels of beans, at $1.75 a bushel, delivered August 15, 1831; 7 cwt. of hay, at $1 a cwt., delivered Sept. 1; 48 bushels of potatoes, at 30 cts. a bushel, delivered Oct. 29; 16 bushels of corn, at $1.12½ a bushel, delivered Jan. 2, 1832; and he finds them charged $5 for breaking his cart, Jan. 3, 1832; how should he make out his bill, and acknowledge the payment, Feb. 1, 1832?

Explanation. The last item may be put in thus; To breaking cart $5.

5. How should Joseph Smith make out the bill of Jeremiah Dustwich, from the preceding account book, and acknowledge the receipt of what is due, July 5, 1834?

What is said of signing a bill?

LESSON 235.

DUE BILLS.

Trenton, December 4, 1826.
Due John James, 5 dollars.
Henry Sargent.

Note. Due Bills are only memorandums given for small sums.

1. How should a due bill for $4.50, from Edward Jones of Auburn, to Calvin Stoop, be written, June 15, 1832?

2. How should a due bill for $8.37, from William Brown of Portland, to Sylvester Freeman, be written, March 4, 1836?

NOTES.

No. 1.

Note on Demand.

$50 Providence, January 3, 1838.

For value received, I promise to pay John Davis, or order, fifty dollars on demand, with interest.

Witness, Abram Horne. Benj. Homans.

No. 2.

Note Payable to Bearer.

$100 Louisville, May 3, 1836.

For value received, I promise to pay Simeon Fowle, or bearer, one hundred dollars on demand, with interest.

Eben. Earle.

No. 3.

Note by Two Persons.

$200 Portsmouth, July 15, 1838.

For value received, we jointly and severally promise to pay Caleb Callahan, or order, two hundred dollars on demand, with interest.

Samuel Slater.
Darius Bird.

What are due bills?

No. 4.

Note payable after a certain time.

$127.33 Brooklyn, May 23, 1837.

For value received, I promise to pay Cyrus Jackson, or order, one hundred and twenty-seven dollars and thirty-three cents, six months after date.

Daniel Williams.

No. 5.

Note at Bank.

$300 Boston, June 1, 1838.

Ninety days from date, I promise to pay Timothy Stearns, or order, at the Columbian Bank, three hundred dollars, for value received.

Hiram Hobart.

Note. Timothy Stearns indorses the note, that is, writes his name on the back of it, thereby binding himself to pay it if Hobart is unable, and Hobart then proceeding to the bank with the note obtains the money.

OBSERVATIONS ON NOTES.

Unless the note has the words *or order*, or the words *or bearer*, it is not negotiable, that is, it cannot be sold, exchanged, or traded with, because it is payable to only one person.

If a note is payable to a certain person *or order*, say to John Davis or order, see No. 1, then John Davis can sell this note to whom he pleases, provided he indorses it, thereby binding himself to pay the note if the purchaser is unable to collect it of the signer, Benj. Homans.

Several persons sometimes indorse a note, then each indorser is a security for the payment of it.

If a note is payable to a certain person *or bearer*, say to Simeon Fowle *or bearer*, see No. 2, then Simeon Fowle

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By the preceding note at bank, how is the money obtained?

What if a note has not the words *or order*, or the words *or bearer*, on it?

If a note is payable to a certain person, *or order*, say to John Davis or order, on what terms can John Davis sell it?

What if several persons indorse a note?

If a note is payable to a certain person, or bearer, say to Simeon Fowle, or bearer, to whom can Simeon Fowle sell it, and of whom can *the purchaser collect* it?

can sell it to whom he pleases, and the purchaser can collect it only of Eben. Earle, the signer.

A note payable at a certain time, is on demand on or after that time ; and if a note mention no time of payment, it is always on demand, whether the words *on demand* be expressed or not.

If no mention is made of interest, a note on demand draws interest as soon as payment is demanded ; and a note payable at a certain time, draws interest after that time.

It is unnecessary to mention the rate, unless a rate lower than that fixed by law be agreed upon.

If several persons sign a note promising jointly and severally to pay a certain sum, like No. 3, this sum can be collected of either of the persons.

If a note is payable within a certain time, in a particular article, say wheat, the holder is not obliged to receive the wheat after the expiration of the time, but can then demand money.

Lesson 236.

1. Write a note, dated Hartford, April 29, 1837, signed by Charles Lincoln, promising to pay Caleb Hunt, or order, $80 on demand, with interest.

2. Write a note, dated Taunton, June 3, 1838, signed by Ezekiel Goodhue, and witnessed by Jonathan Tyler, promising to pay Ezra Dean, or bearer, $251.42, in one year from date.

3. Write a note, dated Cincinnati, October 17, 1831, signed by John Sargent, E. Tucker, and Alfred Shaw, promising jointly and severally to pay Baird & Bowman, or order, $500 on demand, with interest.

4. Write a note, dated New York, December 18, 1829, indorsed by Horace Smith and Warren Davis, and signed by James Green, promising to pay Horace Smith, or

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When is a note payable at a certain time, on demand?

What if a note mention no time of payment?

If no mention is made of interest, when does a note on demand draw interest, and when does a note payable at a certain time draw interest?

Is it necessary to mention the rate of interest?

What if several persons sign a note?

If a note is payable within a certain time, in a particular article, say *wheat,* when can the holder demand money?

order, at the Merchants' Bank, $2,500 in sixty days from date.

5. Write a note, dated Pittsburg, August 7, 1835, signed by David Wilder, and witnessed by Alfred Davidson, promising to pay John Stone, or order, $1,000 eight months from date.

6. Write a note, dated Bangor, February 15, 1831, signed by Abraham Nichols, promising to pay Benjamin Murdock, or bearer, $200 on demand, with interest.

ORDERS.

No. 1.

Order for Money.

Lowell, June 5, 1830.

Mr. John Ryan,

Please pay Mr. Phineas Stone, or order, twenty-five dollars on my account.

Alexander Bernard.

No. 2.

Order for Goods.

Hallowell, April 15, 1833

Messrs. Swan, Porter & Co.,

Please pay Mr. Julian Long, or order, forty-five dollars in goods, and charge the same to me.

George Crosby.

Observations on Orders. When an order is made payable to a certain person *or order*, like the preceding, this person can sell it by indorsing it, thereby making himself security for the payment. An order made payable to a certain person *or bearer*, can be sold like an article of merchandise, without any indorsement, and the holder can collect it. Making an order is called *drawing* an order.

7. How should Philip Barber of Natchez, draw an order on Isaac Shaw, June 29, 1835, directing him to pay John Black, or order, $35 on account of said Barber ?

When an order is made payable to a certain person, or order, how can it *be sold ?* How can an order, made payable to a certain person, *or bearer, be sold* and collected ? What is making an order called ?

8. How should Goodwin & Clapp of New York, draw an order on Cyrus Holman, December 2, 1837, for $100, in favor of James Cook, or bearer.

9. How should Hall & Rogers of Providence, draw an order on Samuel Perry, February 14, 1825, directing him to pay George Rice, or order, $180 in goods ?

LESSON 237.

RECEIPTS.

No. 1.

Receipt for Money on a Note.

If Samuel Reed owes John Blake of Boston, a note for $100, and pays him $15, it is customary to write a receipt on the back of his note, thus ;

Boston, March 20, 1835. Received fifteen dollars.

If the note be not at hand, a receipt is made in this way ;

Boston, March 20, 1835.

Received of Samuel Reed fifteen dollars on his note for one hundred dollars, dated August 15, 1834.

John Blake.

No. 2.

Receipt for Money on Account.

Charleston, July 7, 1838.

Received of Solomon Jenkins ninety-four dollars on account.

Ezra Blair.

No. 3.

Receipt for Money obtained on an Order.

The receipt is indorsed on the order thus ;

Utica, January 15, 1835.

Received of Mr. Silas Derby, twenty dollars, the amount of the within order.

John Dory.

No. 4.

Receipt for Goods received on an Order.

The receipt is indorsed on the order thus;

Albany, October 1, 1831.

Received of Charles Quackenboss, eighty dollars in goods, the amount of the within order.

Thomas Kelder.

Note. When a part only of an order is paid, the sum can be indorsed on the order the same as the receipt on the back of a note.

No. 5.

General Receipts.

Louisville, November 18, 1830.

Received of Thomas Miller twenty-five dollars in full of all accounts. John Partridge.

Louisville, November 18, 1830.

Received of John Partridge twenty-five dollars in full of all accounts. Thomas Miller.

Observations on Receipts. Receipts in full of *all accounts*, like the preceding, cut off *accounts* only, but if they are given in full of *all demands*, they cut off all accounts, and all demands of every kind. Such receipts are usually exchanged between persons who have had business with each other when a final settlement is made.

No. 6.

Receipt for Money paid before it is due.

Boston, May 6, 1836.

Received of Joseph Hurd two hundred and fifty dollars, in full for the rent of my house from April 1, 1836, to April 1, 1837. Jacob Parker.

1. Write a receipt on the back of a note for $5, paid in Worcester, October 7, 1837.

2. Write a second receipt on the back of the same note for $13, paid in Worcester, January 15, 1838.

3. How should Joshua Day write a receipt for $81, paid him by Jonas Bridge in Troy, March 4, 1837, Day having

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What if a part only of an order is paid?
What is said of receipts in full of all accounts? In full of all demands?
When are such receipts usually exchanged?

a note against Bridge for $300, dated July 6, 1836, the note not being at hand ?

4. How should Henry Gay write a receipt for $75, paid him by Samuel Drake in Concord, July 16, 1838; Gay having a note against Drake for $150, which note was not at hand ?

5. How should Edward Clough of Boston, write a receipt for $50, paid him on account by David Clark, August 21, 1837, in Albany ?

6. Ezekiel Snow of Newbury, has an order on John Hermann for $80; if he is paid August 15, 1831, how and where should he write a receipt ?

7. Mark Doyle of Norwich has an order on Walter Lewis for $45 in goods; if he is paid February 5, 1831, how and where should he write a receipt ?

8. If Ezekiel Snow, in example 6, received only $30, August 15, 1831, how and where should he write a receipt ?

9. How do George Perley and Stephen Guild of Harrisburg write receipts in full of all accounts to be exchanged February 28, 1830 ?

10. Write receipts for $10 in full of all demands, to be exchanged between Daniel Wood and James Stockton, in St. Louis, April 13, 1832.

11. Write a receipt for $95, paid in Hudson, June 20, 1832, by Joseph Doane to Frederick Swift, for the yearly rent of a farm; the year ending June 1, 1834 ?

LESSON 238 AND 239.

DRAFTS AND BILLS OF EXCHANGE.

No. 1.

Form of a Draft.

750\frac{75}{100}$ New York, August 11, 1837.

Ninety days after date, pay to the order of Samuel Rogers, seven hundred and fifty dollars $\frac{75}{100}$, value received, and charge the same to the account of

John Dillon.

To Messrs. Davis & Tindal,
Merchants, Philadelphia.

No. 2.

Form of a Bill of Exchange.

Boston, July 17, 1838.

Exchange for 11,000 francs.

At sixty days sight of this my first of exchange, (second and third of the same tenor and date not paid,) pay to James Parke or order, eleven thousand francs, with or without further advice from me.

Charles Brown.

Mr. Pierre Davenant,
Merchant, Paris.

Note. In order to guard against accident or miscarriage, it is usual to draw three copies of a bill of exchange, and send them by different conveyances; either of the copies being paid, the others become void. The first copy is written like the preceding; in the second, say *my second of exchange* (*first and third of same tenor and date not paid,*) and in the third, say *my third of exchange,* (*first and second of same tenor and date not paid.*)

1. Stephen Lansing wishes for a draft from John Van Blarcom of Troy, for $625, on Skinner & Dustwich of Baltimore, payable thirty days after date; how should Van Blarcom write it, January 15, 1836?

2. How should Van Blarcom write it so as to be payable at sight?

Explanation. Say, at sight of this, pay to the order, &c.

3. How must Horace Harper of Charleston, draw a bill of exchange for £2,000 sterling on Joseph Pettingill of London, in favor of Norman Phillips, August 2, 1828, payable thirty days after sight?

Explanation. Recollect there are three to be drawn.

4. How should Sylvanus Brothwick of Baltimore, draw a bill of exchange for 10,000 reals on Diego Lana of Matanzas, June 20, 1831, to be paid to Manual Morena, or bearer, at sight.

OBSERVATIONS ON DRAFTS AND BILLS OF EXCHANGE.

A draft or a bill of exchange is nothing but a formal order. The distinction between orders, drafts and bills of

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What is done with regard to bills of exchange to guard against accident or miscarriage? How is the first copy written? Second? Third?
What is a draft, or bill of exchange?

exchange, is this; orders are used in the transaction of business in the neighborhood, drafts in the transaction of business in a distant town or city, and bills of exchange in the transaction of business in a foreign nation. Moreover, the presentation, payment, &c., of drafts, and especially bills of exchange, are accompanied with much more ceremony than the presentation, payment, &c., of orders.

Usages with regard to drafts and bills of exchange are very much alike. Drafts are sometimes called *inland bills of exchange*, and other bills, *foreign bills of exchange*. Bills of exchange are often bought, sold, and transferred from one man to another several times before they become due. Banks are in the habit of buying bills.

The person who makes or draws a bill, is called the *drawer* or *seller* of the bill.

The person on whom the bill is drawn, is called the *drawee*; he is also called the *acceptor*, if he accepts the bill; that is, agrees to pay it.

The person to whom the bill is ordered to be paid, is called the *payee*.

The person who has the bill at any time in his possession, is called the *holder*.

The buyer of the bill who remits it to the drawee, is called the *remitter*.

When a bill is payable to a certain person *or bearer*, it can be sold, and transferred like an article of merchandise, or like a bank note.

When a bill is payable to a certain person *or order*, this person, who is the payee, can sell and transfer it to whom he pleases, by indorsing it, thereby making himself security for the payment.

An indorsement may be *blank* or *special*. A blank in-

What is the distinction between orders, drafts, and bills of exchange? What further difference is there?

What is said of the usages with regard to drafts and bills of exchange? What are drafts sometimes called? Other bills? What is said of buying, selling and transferring bills of exchange?

What is the person who makes or draws a bill called?

What is the person on whom the bill is drawn called?

What is the person to whom the bill is ordered to be paid called?

Who is the holder? The remitter?

When a bill is payable to a certain person, or bearer, how can it be sold and transferred?

When a bill is payable to a certain person, or order, how can it be sold and transferred?

What may an indorsement be?

dorsement consists of the indorser's name only, and then the bill can be transferred from one person to another by simple delivery. A special indorsement is an order from the holder directing the money to be paid to a particular person, called the *indorsee*, who must also indorse the bill if he sells it. A blank indorsement may always be filled by the holder with any person's name so as to make it special.

Any person may indorse a bill, and every indorser, as well as the acceptor is security for the payment of the bill. An indorsement may be made at any time after the bill is drawn, even after the day of payment named in it.

In reckoning to find when a bill payable in a certain number of days after date becomes due, we omit the day on which it is dated. When the time is expressed in months, *calendar* months are always understood.

In the United States and Great Britain three days of grace are allowed after the bill is due ; but a bill payable at sight, must be paid the day it is presented.

A bill should be presented for acceptance or payment during the usual hours of business ; and a bill should be presented for payment on the third day of grace.

To accept a bill, that is, to agree to pay it, the drawee usually writes his name at the bottom, or across the body of the bill, with the word *accepted*.

When the drawee refuses to accept or to make payment, the holder of the bill should give regular and immediate notice to all parties to whom he intends to resort for payment ; for if he does not, they will not be liable to pay.

With regard to inland bills or drafts, when the drawer and drawee live in the same state, the holder has merely to send an ordinary notice by letter or otherwise, to the

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What is said of a blank indorsement? What is said of a special indorsement? What may always be done to a blank indorsement?

May any person indorse a bill? What is the consequence? When may an indorsement be made?

In reckoning to find when a bill payable in a certain number of days after date, becomes due, what do we omit? What if the time is expressed in months?

What is allowed in the United States and Great Britain? When must a bill, payable at sight, be paid?

During what hours should a bill be presented for acceptance or payment? On what day should a bill be presented for payment?

How does the drawee usually accept a bill?

What if the drawee refuses to accept, or to make payment?

drawer and indorsers, that acceptance or payment, as the case may be, is refused, and that he does not intend to give credit to the drawee. But with regard to foreign bills, or bills from one state to another, a *protest* is absolutely necessary, and is made as follows. A Public Notary appears with the bill, and demands either acceptance or payment, as the case may be; and on being refused, draws up an instrument called a protest, expressing that acceptance or payment, as the case may be, has been demanded and refused, and that the holder of the bill intends to recover any damages he may sustain in consequence. Such an instrument is admitted in all countries as a proof of the fact of refusal.

The protest should be sent as soon as possible to the drawer and indorsers, and if it be for non-payment, the bill must be sent with the protest.

If the drawee absconds or cannot be found, protest is to be made, and notice given in the same manner as if acceptance had been refused.

When acceptance is refused, and the bill is returned by protest, an action may be commenced immediately against the drawer, though the regular time of payment be not arrived. The debt in such case is considered as contracted the moment the bill is drawn.

The preceding observations contain some of the principal rules which govern merchants in the use of bills; the other laws and customs on this subject are very numerous, and must be learned by practice.

How does he give notice with regard to inland bills or drafts, when the drawer and drawee live in the same state? What is necessary with regard to foreign bills, or bills from one state to another? How is a protest made? What is this instrument admitted as a proof of?

What should be done with the protest and bill?

What if the drawee absconds, or cannot be found?

When may an action be commenced against the drawer? When is the debt in such a case considered as contracted?

NOTES.

NOTE 1.

SIGNS.

Certain algebraic signs are employed in many arithmetics, for the purpose, it is said, of abridging common language. They are never used in the transaction of business, but as they are occasionally seen in books, the following description of them should be studied after Division. The scholar should then be occasionally required to explain by these signs how he performs an example.

$=$ Two horizontal lines are the sign of equality. They show that numbers with this sign between them are equal; thus, 5 added to $3 = 8$. They may be read, *is equal to;* thus, 5 added to $3 = 8$, may be read, 5 added to 3 *is equal to* 8.

$+$ An upright cross is the sign of addition. It shows that two numbers with this sign between them are added together; thus, $5 + 3 = 8$. It may be read, *added to;* thus, $5 + 3 = 8$, may be read, 5 *added to* 3 *is equal to* 8.

This sign is sometimes put at the right of decimals, to show that the entire number is not written, the figures neglected being of small value; thus, the value of $\frac{1}{3}$ being in decimals .3333, &c., we can write 333+, instead of writing .333 about.

$-$ A horizontal line is the sign of subtraction. It shows that the number after it is to be subtracted from the number before it; thus, $5 - 3 = 2$. It may be read, *less by;* thus, $5 - 3 = 2$ may be read, 5 *less by* 3 *is equal to* 2.

This sign is sometimes put at the right of decimals, to show that a number a little too large is written, the last figure being increased by 1, because the first one neglected is 5 or more; thus, the value of $\frac{2}{3}$ in decimals being .6666, &c., we can write .667—, instead of writing .667 nearly.

$\times$ An inclined cross is the sign of multiplication; it shows that two numbers with this sign between them, are multiplied together; thus, $3 \times 5 = 15$. It may be read, *multiplied by;* thus, $3 \times 5 = 15$, may be read, 3 *multiplied by* 5 *is equal to* 15.

$\div$ A horizontal line with a dot above it and another below it, is the

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For what purpose are certain algebraic signs employed in many arithmetics? Are they ever used in the transaction of business?

What are two horizontal lines? What do they show? Give an example. How may they be read? Give an example.

What is the sign of addition? What does it show? Give an example. How may it be read? Give an example.

Where is this sign sometimes put, and for what purpose? Give an example.

What is the sign of subtraction? What does it show? Give an example. How may it be read? Give an example.

Where is this sign sometimes put, and for what purpose? Give an example.

What is the sign of multiplication? What does it show? Give an example? How may it be read? Give an example.

What is the sign of division?

sign of division; it shows that the number before it is divided by the number after it; thus, 15 ÷ by 3 = 5. It may be read, *divided by*; thus, 15 ÷ 3 = 5 may be read, 15 *divided by* 3 *is equal to* 5.

EXERCISES.

How do you read 4 + 9 = 13? 16 — 7 = 9? 11 × 5 = 55? 24 ÷ 8 = 3? 18 + 25 = 43? 20 — 5 = 15? 15 × 15 = 225? 320 ÷ 16 = 20? 6 + 2 = 15 — 7? 39 — 4 = 7 × 5? 4 + 5 = 99 ÷ 11?

How do you write by means of the preceding signs, 6 and 7 are 13, that is, 6 *added to* 7 *is equal to* 13? 11 from 23 leaves 12? 25 times 33 are 825? 7 is in 385, 55 times? 20 and 45 are 65? 73 from 137 leaves 64? 12 times 12 are 144? 16 is in 480, 30 times? 9 added to 5 is equal to 20 less by 6? 27 added to 5 is equal to 4 multiplied by 8? 11 less by 2 is equal to 108 divided by 12?

Many numbers are sometimes connected together by these signs; thus, 5 + 8 — 4 × 8 ÷ 6 = 9 × 4 ÷ 12 × 4. These numbers may be read, 5 added to 8, the result less by 4, this result multiplied by 8, and this result divided by 6, is equal to 9 multiplied by 4, the result divided by 12, and this result multiplied by 4.

Though these signs are of no advantage in arithmetic they are very useful in algebra, indeed algebra is a language of signs, but then they are usually employed for different purposes from what they are in arithmetic.

Note 2.

Repeating Decimals, *often called Circulating Decimals.*

Decimals consisting of figures continually repeated, are called *repeating decimals* or *repetends;* as .333, &c., .2121, &c.

When only one figure is repeated the fraction is called a *single repetend*, as .333, &c.

When two or more figures are repeated the fraction is called a *compound repetend*; as .2121, &c.

When other decimals come before the repeating decimals the fraction is called a *mixed repetend;* thus, .1666, &c. is a mixed single repetend, and .2304304, &c. is a mixed compound repetend.

Mathematicians sometimes write but one figure in a single repetend, and place a dot over it; thus, .333, &c. is written $.\dot{3}$. In a compound repetend they write the repeating figures but once, and place a dot over the first and last; thus, .2121, &c. is written $.\dot{2}\dot{1}$, and .2304304, is written $.\dot{2}30\dot{4}$.

What does it show? Give an example. How may it be read? Give an example.

Are these signs of any advantage in arithmetic? What is said of them in algebra?

What are called repeating decimals or repetends? Give some examples.

What is called a single repetend? Give an example.

What is called a compound repetend? Give an example.

What is called a mixed repetend? Give examples of a mixed single repetend, and of a mixed compound repetend.

How do mathematicians sometimes write a single repetend? Give an example. How do they write a compound repetend? Give some examples.

For all practical purposes, repeating decimals can be employed just like any other decimals. But as some of them must always be omitted in calculations, the results will not be *perfectly* correct.

Let us see if we can change them to common fractions with perfect accuracy.

Changing $\frac{1}{9}$ to decimals we get .1111, &c. or $.\dot{1}$.

So .1111, &c. or $.\dot{1}$ is $\frac{1}{9}$ in common fractions. Also $.\dot{2}$, or 2 times $\dot{1}$ is $\frac{2}{9}$ in common fractions, $.\dot{3}$, or 3 times $.\dot{1}$ is $\frac{3}{9}$, $.\dot{4}$ is $\frac{4}{9}$, $.\dot{5}$ is $\frac{5}{9}$, &c.

Changing $\frac{1}{99}$ to decimals we get .0101, &c. or $\dot{0}\dot{1}$.

So .0101, &c. or $.\dot{0}\dot{1}$ is $\frac{1}{99}$ in common fractions. Also $.\dot{0}\dot{2}$, or 2 times $.\dot{0}\dot{1}$ is $\frac{2}{99}$ in common fractions, $.\dot{0}\dot{3}$, or 3 times $.\dot{0}\dot{1}$ is $\frac{3}{99}$, $.\dot{0}\dot{4}$ is $\frac{4}{99}$, $.\dot{1}\dot{5}$ is $\frac{15}{99}$, $.\dot{7}\dot{6}$ is $\frac{76}{99}$, &c.

Changing $\frac{1}{999}$ to decimals we get .001001, &c. or $.\dot{0}0\dot{1}$.

So .001001, &c. or $.\dot{0}0\dot{1}$ is $\frac{1}{999}$ in common fractions. Also $.\dot{0}0\dot{2}$, or 2 times $.\dot{0}0\dot{1}$ is $\frac{2}{999}$ in common fractions, $.\dot{0}0\dot{3}$, or 3 times $.\dot{0}0\dot{1}$ is $\frac{3}{999}$, $.\dot{0}0\dot{4}$ is $\frac{4}{999}$, $.\dot{0}2\dot{6}$ is $\frac{26}{999}$, $.\dot{2}3\dot{5}$ is $\frac{235}{999}$, &c.

Therefore, to change a repeating decimal to a common fraction,

Make the repetend a numerator, and write as many 9s *for a denominator as there are repeating figures.*

EXAMPLES.

What common fraction is equal to $.\dot{3}$, $.\dot{5}$, $.\dot{8}$, $.\dot{3}\dot{6}$, $.\dot{4}\dot{9}$, $.\dot{1}3\dot{7}$, $.\dot{4}0\dot{3}$, $.\dot{7}$, $.\dot{1}\dot{2}$, $.\dot{4}$, $.\dot{7}3\dot{7}$, $.\dot{8}53\dot{4}$, $.\dot{8}\dot{2}$, $.\dot{6}$, $.\dot{1}2\dot{3}$.

We can now change a mixed repetend to a common fraction with ease.

For example, to change $.1\dot{6}$ to a common fraction, we proceed thus;

Value of .1.	Value of $.0\dot{6}$.
$\frac{1}{10}$	$\frac{6}{90}$

Changing to a common denominator, and adding, we get, $\frac{150}{900}$ or $\frac{1}{6}$ Ans.

Explanation. $.1\dot{6}$ is composed of .1 and $.0\dot{6}$; the value of .1 in common fractions is $\frac{1}{10}$, and the value of $.0\dot{6}$ is evidently $\frac{6}{90}$, since $.0\dot{6}$ is one tenth of $.\dot{6}$ whose value is $\frac{6}{9}$. Adding $\frac{1}{10}$ to $\frac{6}{90}$, we get $\frac{150}{900}$ or $\frac{1}{6}$.

After changing repeating decimals to common fractions, we can add, subtract, multiply, and divide in them without any necessary errors.

EXAMPLES.

1. Add .5, $.8\dot{3}$, .625, and $.\dot{6}$. Ans. 2.625.
2. Subtract $.\dot{7}3\dot{5}$ from $2.\dot{3}$. Ans. $1\frac{597}{999}$ or $1.\dot{5}9\dot{7}$.
3. Multiply $.\dot{0}\dot{3}$ by $.\dot{2}$. Ans. $\frac{6}{891}$.
4. Divide $.\dot{4}$ by $.\dot{1}2\dot{3}$. Ans. $\frac{3996}{1107}$.

For all practical purposes, how can repeating decimals be employed? Will the results be perfectly correct? Why?

Explain how we can change repeating decimals to common fractions with perfect accuracy.

How do we change a repeating decimal to a common fraction?

Explain how we proceed to change a mixed repetend, say $1\dot{6}$, to a common *fraction?*

After changing repeating decimals to common fractions what can we *do?*

Note 3.

Duodecimals.

Arithmetics usually contain a rule called Duodecimals. It shows how to multiply feet and inches together without changing them to the same denomination. The rule is difficult and useless, but as many persons employ this method in measuring wood, the following description of it should be studied.

1. What is the surface of the end of a load of wood 3 ft. 10 in. wide, and 4 ft. 8 in. high?

OPERATION.

ft.	in.	
4	8	
3	10	
3	10	8
14	0	
17	10	8

Ans. 17 sq. ft. $\frac{10}{12}$ and $\frac{8}{144}$.

Explanation. Multiplying 8 in., or $\frac{8}{12}$ of a foot, by 10 in., or $\frac{10}{12}$, we get $\frac{80}{144}$, or $\frac{6}{12}$ and $\frac{8}{144}$; we put down the $\frac{8}{144}$, and multiplying 4 ft. by $\frac{10}{12}$, get $\frac{40}{12}$, to which we add the $\frac{6}{12}$, and obtain $\frac{46}{12}$, or 3 ft. $\frac{10}{12}$. Now multiplying the $\frac{8}{12}$ by 3 ft., we get $\frac{24}{12}$, or 2 ft. $\frac{0}{12}$; we put down the 0 under the 12ths, and add the 2 ft. to the 12 ft. found by multiplying the 4 ft. by 3 ft.

2. What quantity of wood is there in the preceding load, it being 7 ft. 1 in. long?

OPERATION.

sq. ft.				
17	10	8 surface of end.		
	7	1		
1	5	10	8	
125	2	8		
126	8	6	8	

Ans. 126 cubic ft. $\frac{8}{12}$, $\frac{6}{144}$ and $\frac{8}{1728}$.

Explanation. We proceed as in the preceding example, carrying the 12s in each column one place to the left.

In Duodecimals, therefore,

Proceed as in decimal numbers, but carry 1 *for every* 12 *instead of* 1 *for every* 10.

Most arithmetics require the scholar to multiply in Duodecimals first by the feet, and then to put the product by the inches beneath the first product one place to the right. For instance;

What does the rule called Duodecimals show? Is the rule easy and useful?

Explain how example 1 is performed.

Explain how example 2 is performed.

How do we proceed in Duodecimals?

How do most arithmetics require the scholar to multiply in Duodecimals?

Example 1 is performed thus,

4 8
3 10

14 0
3 10 8

17 10 8

and example 2 thus;

17 10 8
7 1

125 2 8
1 5 10 8

126 8 6 8

This course evidently gives the same result as before. It produces no benefit however, but renders the process more obscure.

3. What is the value of the preceding load of wood at $½ a ft.?

OPERATION.

12)8 1728ths.

12)6.666, &c. 144ths.

12)8.555, &c. 12ths.

126.7129 cubic ft.

2
16)126.7129(7.92 ft. of wood.
112

147
144

31
32

$3.96 value. Ans.

The long and difficult operations in examples 1, 2, and 3, are easily performed by decimals, according to lesson 173.

4.7 height.
3.8 width.

376
141

17.86 sq. ft., surface of end.
7.1 length.

1786
12502

126.806 cubic ft. in load.

2
16)126.806(7.92 ft. of wood.
112

148
144

40
32

$3.96 value. Ans.

In many cases it is necessary to take the dimensions within less than an inch, and then the operation by duodecimals becomes so tedious that decimals must be employed.

EXAMPLE.

4. What is the value of a polished marble block 5.45 ft. or 5 ft. 5⅜ in. long, 3.64 ft., or 3 ft. 7⅝ in. wide, and 2.99 ft. or 2 ft. 11⅞ in. high, at $4 a cubic foot? Ans. $237.26.

Perform the following examples by duodecimals.

5. Example 6, lesson 107. Ans. 26.58, about.
6. Example 7, lesson 107. Ans. 405.
7. Example 8, lesson 107. Ans. 7 C. 13.5 cubic ft.
8. Example 9, lesson 107. Ans. .8125.
9. Example 7, lesson 173. Ans. $16.05.

What is said of this course?

By what are the long and difficult operations in examples 1, 2 and 3 easily performed?

In many cases how is it necessary to take the dimensions? *What is the consequence?*

NOTE 4.

PROPORTION.

The examples in the Rule of Three which are performed so easily by common sense, are often solved in a difficult manner by means of algebraic proportion. As many persons employ this method, the following description of it should be studied, and used in solving some examples in the Rule of Three.

1. If 2 pears cost 6 cents, what will 4 pears cost? Ans. 12 cents.

In this example, we see that 2 pears bear the same proportion to 4 pears, as 6 cents to 12 cents. To show that such is the case, mathematicians write the four numbers thus;

$$\overset{\text{pears.}}{2} : \overset{\text{pears.}}{4} : : \overset{\text{cents.}}{6} : \overset{\text{cents.}}{12}$$

and read them, so written, thus; 2 pears are to 4 pears as 6 cents are to 12 cents.

Any four numbers in which the first bears the same proportion to the second as the third does to the fourth, when written like the preceding, form an algebraic proportion, commonly called a proportion.

We see that the ratio of 2 to 4 is the same as the ratio of 6 to 12, each being $\frac{1}{2}$. So a proportion may be considered as formed of two equal ratios.

The four numbers constituting a proportion, as 2, 4, 6, and 12 in the preceding one, are called the *terms*.

The two outside terms, as 2 and 12, are called the *extremes*.

The two middle terms, as 4 and 6, are called the *means*.

In the preceding proportion, the ratio of the first term to the second, is $\frac{2}{4}$, and the ratio of the third term to the fourth, is $\frac{6}{12}$. Changing these fractions to a commom denominator we get,

$$\frac{24}{48} \text{ and } \frac{24}{48}.$$

In changing to a common denominator, the numerator of the first fraction is formed by multiplying 2 and 12, or the two extremes of the proportion; and the numerator of the second fraction is formed by multiplying 4 and 6, or the two means. Now the two ratios forming a proportion always being equal, the two fractions, when changed to a common denominator must always have equal numerators.

What is said of performing the examples in the Rule of Three by common sense, and by algebraic proportion?

In example 1, what proportion do 2 pears bear to 4 pears? How do mathematicians show that such is the case? How do they read them so written?

What form an algebraic proportion?

What is the ratio of 2 to 4 the same as? What then may a proportion be considered as formed of?

What are called the terms? The extremes? The means?

In the preceding proportion what is the ratio of the first term to the second? Of the third term to the fourth?

In changing these fractions to a common denominator, how is the *numerator* of the first fraction formed? How is the numerator of the *second fraction* formed? Why must the two fractions when changed *to a common* denominator, always have equal numerators?

Therefore in a proportion,

The product of the two extremes is equal to the product of the two means. So if we divide the product of the two means by one extreme, the quotient will be the other.

In order then, to perform example 1 by algebraic proportion, we may write the first three terms thus;

2 : 4 : : 6 :

and to get the fourth term, or the answer, we divide the product of 4 and 6 by 2, thus;

```
   6
   4
   —
 2)24
   —
   12 fourth term, or Ans.
```

2. If 3 men can build a stone wall in 4 days, how long will it take 6 men to build it? Ans. 2 days.

In example 2 we see that

men.		men.		days.		days.
6	:	3	: :	4	:	2

Now, in example 1, the more pears the more they cost, but in example 2, the more men the fewer days will they be in doing the work. The principle in the first proportion is inverted in the second; so the first is called a *direct proportion*, and the second an *inverse proportion*.

From what precedes we derive the following rule for performing an example in the Rule of Three by algebraic proportion.

Make that number which is of the same kind as the answer sought the third term of the proportion. Then if the answer must be larger than the third term, make the larger of the other two numbers the second term, and the smaller the first term, but if the answer must be smaller than the third term, make the smaller of the other two numbers the second term, and the larger the first term. Afterwards, to get the answer, divide the product of the second and third terms by the first.

In many examples, however, it is by the rule difficult or impossible to state the question, that is, to write down the proper numbers for the third, second, and first terms. Such examples cannot be performed *by the rule*, but must be solved by common sense.

The following are examples of this kind.

1. Four boys, A, B, C, and D, have a number of cents; A has 1 cent, B 2 cents, and C 3 cents; now D's money bears the same proportion to C's as B's does to A's; how many cents has D?

In a proportion what is the product of the two extremes equal to? How can we get one of the extremes?

How then may we perform example 1 by algebraic proportion?

In example 2 what do we see?

What is the difference between the proportion in example 1, and the proportion in example 2? What is the first called? The second?

From what precedes, what rule do we derive for performing an example in the Rule of Three by algebraic proportion?

In many examples, however, what is it difficult or impossible to do *by the rule*? What is said of such examples?

2. A bin containing 60 bu. of wheat diminished by drying to 58.5 bu.; how much then should a bin containing 400 bu. diminish from the same cause?

3. There are four numbers, the second bearing the same proportion to the first, as the fourth does to the third; the first number is 22, the second 2, and the fourth 11; what is the third?

Examples in the Rule of Three Compound, may be performed by using two proportions. For instance, in example 1, Rule of Three Compound, we can first get the time it will take the interest of $675 to amount to $18, and then the time it will take the interest to amount to $162. In many examples, however, it is difficult or impossible to state the questions, and they cannot then be performed this way.

How may examples in the Rule of Three Compound be performed? Give an instance. In many examples, however, what is it difficult or impossible to do? What is said of such examples?

THE END

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MAN [illegible] PHYSICAL STRUCTURE AND ADAPTATION. By [illegible]

LIFE OF WILLIAM COWPER. By [illegible]

PRACTICAL [illegible] OF FARMING AND GARDENING. [illegible] [illegible] and Vegetables [illegible] By [illegible]

THE COMPLETE FARMER AND RURAL ECONOMIST. [illegible] [illegible] Rural Economy. By [illegible] FESSENDEN. [illegible] [illegible] & CO.